Optimizing delivery routing problem for logistics companies based on Integer Linear Programming method

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Abstract

Currently, issues related to freight at Vietnamese logistics companies are becoming more and more urgent because of typical problems in Vietnam such as traffic, infrastructure, and application of information technology. This problem has been studied by applying many different approaches such as Integer Programming (LP), Mixed Integer Programming (MIP), hybrid, meta search, ... In this paper, we applied the ILP model in order to deal with the VRP problem in a small size logistics company which is very popular in Vietnam. The experiments showed promising results with some optimal solutions with some small extra costs.

Keywords: Vehicle Routing Problem (VRP); Linear programming (LP), Integer Linear Programming (ILP).

1. Introduction

Despite a rather long development process, the transport industry has recently seen many strong developments due to appropriate government policies, upgraded infrastructure, international integration as well as strong competition from foreign shipping companies. Currently, there are many companies that have been established or entered the freight transport market, but most of them are small companies with limited potential and have not been optimized in terms of operational processes and transportation capacity. Most of these companies are still operating according to the traditional model, and the application of IT to increase production efficiency has not been given much attention. However, with the trend of digital transformation, along with the participation of many international transport companies, there has been a great competitive pressure on domestic transport companies. The research and application of information technology is concentrated on large logistics companies, but also in small units, so it has revealed many limitations such as not being computerized, few people using information technology with high-level information technology. this object class.
The difficulty of the research on optimization with LP is to understand and know how to model the real problems we are facing into linear programming problems as well as the compatible methods to solve them, thereby making best use of linear programming and interpreting the results obtained. New research works on LP are shown in [1][2][3][5].

The most challenging problem these logistics companies face is how to serve the maximum number of customers while minimizing the cost of resources such as vehicles, time, and humans. This problem is scientifically formulated as a VRP (Vehicle Routing Problem) problem. There are many methods available to solve the VRP which are basically classified into three groups: heuristics, and meta-heuristics and exact methods [1][6][7][8].

Recently, several sophisticated algorithms have emerged aiming at solving VRP with more and more instances. Yet, much effort has been put in, only problems with relatively 200 customers can be solved optimally and the required computing time is high [9]. That’s why heuristics are necessary in practice, where a variety of objectives and side constraints are handled. Constructive Heuristics is one of the most employed methods to provide a starting solution (an initially complete tour) by iteratively extending a connected partial tour. Some particular constructive-heuristic methods that can be mentioned are Greedy Heuristic, Quick-Borůvka Heuristic, and Savings Heuristic [10][11][12][13]. However, the best practice to obtain an optimal route is to carry out two-phase techniques. A constructive method is firstly applied then the result is refined with an improvement algorithm; construction produces a feasible solution then it is improved by improvement heuristics that employ intelligent search techniques to find a high-quality solution, neighborhood solutions (λ-OPT exchanges[14]) for example. In fact, most of constructive heuristics have now been left in disuse, due to the robustness of current metaheuristics.

Current meta-heuristics for the VRP are basically classified into local search and population-based heuristics. Local search methods explore the solution space by moving from a solution to another neighbor solution at each iteration. Some famous algorithms are simulated annealing (Kirkpatrick, Gelatt, and Vecchi[15]), deterministic annealing (Dueck and Scheuer [16]), tabu search (Glover [17]), and neighborhood search variants. Population-based heuristics evolve a population of solutions which may be combined together in the hope of generating better ones. These include ant colony optimization, genetic algorithms, scatter search,... Notably, a wide variety of powerful hybrids of these algorithms have emerged. The most well-known family of hybridizations is Population-based search and Local Search. (Ho, Ang, & Lim, 2001), aim to minimize the total number of vehicles, total distance traveled and total waiting time of vehicles creating a hybrid heuristics method that combines the strengths of both tabu search (TS) and genetic algorithm (GA).

The exact methods are considered to have the most optimal results, especially beneficial for small and medium-scaled problems. In practice, ILP is a very useful tool regarding the ability to represent a number of real-world situations that involve multiple variables and constraints. Therefore, within the scope of our article, we consider ILP the most appropriate approach that solves our problem radically.

