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Cyclotron Resonance of the Wannier-Landau Transition System Based on the Ensemble Projection Technique

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Abstract We study the linear-nonlinear quantum transport theory of Wannier-Landau transition system in the confinement of electrons by a square well confinement potential. We use the projected Liouville equation method with the ensemble density projection technique. We select the dynamic value under a linearly oscillatory external field. We derive the dynamic value formula and the memory factor functions in three electron phonon coupling systems and electron impurity coupling systems of two transition types, the intra-band transitions and inter-band transitions. We obtain results that can be applied directly to numerical analyses. For simple example of application, we analyze the absorption power and line-widths of ZnO, through the numerical calculation of the theoretical result in the Landau system.

Keywords Cyclotron resonance, Wannier-Landau transition, line-widths, absorption power, memory factor function, electron-phonon interaction

Introduction

It is well known the Landau splitting system is that the electronic state in a crystal, in the presence of static magnetic field applied along the z-axis, is quantized in the x–y plane.[1,2] In addition, an electromagnetic wave is applied to the system, the electrons absorb the proper photon energies to make magneto-optical transitions, among which cyclotron resonance (CR) is the most typical one.[3] So far quite many theories have been reported for CR electron interacting with phonon (or impurity) in semiconductor. The Wannier-Landau transition system (WLTs) is that the electronic state in a crystal, in the presence of static magnetic field applied along the z-axis and static electric field applied along the x-axis, is quantized in the x–y plane with the x directional electric field dependency. Since the WLTs has the magnetic field and also electric field dependency, research in quantum transition theory of WLTs may be important in investigating the microscopic scattering phenomena of many condensed matter systems. The main purpose of this work is to investigate the linear-nonlinear effect of the induced dynamic value due to polarizability. Early studies were mainly based on linear schemes and nonlinear response studies were performed only in limited.

Among those low-dimensional systems, semiconductor optoelectronic devices, such as laser diodes, optical waveguides, and photo detectors, have important applications in optical communication systems. In order to understand the physics and the operational characteristics of these optoelectronic devices, we need to develop a quantum statistical

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method. Thus the quantum statistical study of low-dimensional electron systems is of great importance in condensed material systems. Theoretical studies of CR in quasi-two dimensional (Q2D) quantum well structures have been in active progress over the last several years.[4-7] There are several methods to obtain line-widths in response function.[8-14] The theoretical studies performed so far on high electric field transport are usually based on the following methods: the Green's function approach, Feynman's path integral approach, the Wigner representation approach, and the ensemble density projection technique (EDPT).[15-20] Despite the fact that all these methodologies are quite reasonable, the nonlinear behavior has been investigated in limited schemes. Using the EDPT, we will derive the integrodifferential equation of a dynamic variable, and obtain a response function in Fourier-Laplace transformed space. In first step, the result contains a nonlinear response term which is not determined. We will expand the linear memory factor function (MFF) in a series form. Through the continuous approximation of quantum state integration, we obtain final result of the dynamic value and MFF in integration form.

The interest in zinc oxide (ZnO) is fueled and fanned by its prospects in optoelectronics applications owing to its direct wide band gap ($E_g \approx 3.3$ eV). For simple example of application, we analyze the absorption power and line-widths of ZnO, through the numerical calculation of the theoretical result in the Landau splitting system. Finally, we shall be devoted to some discussion and draw conclusions.

The Expansion of the Memory Factor Function

If we consider the case that the induced the dynamic value direction k (x, y, z, etc) is same to the external field direction l (x, y, z, etc), as $k = l$, $\Lambda_{kl} \equiv Tr\{J_k L_l' \rho_s\} = 0 \quad (If, \quad k = l)$, and $J_k(\omega_l) = 0$. Thus we must consider the case that the induced dynamic value direction k (x, y, z, etc) is differ to the external field direction l (x, y, z, etc), as $k \neq l$. Since H_s contains the diagonal Hamiltonian (the main particles Hamiltonian and the background particles Hamiltonian), H_d , and the non-diagonal Hamiltonian (the interaction Hamiltonian between the main particles and background particles), H_v, we can divide the corresponding Liouville operator as, $L_s \equiv L_d + L_v$. So, we can divide the MFF into four terms as

