

Short-term Fairness Analysis of Connection-based Slotted-Aloha

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Abstract

Slotted-Aloha (S-Aloha) has been widely employed in random access networks owing to its simple implementation in a distributed manner. To enhance the throughput performance of the S-Aloha, connection-based slotted-Aloha (CS-Aloha) has been proposed in recent years. The fundamental principle of the CS-Aloha is to establish a connection with a short-sized request packet before transmitting data packets. Subsequently, the connected node transmits long-sized data packets in a batch of size M . This approach efficiently reduces collisions, resulting in improved throughput compared to the S-Aloha, particularly for a large M . In this paper, we address the short-term fairness of the CS-Aloha, as quantified by Jain's fairness index. Specifically, we evaluate how equitably the CS-Aloha allocates time slots to all nodes in the network within a finite time interval. Through simulation studies, we identify the impact of system parameters on the short-term fairness of the CS-Aloha and propose an optimal transmission probability to support short-term fairness.

Keywords: Slotted-Aloha, Connection Establishment, Random access, Short-term Fairness

1. Introduction

Slotted Aloha (S-Aloha) is known as a medium access control protocol designed to facilitate packet transmissions from a node to an access point (AP) in a distributed manner. At the beginning of a slot, each node in the S-Aloha network attempts to transmit a packet with probability r (called transmission probability). Hence, the total number of attempts varies randomly across slots. When the number of attempts is equal to one, a successful packet transmission occurs during the slot. Otherwise, multiple attempts give rise to collisions, while the absence of any attempts results in underutilization of resources. Owing to its simple implementation and efficient operation, the S-Aloha has been widely employed in various wireless networks [1].

There have been extensive studies on the performance of the S-Aloha in terms of throughput, stability, and delay (see [2] and references therein). It is well established that the throughput of the S-Aloha can be low due to frequent collisions occurred by concurrent packet transmissions, resulting in a maximum achievable throughput of only $e^{-1} \approx 0.37$ [3]. To enhance the throughput performance of the S-Aloha, connection-based slotted-Aloha (CS-Aloha) was proposed recently by Gao and Dai [4]. A notable feature that distinguishes

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the CS-Aloha from the S-Aloha is that each node in the network attempts to transmit a (short-sized) request packet at the beginning of each slot with probability r before transmitting a (long-sized) data packet. When multiple nodes or no nodes send a request in a slot, the attempt process is carried over to the next slot. When only one node sends a request, this node is granted permission to transmit M data packets to the AP, while other nodes refrain from packet transmission. After the completion of data packet transmissions, the attempt process restarts in the next slot.

In [4], the throughput performance of the CS-Aloha is analyzed and compared to that of the S-Aloha. The analysis demonstrates that the throughput of the CS-Aloha increases with an increase in M . In particular, the CS-Aloha can achieve a higher throughput than the S-Aloha when M exceeds a certain threshold. In [5], the delay performance and stability of the CS-Aloha are analyzed using a vacation queueing model. The analysis shows that increasing M can decrease the mean delay and the delay jitter while enlarging the bounded delay region. In [6], the throughput and the mean queueing delay of the CS-Aloha are characterized, optimized, and compared with those of connection-free S-Aloha. As such, recent works on the CS-Aloha primarily focus on throughput, delay, and stability as the main performance measures.

Short-term fairness is also an important measure for evaluating the performance of a random-access protocol in a wireless network, particularly in scenarios where multiple nodes contend for limited resources such as time slots, bandwidth, and power [7]. In this competitive environment, short-term fairness quantifies the ability of the access protocol to allocate resources equitably to all nodes over a finite time interval, without discrimination. Consequently, supporting short-term fairness becomes a critical design principle in wireless networks, especially for guaranteeing QoS (Quality-of-Service) and enhancing user satisfaction within a finite time frame. In this paper, we investigate the impact of system parameters (such as M and r) on the short-term fairness of the CS-Aloha. It is worthwhile to note that the throughput, delay, and stability addressed in previous works are associated with ergodic performances achievable over a sufficiently long period of time. In contrast, the short-term fairness addressed in this paper pertains to a transient performance achievable within a relatively short period of time. Therefore, our analysis contributes to a comprehensive understanding of the CS-Aloha, offering insights from various perspectives and suggesting a proper choice for the system parameters based on the targeted time scale.