The remaining part of this paper is organized as follows. Section 2 introduces the ILP method's idea and points out its advantages and real-life applications. Our problem and the proposed model are presented in Section 3. In Section 4, we detail our experiment regarding the programming environment, collected dataset, and obtained results. We also present a comparison between the traditional heuristic method and our proposed method, indicating that the proposed ILP approach gives much less time to solving and fewer resources. Finally, we show our conclusions and suggest future work in Section 5.
2. Integer Linear Programming Model

Principle of ILP

In integer linear programming, an ILP in standard form is expressed as \[^{[17]}\]:

Maximize or Minimize \[ c^T x \]
Subject to \[ Ax \leq b, \]
\[ X \geq 0, \]
And \[ x \in \mathbb{Z}^n \]

where \( x \) vector is decided variable; \( c \) and \( b \) are vectors and \( A \) is a matrix, where all entries are integers.

This model allows solving problems with very large dimensions, from tens to hundreds of thousands of variables in acceptable time. Solving a linear programming problem to find the result is an optimal solution according to the set goal and satisfying the established constraints. This result will support decision making for the day-to-day business of logistics enterprises.

Linear programming has many applications such as:

- Select the lowest cost input combination for the output product
- Determine the optimal budget
- Deciding the optimal portfolio (or asset allocation)
- Allocating advertising budgets to media
- Plan to use machines
- Decide on the lowest cost shipping method
- Plan fleets of vehicles
- Optimal distribution of manpower
- Choose the most suitable factory location
3. The proposed Approach

The system framework is shown in Figure 2. Accordingly, the input to the system is customer information, warehouse and customer demand list. The output of the system is a report on the number of vehicles, the number of customers, information on each route, the list of customers on each route.

The vehicle routing problem (VRP) is a very important problem. It focuses on finding the route for vehicles while distributing commodities from warehouses of a supplier to its customers [18]. Mathematically, the VRP problem can be represented by a set $N$ of customers and a directed graph $G$. Graph $G$ has $|V|+2$ vertices, the customers are represented by vertices indexed from 1, 2, ..., $n$; vertex 0 represents the starting warehouse and vertex $n+1$ represents the ending warehouse. Each arc $(i, j)$ has a value $c_{ij}$ that describes the distance from vertex $i$ to vertex $j$. Each vertex $i$ ($0 < i < n+1$) has value $d_i$ show that demand of customer $i$. A vehicle starts at the starting warehouse, it provides goods to several customers and ends at the ending warehouse.
warehouse. Each customer is served by a vehicle. The objective of the problem is to find the routes (number of vehicles required) for the fleet of vehicles to satisfy the demand of all customers and minimize the total distance [19].

In order to describe the mathematical model of the problem, we use the parameters as follows:

- \( n \): Total number of customers
- \( N = \{1, 2, \ldots, n\} \): Set of customers
- \( V = \{0, 1, \ldots, n+1\} \): The set of vertices of the graph \( G \)
- \( Q \): capacity of a vehicle
- \( c_{ij} \): Distance from vertex \( i \) to vertex \( j \) \( \forall \{i, j\} \in V \)
- \( d_i \): demand at node customer \( i \) \( 0 < i < n+1 \)

Then we have the two variables in the model:

\[ x_{ij} \in \{0, 1\}, \forall \{i, j\} \in V; i \neq j \]

represent a path from customer \( i \) to customer \( j \)

The objective function of the problem:

\[
\min \sum_{i \in V} \sum_{j \in V} c_{ij} x_{ij}
\]

Constraints of the problem:

\[
\sum_{i \in V} x_{ij} = 1 \quad \forall j \in V, j > 0 \quad (1)
\]
\[
\sum_{j \in V} x_{ij} = 1 \quad \forall i \in V, i \leq n \quad (2)
\]

The Miller-Tucker-Zemlin (MTZ) constraints [20]:

\[
u_j = u_i + d_j \quad \forall x_{ij} = 1 \quad \forall i, j \in V \quad (3)
\]
\[
d_j \leq u_j \leq Q \quad \forall j \in V \quad (4)
\]

In there: \( u_i \) is an additional continuous variable which represents the load left in the vehicle after visiting customer \( i \)

(1), (2): make sure each customer gets exactly a vehicle go in and a vehicle go out.