$$\Xi_{kl}(\omega_l) \equiv \frac{i}{\hbar\ \Lambda_{kl}} Tr\{J_k L_s G_k(\omega_l) Q_k L_s L_l' \rho_s\} \equiv I + II + III + IV\,, \tag{1}$$

where

$$I \equiv (\frac{i}{\hbar\Lambda_{kl}})\langle Tr^e\{J_k L_d G_k(\omega_l) Q_k L_d L_l' \rho_s\}\rangle_B\,, \tag{2}$$

$$II \equiv (\frac{i}{\hbar\Lambda_{kl}})\langle Tr^e\{J_k L_v G_k(\omega_l) Q_k L_d L_l' \rho_s\}\rangle_B\,, \tag{3}$$

$$III \equiv (\frac{i}{\hbar\Lambda_{kl}})\langle Tr^e\{J_k L_d G_k(\omega_l) Q_k L_v L_l' \rho_s\}\rangle_B\,, \tag{4}$$

$$IV \equiv (\frac{i}{\hbar\Lambda_{kl}})\langle Tr^e\{J_k L_v G_k(\omega_l) Q_k L_v L_l' \rho_s\}\rangle_B\,. \tag{5}$$

Using the properties of projection operator, we obtain the relation for an arbitrary operator X ,

$$\begin{aligned}&(\frac{i}{\hbar\Lambda_{kl}})\langle Tr^e\{J_k X Q_k L_d L_l' \rho_s\}\rangle_B \\ &= (\frac{i}{\hbar\Lambda_{kl}})\langle Tr^e\{J_k X L_d L_l' \rho_s\}\rangle_B \\ &- (\frac{i}{\hbar(\Lambda_{kl})^2})\langle Tr^e\{J_k X L_l' \rho_s\}\rangle_B \langle Tr^e\{J_k L_d L_l' \rho_s\}\rangle_B\end{aligned} \tag{6}$$

Then, the first term I and the second term II are zero and

$$\begin{aligned}&(\frac{i}{\hbar\Lambda_{kl}})\langle Tr^e\{J_k L_d Q_k X L_l' \rho_s\}\rangle_B \\ &= (\frac{i}{\hbar\Lambda_{kl}})\langle Tr^e\{J_k L_d X L_l' \rho_s\}\rangle_B \\ &- (\frac{i}{\hbar(\Lambda_{kl})^2})\langle Tr^e\{J_k L_d L_l' \rho_s\}\rangle_B \langle Tr^e\{J_k X L_l' \rho_s\}\rangle_B\end{aligned} \tag{7}$$

then, the third term III is zero. Here $\langle\cdots\rangle_B$ is the ensemble average of background particle states (for example, phonon or impurity state). Since the average of odd background terms are zero, we use the the useful relation as bellows,

$$\langle Tr^e\{Y Q_k L_v L' \rho_s\}\rangle_B = \langle Tr^e\{Y L_v L' \rho_s\}\rangle_B\,. \tag{8}$$

Using the conventional series expansion method, we expand the propagator $G_k(\omega_l)$ as

$$G_k(\omega_l) \equiv \frac{1}{\hbar\omega_l - Q_k L_s} = G_d \sum_n [PL_d G_d]^n \sum_r \{Q_k L_v G_d \sum_n [PL_d G_d]^n\}^r$$
$$= G_d + G_d PL_d G_d + \ldots + G_d Q_k L_v G_d + \ldots \approx G_d, \quad (9)$$

where the diagonal propagator is

$$G_d(\omega_l) = \frac{1}{\hbar\omega_l - L_d}, \quad (10)$$

Thus then, with weak interacting system approximation in pair interacting system, the MFF can be reduced to simple form, as bellows,

$$\Xi_{kl}(\omega_l) \approx (\frac{i}{\hbar\Lambda_{kl}})\langle Tr^e\{J_k L_v G_d(\omega_l) L_v L_l' \rho_s\}\rangle_B. \quad (11)$$

We will apply the MFF in the electron phonon interaction system and the electron-impurity interaction system.