The rest of the paper is organized as follows. In Section 2, we overview the CS-Aloha and describe the system model. In Section 3, we explain the notion of fairness in detail and formulate the short-term fairness. In Section 4, we present simulation studies. In Section 5, we conclude the paper.

2. Overview of the CS-Aloha

In this section, we present an overview of the CS-Aloha and describe the system model. The network consists of n nodes and an AP. The system's time axis is discretized into equal-length slots, and without loss of generality, we assume the slot length to be 1 unit. All nodes and the AP in the network are synchronized to the slot timing.

We consider uplink transmission from nodes to the AP. For uplink transmission, two types of packets are involved: a request packet and a data packet. It consumes 1 slot for transmitting a request packet and L slot(s) for transmitting a data packet. Here, L is assumed to be a positive integer. Typically, we have $L \geq 2$, as the size of the request packet is shorter than that of the data packet.

Let $t \in \{1, 2, 3, \dots\}$ represent the current slot on the system clock. Suppose that the t -th slot is not

allocated to any node for packet transmission; we refer to such a slot as “free.” At the beginning of the t -th slot, each node in the network attempts to transmit a request packet to the AP with probability r . We denote $R(t)$ as the total number of attempts at the beginning of the t -th slot. Then, $R(t)$ is described by a Binomial random variable with parameters n and r . Depending on the value of $R(t)$, the CS-Aloha operates the attempt process as follows:

- (a) $R(t) = 0$: In this case, no nodes transmit the request packet. Consequently, the $(t + 1)$ -st slot is not allocated to any node for data packet transmission and becomes free.
- (b) $R(t) \geq 2$: In this case, multiple nodes transmit the request packet concurrently, and collisions occur. Due to the collision, the AP cannot receive any request packet successfully. As a result, the $(t + 1)$ -st slot is not allocated to any node for data packet transmission and becomes free.
- (c) $R(t) = 1$: In this case, a single node transmits the request packet, enabling the AP to successfully receive it. Subsequently, the AP allocates the next ML slots to this node for data packet transmissions. The node then transmits M data packets to the AP from the beginning of the $(t + 1)$ -st slot to the end of the $(t + ML)$ -th slot, while the remaining $n - 1$ nodes refrain from packet transmission. As a result, the $(t + ML + 1)$ -st slot becomes free.

At every free slot, all nodes repeat the above attempt process, independently of the outcomes in the preceding free slots. Hence, in the cases of $R(t) = 0$ or $R(t) \geq 2$, the attempt process restarts at the beginning of the $(t + 1)$ -st slot. In the case of $R(t) = 1$, the attempt process restarts at the beginning of the $(t + ML + 1)$ -st slot. This attempt process continues until only one node successfully transmits the request packet to the AP. Figure 1 illustrates the operation of the CS-Aloha with $M = 2$ [packets], $L = 2$ [slots], and $n = 2$. The first, 7th, 8th, and 13th slots are free, among which collisions occur in the 7th slot. In the first slot, node 1 is granted permission to transmit two data packets to the AP.