(3), (4): make sure of the customer's cumulative demands at customer \( j \).

The number of vehicles in the solution is the number of different vehicles that come from the warehouse

4. Experiment results

4.1. Environment

Below, the VRP problem-solving ILP model is implemented using the Python programming language. The program runs on a Core i7-E7440 4600U 2.7GHz 8G RAM personal computer. The program uses numpy, docplex, matplotlib libraries, especially the 64-bit Cplex Studio ver2210 library, which is used to increase memory during big data experiments.

4.2. Collected data

The data was collected from a dairy distribution business with a total of approximately 1350 customers. This business currently has 15 trucks for delivery, on average each vehicle will serve the needs of about 28
customers per day.

The data includes information about the coordinates of the warehouse, the coordinates of each customer, the list of needs of each customer. The input data after collection is described into 2 files with the storage structure as shown in Figure 3.

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(a) Geography location of customers (b) Customers’ demand

Figure 3. Data structure of Input files

We have conducted a practical survey at the logistics company, analyzing the number of vehicles of the company, the number of customers taking care of each vehicle and the total number of customers being cared for by day of the week.

With the total number of customers of the company is 1,309 customers with monthly needs; With strong financial resources in February and December (the last month of the year and near the Lunar New Year), customer demand in these months is the highest, in addition, January, June, August, September: when The income of customers is reduced due to spending a lot in the summer, so the demand of customers is also reduced

Currently, the enterprise uses 15 trucks to delivery packs of milk to customers; On average, the business meets the needs of about 218 customers every day (chart 1 and table 1)

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Table 1. Statistical table of customer demands by month
According to the survey results, with a total of 1300 customers, average monthly customer demand ranges from 9 to 16 boxes of goods, peak months are February, December (the high demand these months are the last months of the calendar year and the lunar calendar), the company uses 15 vehicles to serve the demands of about 218 customers, each car travels about 700 units of distance per day (*).

Several questions can be raised up here for optimizing this in order to save enterprise’s cost consisting of

1. How many trucks can be used to solve the current need of the enterprise?
2. How many customers can be served with current numbers of trucks?
3. What is the average distance traveled by each car in a day?

4.3. Experiment

The VRP problem-solving ILP model is implemented using the Python programming language. Table 2 and Figure 4 shows the optimal routing with input data with respect to 218 and 250 customers. The company only needs to use 13 vehicles and 15 vehicles, respectively, to serve the needs of 218 and 250 customers, the total distance of the fleet is 7761 and 8727 units, respectively.

<table>
<thead>
<tr>
<th>Num of customers</th>
<th>Total Distance</th>
<th>Num of vehicles</th>
<th>Time (s)</th>
<th>average distance per a vehicle</th>
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Table 2. Experimental results
These experimental results have shown that, with 218 customers, the system only needs 13 cars to meet all customers' needs, in addition, with 15 cars of the business can meet the needs of 250 customers, each vehicle only runs about 570 to 590 distance units. In addition, the research team also conducted an experiment with a larger number of customers (N=300) than previous research (N <=200). When compared with the survey results in section (*), the experimental results are better in terms of the number of vehicles, saving time and distance traveled by vehicles (table 3).

<table>
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Table 3. Table comparing experimental results with traditional data
Experimental results show that the system is more optimized when applying the ILP method in fleet scheduling.

5. Conclusion

The integer linear programming model provides a very interesting approach to solving fleet planning problems for a small logistics company in Vietnam, which has a limited number of vehicles, number of customers and needs results. We presented the current status of the fleet at a Vietnamese logistics company. Then we tried to model a real-world problem by integer linear programming, and conducted experiments with collected data sets. Even the model could not completely solve the potential issues but some-how it can improve the heavy task of scheduling with a small extra cost. For the near future, we would like to try the current state of art method for VRP problem to deal with this problem.

Acknowledgment

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