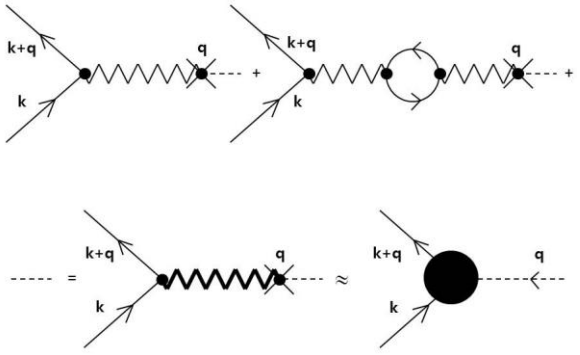

Figure 1. Ring diagram contribution to the scattering system.

The Formula of the Absorption in Wannier -Landau Level System

We suppose that an oscillatory electric field $E(t) = E_0 \exp(i\omega t)$ is applied along the z-axis, which gives the absorption power delivered to the system as $P(\omega) \propto (E_0^2/2)\,\mathrm{Re}\{\sigma(\omega)\}$, where "Re" denotes the real component and $\sigma(\omega)$ is the optical conductivity tensor which is the coefficient of the dynamic value formula. The absorption power can be represented in the optical quantum transition LPs. We will derive the line-widths (or absorption power formula) in interacting system. We consider the electron-phonon interacting system; then, we have the Hamiltonian of the system as

$$H_s = H_0 + H_p + V = \sum_\beta \langle\beta|h_0|\beta\rangle a_\beta^+ a_\beta + \sum_q \hbar\omega_q b_q^+ b_q + \sum_q \sum_{\alpha,\mu} C_{\alpha,\mu}(q) a_q^+ a_\mu \left(b_q + b_{-q}^+\right). \quad (12)$$

Here H_e is the electron Hamiltonian, h_0 is a single-electron Hamiltonian, H_P is the phonon Hamiltonian and V is the electron-phonon (or impurity) interaction Hamiltonian. The $b_1\,(b_2^+)$ are the annihilation operator (creation operator) of boson particle, and $\vec{q}$ is phonon (or impurity) wave vector. The interaction Hamiltonian of electron-phonon (or impurity) interacting system is $V \equiv \sum_q \sum_{\alpha,\mu} C_{\alpha,\mu}(q) a_\alpha^+ a_\mu \left(b_q + b_{-q}^+\right)$ where the coupling matrix element of electron-phonon interaction $C_{\alpha,\mu}(q)$ is $C_{\alpha,\mu}(q) \equiv V_q < \alpha | \exp(i\vec{q}\cdot\vec{r}) | \mu >$, $\vec{r}$ is the position vector of electron and V_q is coupling coefficient of the materials.

We also select the system as the induced dynamic value system under the lineally oscillatory external field since this case during application of the response theory to real system because of that this case contains the more general results than other cases. The dynamic value operator caused by the linearly polarized external field of frequency ω consider as,

$$J_x = \sum_\beta j_{\beta,\beta+1} a_\beta^+ a_{\beta+1} + \sum_\beta j_{\beta+1,\beta} a_{\beta+1}^+ a_\beta \equiv J_x^R + J_x^L, \quad (13)$$

where $j_{\beta,\alpha} \equiv \tilde{g}_{(sys)} \sqrt{N_\beta}\,\delta_{\beta,\alpha+1}$, with $\tilde{g}_{(sys)} \equiv (-ie\hbar/m_e^*)\sqrt{1/l_0^2}$, $l_o^2 \equiv \hbar/eB$. $\tilde{g}_{(sys)}$ can be changed for other systems and external fields., $J_x^R\ (J_x^L)$, represent the right linearly polarization current(RLPC) (left linearly polarization current(LLPC)), which is induced from right to left (from left to right) through x-axis, and the Liouville operator of the external field consider as

$$L_x' X \equiv (-i/\omega)\,[J_x, X] \quad \left(L_x^{L'} X \equiv (-i/\omega)\,[J_x^L, X] \ , \ L_x^{R'} X \equiv (-i/\omega)\,[J_x^R, X]\right) \quad (14)$$

for an arbitrary operator X.