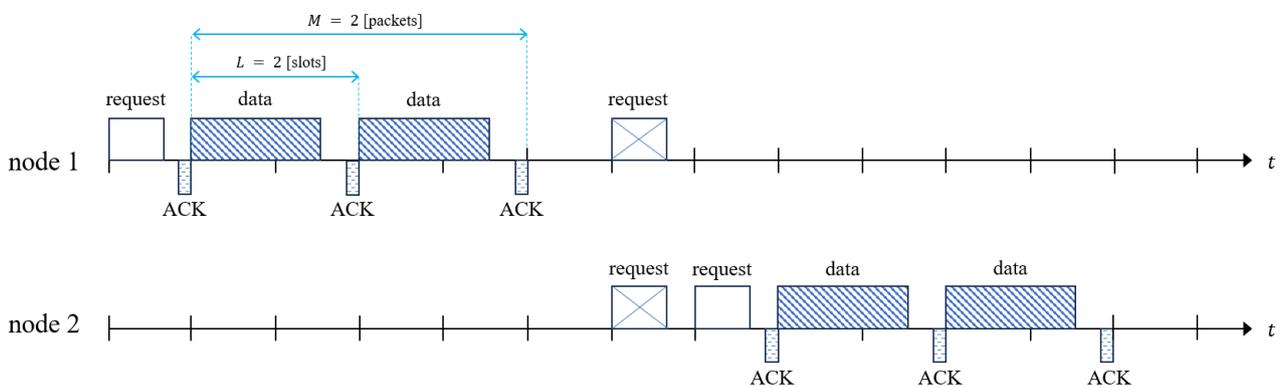


Figure 1. Illustration of the CS-Aloha

In the CS-Aloha, each node transmits data packets in a batch after establishing a connection to the AP. In this manner, the CS-Aloha can efficiently reduce frequent collisions, thereby enhancing the throughput performance. It is shown that the throughput of the CS-Aloha increases with an increase in the back size M . Specifically, the CS-Aloha can achieve a higher throughput than the S-Aloha when M exceeds a certain

threshold. Using the Poisson approximation $P(R(t) = 1) \approx nre^{-nr}$, this threshold is determined by the tuple (L, n, r) as given below [5]:

$$M \geq \frac{1}{L(1 - nre^{-nr})} \quad (1)$$

3. Short-term Fairness

In this section, we explain the notion of fairness in detail and formulate the short-term fairness using Jain's fairness index.

Fairness is a performance measure for evaluating the ability of a random-access protocol to allocate shared resources (e.g., time slots, radio frequencies, bandwidth) equitably to all nodes in the network during a specified finite time interval, without discrimination. In this paper, we specifically concentrate on analyzing the time slot fairness of the CS-Aloha, as it is an important consideration for providing timely and equitable access for all nodes in the network.

Depending on the scale of the observed time interval, time slot fairness can be classified into two categories: long-term fairness and short-term fairness. Long-term fairness assesses the equity of time slot allocation over an infinite time interval. In contrast, short-term fairness assesses the equity of time slot allocation over a finite time interval. It is well established that short-term fairness implies long-term fairness, but the reverse is not necessarily true [7]. Hence, supporting short-term fairness is a more challenging issue, motivating our analysis of the short-term fairness of the CS-Aloha.

Various indices for measuring short-term fairness have been proposed in the existing literature (see [7] and references therein). In this paper, we utilize Jain's fairness index to evaluate the short-term fairness of the CS-Aloha [8]. Jain's fairness index can be expressed mathematically as follows.

Let $w \in \{1, 2, 3, \dots\}$ be a constant representing the length of a time interval, referred to as the *window size*. We observe the time slot allocation of the CS-Aloha over the time interval $I(w) = [t_0, t_0 + 1, \dots, t_0 + w - 1]$ of length w , where $t_0 \in \{1, 2, 3, \dots\}$ denotes the starting point of the observation. For each node $k = 1, 2, \dots, n$, we count the total number of slots allocated to node k during the observed time interval $I(w)$ and denote this number by the random variable $A_k(w)$. Then, the first moment $E[A_k(w)]$, averaged for all nodes in the network, is given by:

$$\mu_1(w) = \frac{1}{n} \sum_{k=1}^n E[A_k(w)] = \frac{1}{n} \sum_{k=1}^n \sum_{a=0}^w a \cdot P(A_k(w) = a)$$

Similarly, the second moment $E[(A_k(w))^2]$, averaged for all nodes in the network, is given by:

$$\mu_2(w) = \frac{1}{n} \sum_{k=1}^n E[(A_k(w))^2] = \frac{1}{n} \sum_{k=1}^n \sum_{a=0}^w a^2 \cdot P(A_k(w) = a)$$

Then, Jain's fairness index for window size w is defined as follows [8]:

$$J(w) = \frac{(\mu_1(w))^2}{\mu_2(w)} \quad (2)$$

The following inequality shows a key characteristic of Jain's fairness index:

$$0 \leq J(w) \leq 1 \quad \text{for all } w = 1, 2, 3, \dots$$

That is, Jain's fairness index is always bounded between 0 and 1 for any window size w . In particular, the

cases of $J(w) = 0$ and $J(w) = 1$ signify total unfairness (i.e., one node monopolizes all resources) and total fairness (i.e., resources are evenly distributed among all nodes), respectively. Accordingly, as Jain's fairness index $J(w)$ approaches 1, it indicates that the CS-Aloha can provide nearly equal access to all nodes in the network over a finite time interval of length w . Conversely, as Jain's fairness index $J(w)$ approaches 0, it indicates that the CS-Aloha results in uneven access to the nodes in the network, allowing a specific subset of nodes to monopolize the channel during a finite time interval of length w .