The MMF of right(or left) linear polarization term, $\Xi_{kl}^{(R)}(\omega_l)$ (or $\Xi_{kl}^{(L)}(\omega_l)$) is complex as, $\Xi_{kl}^{(R)}(\omega_l)=\mathrm{i}\Delta_{total}^{(R)}+\gamma_{total}^{(R)}(\omega_c)$ (or $\Xi_{kl}^{(L)}(\omega_l)=\mathrm{i}\Delta_{total}^{(L)}+\gamma_{total}^{(L)}(\omega_c)$). In most case, the imaginary part of the MMF, $\Delta_{total}^{(R)}\equiv \mathrm{Im}\,\Xi_{kl}^{(R)}(\omega_l)$ (or $\Delta_{total}^{(L)}\equiv \mathrm{Im}\,\Xi_{kl}^{(L)}(\omega_l)$) give the line-shift of response type formula and the real part of the MMF, $\gamma_{total}^{(R)}(\omega_c)\equiv \mathrm{Re}\,\Xi_{kl}^{(R)}(\omega_l)$ (or $\gamma_{total}^{(L)}(\omega_c)\equiv \mathrm{Re}\,\Xi_{kl}^{(L)}(\omega_l)$) give the half-width of response type formula. In most case, the imaginary part of the MMF, $\Delta_{total}^{(R)}\equiv \mathrm{Im}\,\Xi_{kl}^{(R)}(\omega_l)$ (or $\Delta_{total}^{(L)}\equiv \mathrm{Im}\,\Xi_{kl}^{(L)}(\omega_l)$) are neglected in real system as a small vale term

Using this interaction parameter, we obtain the final result for the line-shift, In continuous approximation, the interaction matrix part in the square quantum well potential (SQWP) system are as below,

$\tilde{C}_{N_\beta,N_\alpha}^{\mp N_\alpha,N_\beta}=\left|V_q\right|^2 K_{\beta,\alpha}^{\alpha,\beta}(t)\,|\,F_{\alpha,\beta}(q)\,|^2\,\delta_{k_{\beta_z},k_{\alpha_z}\pm q_z}$,

$\tilde{C}_{N_\alpha,N_\beta}^{\mp N_\beta,N_\alpha}=\left|V_q\right|^2 K_{\alpha,\beta}^{\beta,\alpha}(t)\,|\,F_{\beta,\alpha}(q)\,|^2\,\delta_{k_{\beta_z},k_{\alpha_z}\mp q_z}$, and, if $N_\alpha\langle N_\beta$ and $N_\kappa\langle N_\lambda$, the K-matrix is

where the Legendre function is $L_n^m(t)=(n!)^{-1}\exp(t)t^{-m}(d^n/dt^n)[t^{n+m}\exp(-t)]$ and $t\equiv(l_c^2(q_{\perp n}^{\mp\kappa\lambda})^2)/2$. The matrix element of the confinement potential is

$$\tilde{K}_{\beta\alpha}^{(el)}\equiv\frac{2m^*\omega}{\hbar}-\frac{2m^*\omega_c}{\hbar}(N_\beta-N_\alpha) \tag{15}$$

Then, through the continuous approximation, we obtain final result of the dynamic value formula as,

$$\tilde{K}_{\beta\alpha}^{(cfp)}\equiv\frac{2m^*\tilde{w}_0}{\hbar}\left[(N_\beta+1)^2-(N_\alpha+1)^2\right] \tag{16}$$

In the ohm formula as, $J(\omega)=\sigma(\omega)E(\omega)$, here $\sigma(\omega)$ is the conductivity , and the absorption power, $P(\omega)\propto(1/2)\sigma(\omega)E(\omega)^2$ is,

$$V(q_z,q_{\perp n}^{\mp\kappa\lambda})^2=\frac{\bar{K}^2\hbar\upsilon_s e^2}{2\chi\varepsilon_0 V}\frac{1}{\sqrt{q_z^2+(q_{\perp n}^{\mp\kappa\lambda})^2}} \tag{17}$$

and the line-width, $\gamma_{total}^{(R)}(\omega_o)$ and $\gamma_{total}^{(L)}(\omega_o)$, as compact representation $\gamma_{total}^{(RL)}(\omega_c)$

$$\gamma_{total}^{(RL)}(\omega_c)\equiv\frac{\sum_{\mp}\sum_{N_\alpha=0}\sum_{N_\beta=0}\tilde{A}_{sys}\int_{-\infty}^{\infty}dk_{y\alpha}\int_{-\infty}^{\infty}dk_{z\alpha}\int_{-\infty}^{\infty}dq_z Y_{\alpha,\beta}^{(RL)\mp}}{\sum_{N_\alpha=0}^{\infty}\int_{-\infty}^{\infty}dk_{y\alpha}\int_{-\infty}^{\infty}dk_{z\alpha}(N_\alpha+1)(f_{\alpha c}-f_{\alpha v})} \tag{18}$$

where $\tilde{A}_{sys}\equiv\left(\frac{\Omega}{4\pi\hbar^2\upsilon_s}\right)\left(\frac{\pi}{L_z}(2+\delta(n_\alpha,n_\beta))\right)$ an the values of $\tilde{A}_{sys}$ can be change in other systems. The integrand-factor is

$$Y_{\alpha,\beta}^{(RL)\mp}\equiv\begin{Bmatrix}Y_{\alpha,\beta}^{(R)\mp}\\ Y_{\alpha,\beta}^{(L)\mp}\end{Bmatrix},$$

$$\sqrt{(N_\alpha+2)(N_\alpha+1)}J_{\beta,\alpha+1}^{A\mp}\binom{\alpha+2,\beta}{\beta,\alpha}(f_{\alpha+1}-f_{\alpha+2})-(N_\alpha+1)J_{\beta,\alpha+1}^{A\mp}\binom{\alpha,\beta}{\beta,\alpha}(f_\alpha-f_{\alpha+1})$$

$$-(N_\alpha+1)\left\{J_{\alpha,\beta}^{D\mp}\binom{\beta,\alpha-1}{\alpha+1,\beta}+J_{\alpha,\beta}^{D\mp}\binom{\beta,\alpha+1}{\alpha+1,\beta}\right\}(f_\alpha-f_{\alpha+1}) \tag{19}$$

and

$$Y_{\alpha,\beta}^{(L)\mp}\equiv$$

$$\left\{\sqrt{N_\alpha(N_\alpha+1)}J_{\beta,\alpha}^{A\mp}\binom{\alpha-1,\beta}{\beta,\alpha+1}-(N_\alpha+1)J_{\beta,\alpha1}^{A\mp}\binom{\alpha+1,\beta}{\beta,\alpha+1}\right\}(f_\alpha-f_{\alpha-1})$$

$$-(N_\alpha+2)J_{\alpha+1,\beta}^{D\mp}\binom{\beta,\alpha+2}{\alpha,\beta}(f_{\alpha+1}-f_{\alpha+2})$$

$$+(N_\alpha+1)J_{\alpha+1,\beta}^{D\mp}\binom{\beta+1,\alpha+1}{\alpha,\beta}(f_\alpha-f_{\alpha+1}), \tag{20}$$

where, the terms of electron-phonon interacting parts are

$$q_{\perp1}^{\mp\beta\alpha}\equiv+\sqrt{\left(\frac{\pm}{\hbar v_s}\right)^2\left\{\tilde{E}_{\beta\alpha}^{(el)}-\tilde{E}_{\beta\alpha}^{(cfp)}-\tilde{E}_1^{(k\varepsilon)}\right\}^2-\tilde{q}_{sys}^2}\equiv-q_{\perp2}^{\mp\beta\alpha},$$

$$q_{\perp3}^{\mp\beta\alpha}\equiv+\sqrt{\left(\frac{\pm}{\hbar v_s}\right)^2\left\{\tilde{E}_{\alpha\beta}^{(el)}-\tilde{E}_{\alpha\beta}^{(cfp)}+\tilde{E}_3^{(k\varepsilon)}\right\}^2-\tilde{q}_{sys}^2}\equiv-q_{\perp4}^{\mp\beta\alpha} \tag{21}$$

with the interacting matrix, the $\tilde{S}$ -matrix, being

$$\tilde{S}_{\beta,\alpha}^{\mp}(\kappa,\eta;q_z,q_{\perp n}^{\mp\beta\alpha})\equiv V(q_z,q_{\perp n}^{\mp\beta\alpha})^2\frac{\sqrt{(q_{\perp n}^{\mp\beta\alpha})^2+q_z^2}}{|\hbar v_s q_{\perp n}|}K_{\mu,\nu}^{\kappa,\eta}\left(\frac{r_0^2}{2}(q_{\perp n}^{\mp\beta\alpha})^2\right). \tag{22}$$