4. Simulation Studies

In this section, we present simulation studies analyzing the short-term fairness of the CS-Aloha. We developed a simulator using MATLAB to conduct Monte Carlo simulations with 10^7 random seeds in each run. We computed averaged values and depicted them in each figure.

4.1 Parameter setup

We consider a network consisting of $n \in \{10, 20, 50\}$ nodes and an AP, operating under a saturation condition where each node always has data packets in its queue destined for the AP. The transmission probability r for each node is set to n^{-1} for values of $n \in \{10, 20, 50\}$, ensuring a constant average number of attempts per slot (known as the attempt rate G) at $G = 1$. The transmission of a data packet is assumed to consume $L = 2$ [slots]. Using Equation (1), we can determine the threshold on the batch size M , expressed as $\frac{1}{L(1-nre^{-nr})} = \frac{1}{2(1-e^{-1})} \doteq 0.79$. Consequently, any implementation of M in the CS-Aloha results in a higher throughput than the S-Aloha.

4.2 Impact of the window size w on the short-term fairness of the CS-Aloha

We first investigate the impact of the window size w on the short-term fairness of the CS-Aloha. To this end, we compute Jain's fairness index $J(w)$, as given in Equation (2), for varying window sizes, specifically, $w = 10, 100, 200, \dots, 1000$ [slots], with a fixed batch size $M = 3$ [packets]. The results are illustrated in Figure 2. From the figure, it is evident that Jain's fairness index $J(w)$ increases with a larger window size w . This behavior is expected, as larger window sizes provide nodes with more exposure to free slots. Furthermore, the figure reveals that, for each fixed window size w , Jain's fairness index $J(w)$ tends to increase as the network size n decreases. This observation suggests that admission control can serve as an efficient tool for supporting the short-term fairness of the CS-Aloha.

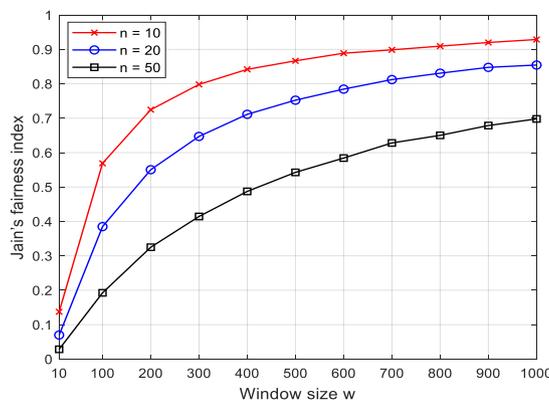


Figure 2. Jain's fairness index for varying window size

4.3 Impact of the batch size M on the short-term fairness of the CS-Aloha

Next, we investigate the impact of the batch size M on the short-term fairness of the CS-Aloha. To this end, we compute Jain's fairness index $J(w)$ with a fixed window size $w = 100$ [slots] for varying batch sizes, specifically, $M = 1, 2, \dots, 15$ [packets]. The results are shown in Figure 3. As shown in the figure, Jain's fairness index $J(100)$ decreases as the batch size M increases, approaching zero. This observation indicates that an increase in the batch size M deteriorates the short-term fairness of the CS-Aloha. Notably, in [4], it is demonstrated that increasing M enhances the throughput of the CS-Aloha. In summary, there exists a trade-off between throughput and short-term fairness in the CS-Aloha, and the batch size M can work as a control knob for this trade-off.