We use $m^*=0.19m_0$ and $\bar{m}=0.7m_0$ which are the effective masses of constants of ZnO are $\rho=4.82\times10^3\ \mathrm{kg/m^3}$, $v_{sl}=4.28\times10^3\ \mathrm{m/s}$ is the longitudinal sound velocity, $v_{st}=1.81\times10^3\ \mathrm{m/s}$ is the transverse sound velocity,

$k = 8.58\times10^{-4}$ eV/K, $\xi = 235$ K and $|K|^2 = 2.98\times10^{-2}$. The speed of sound v_s in Eq. (20) shall be replaced by the average value v_s of v_{sl} and v_{st}, as $\overline{v}_s = (v_{sl}+v_{st})/2$ and the energy gap $\varepsilon_g(T)$ replaced by $\overline{\varepsilon}_g = 2.56$ eV in approximation by noting that the variation against the temperature is very small. More accurate value of $\varepsilon_g(T)$ can be obtained by Eq. (22) if the characteristic constants k and ξ are available. We choose $\varepsilon_0 = 8.854\times10^{-12}$ c²/Nm².[21]

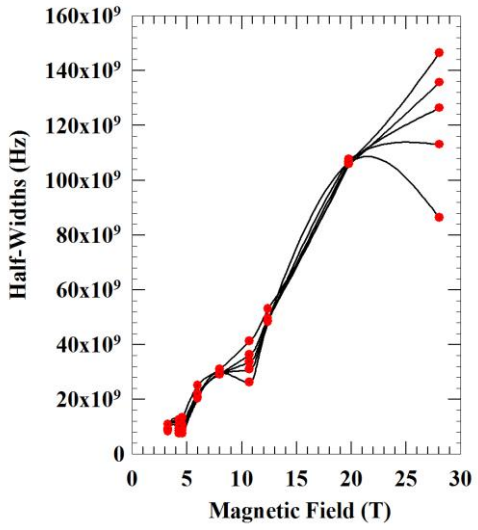

Figure 2. The magnetic field dependence of line-widths, $\gamma(B)$ of ZnO for T=50, 70, 90, 120, 210 K

Results

We analyze absorption power and line-widths of ZnO, under a right circularly polarized external field. It is well known that the piezoelectric potential scattering is a dominant scattering process in ZnO. The piezoelectric potential is $V(q)^2 = (\overline{K}^2\hbar\upsilon_s e^2/2\chi\varepsilon_0 V)(1/q)$.

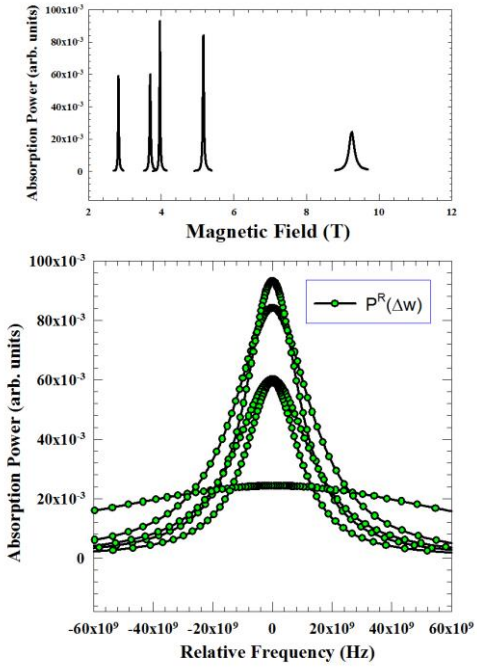

Figure 3. The relatively frequency $\Delta(\omega)$ dependence of the absorption power $P^{(R)}(\Delta\omega)$ of ZnO for $\lambda = 220$, 394, 513, 550, 720 at T=50 K.

In Fig. 2, we plotted the magnetic field dependence of the line-widths, $\gamma(B)$ of ZnO, at T= 50, 70, 90, 120 and 210 K. The results indicate that increase as the magnetic field. This result implies that the scattering effect of phonons enlarges with the increasing temperatures and the increasing magnetic field in ZnO. Even though we cannot separate experimentally the scattering effects of the phonon emission and absorption transition, the analysis of the relation between the total scattering effect and the scattering effect of two processes represents the thermal characteristic of the scattering effect of the system.[22, 23] Also, comparisons of the magnetic field dependence of line-widths, $\gamma(B)_{total}$, $\gamma(B)_{em}$ and $\gamma(B)_{ab}$ of ZnO, at T=50K is shown. The line-widths, $\gamma(B)_{total}$, $\gamma(B)_{em}$ and $\gamma(B)_{ab}$ increase as the magnetic field while $\gamma(B)_{ab}$ decreases as magnetic field increase at the region 18<B<26 Tesla. The contributions of two processes can be appeared

differently in various cases in various systems.

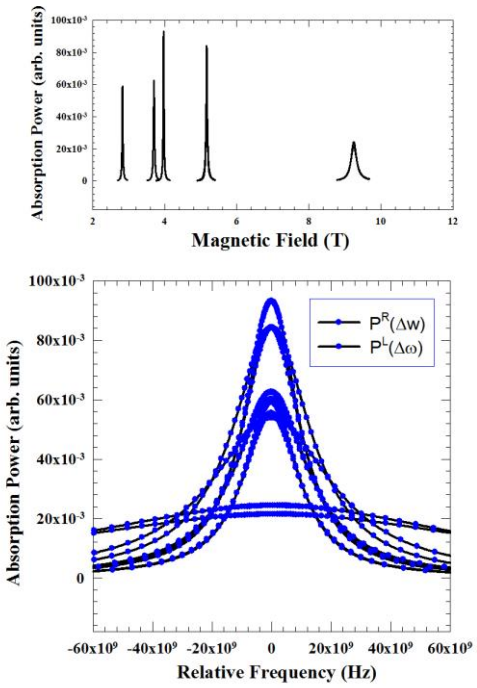

Figure 4. The relatively frequency $\Delta(\omega)$ dependence of the absorption power $P^{(L)}(\Delta\omega)$ of ZnO for $\lambda = 220$, 394, 513, 550, 720 at T=50 K.

Fig. 3, represents the magnetic field dependence of the absorption power $P(B)$ of the line-widths of ZnO for the external field wavelength λ =393 μm at several temperatures, T=50, 70, 90, 120 and 210 K. In order to compare the line of line-widths in the same graph, we plot the value of the rational absorption power, $P_{nr}(B) = \alpha P(B)$ here $\alpha \equiv ((T/10)/P_S(B))$. The $P_S(B)$ is the maximum value at T=30. As seen in Fig. 3, $P(B)$ increases as the temperature increases. Also, the line-widths increases with the increasing temperatures. The results explain the resonant phenomena in the electron-piezoelectric potential interacting system because the collision effect of phonons due to the thermal lattice vibration is expected to become larger with increasing temperatures.

Fig.4, we can read the magnetic-field dependence of the maximum absorption power in upper figure. The bellow figure of Fig. 4 shows the relative frequency dependence of the absorption power, $P(\Delta\omega)$ of ZnO, with λ =220, 394, 513, 550 and 550 μm at T=50 K. The analysis of the relative frequency dependence of the absorption power represents the magnetic field dependency property of the absorption power given for an external field wavelength and the conditions of the system. The results of this work will help to analyze experimental the scattering mechanisms in the electron- piezoelectric potential interacting materials.

Concluding Remarks

As a brief summary, the quantum transition of the EDPT is a useful method to investigate the linear-nonlinear effect of the induced dynamic value due to polarizability on the transition mechanism in the WLT system. We derived the dynamic value formula and the MFF in electron phonon coupling systems and electron impurity coupling systems of two transition types, the intra-band transitions and inter-band transitions. We obtained results of the WLT system that can be applied directly to numerical analyses. The easy analysis of each quantum transition processes are the merits of our theory. In this work, we analyzed the line-shape and the line-width of the electron-piezoelectric phonon interacting system. We selected a system in the confinement of electrons by square-well confinement potential. Because the analysis results shown in Fig. 2-4 are reasonable for explaining the quantum transition phenomena, our results indicate that the quantum transport theory of EDPT is a useful method for explaining the resonant phenomena on the basis of quantum transition and the scattering effect in a microscopic view. The results of this work will help to analyze experimental the scattering mechanisms.

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