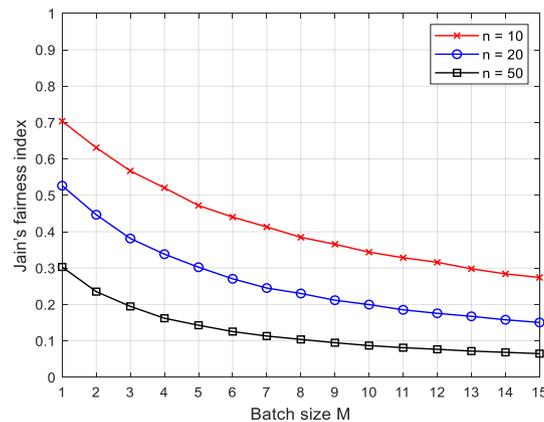


Figure 3. Jain's fairness index for varying batch size

4.4 Impact of the transmission probability r on the short-term fairness of the CS-Aloha

Lastly, we investigate the impact of the transmission probability r on the short-term fairness of the CS-Aloha. To this end, we compute Jain's fairness index $J(w)$ for varying values of $r \in [0.01, 0.9]$ with a fixed window size $w = 100$ [slots] and two different batch sizes, namely, $M = 3$ and $M = 10$ [packets]. The results are presented in Figure 4(a) and Figure 4(b) for $M = 3$ and $M = 10$, respectively. From both figures, we observe that Jain's fairness index $J(100)$ increases and then decreases as the transmission probability r increases. Specifically, we find that Jain's fairness index is maximized at $r = 0.1, 0.05, 0.02$ for $n = 10, 20, 50$, respectively, for both $M = 3$ and $M = 10$. Notably, these probabilities result in an attempt rate $G = nr = 1$, which also maximizes the throughput of the CS-Aloha. This observation suggests that, to support the short-term fairness of the CS-Aloha, we can optimize the transmission probability r by setting it adaptively based on the network size n .

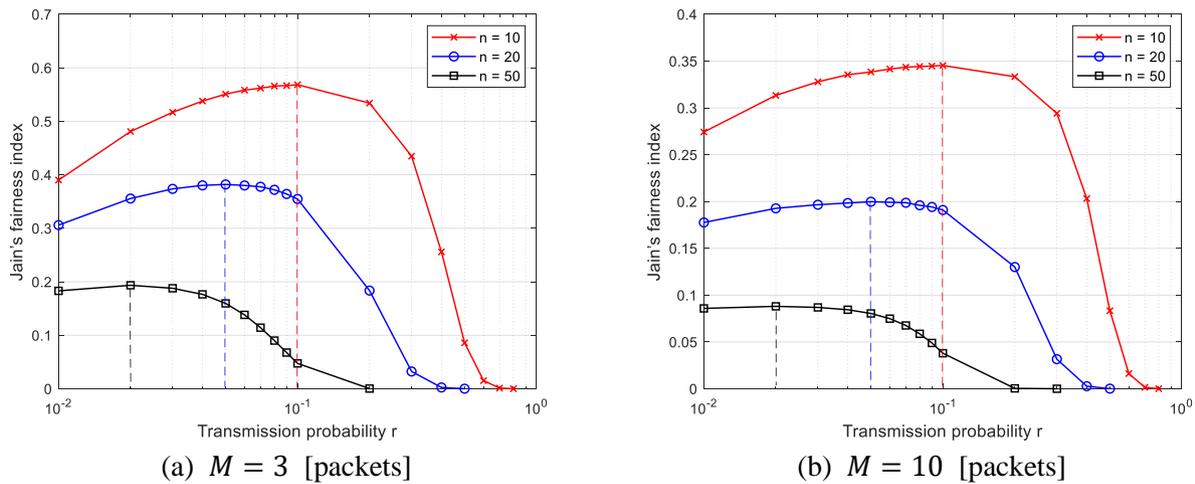


Figure 4. Jain's fairness index for varying transmission probability

5. Conclusion

In this paper, we analyze the short-term fairness performance of the CS-Aloha. We investigate the behavior of Jain's fairness index for various system parameters through simulation studies. Our studies suggest the following engineering guidelines for the operation of the CS-Aloha: (i) Admission control can serve as an efficient tool for supporting the short-term fairness of the CS-Aloha. (ii) There exists a trade-off between throughput and short-term fairness in the CS-Aloha, and the batch size M is a control knob for this trade-off. (iii) We can optimize the transmission probability r to maximize Jain's fairness index by adaptively setting it based on the network size n .

A possible future research direction is to consider a more practical scenario, such as an unsaturated condition, where a node may not have a data packet in its queue intermittently, and thus, does not participate in the attempt process during this period. In this case, we can extend our analysis to optimize the transmission probability r by jointly considering the traffic rate λ and the network size n . In addition, due to dynamic evolution in queue length, a node may not fully utilize the ML slots even when a connection is established. Therefore, it is also important to extend the analysis to optimize the batch size M adaptively based on the traffic rate λ . We leave these problems for future work.

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