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THE INDEPENDENCE AND INDEPENDENT DOMINATING NUMBERS OF THE TOTAL GRAPH OF A FINITE COMMUTATIVE RING

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Abstract. Let R be a finite commutative ring with nonzero unity and let $Z(R)$ be the zero divisors of R. The total graph of R is the graph whose vertices are the elements of R and two distinct vertices $x, y \in R$ are adjacent if $x + y \in Z(R)$. The total graph of a ring R is denoted by $\tau(R)$. The independence number of the graph $\tau(R)$ was found in [11]. In this paper, we again find the independence number of $\tau(R)$ but in a different way. Also, we find the independent dominating number of $\tau(R)$. Finally, we examine when the graph $\tau(R)$ is well-covered.

1. Introduction

Let R be a commutative ring with nonzero unity and $Z(R)$ be the set of zero divisors of R. The total graph, denoted by $\tau(R)$, was first introduced and studied in [4]. The vertices of the graph $\tau(R)$ are the elements of R and two distinct vertices $x, y \in R$ are adjacent in the graph $\tau(R)$ if and only if $x + y \in Z(R)$. Without assuming R is finite, Anderson and Badawi studied in [4] some of the properties of the graph $\tau(R)$ such as the diameter and the girth. Akbari et al. [2] showed that if R is finite and $\tau(R)$ is connected, then $\tau(R)$ is Hamiltonian. Maimani et al. [9] determined all isomorphism classes of finite rings whose total graph has genus at most one (i.e., a planar or toroidal graph). In addition, they have shown that, given a positive integer g, there are only finitely many finite rings whose total graph has genus g. For finite rings, Shekarriz et al. [14] determined when the graph $\tau(R)$ is Eulerian. Also, they computed the domination number of the graph $\tau(R)$. Variations of the total graph of commutative rings were introduced and studied, for more details see [1, 5, 6].

Let G be a graph. A set of vertices of G is called an independent set if no two vertices in the set are adjacent, i.e., the induced subgraph on this set of

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vertices is the null graph. The independence number of G, denoted by $\alpha(G)$, is the maximum cardinality of an independent set in G. A dominating set of G is a set D of vertices of G such that every vertex in G that is not in D is adjacent to a vertex in D. The domination number of a graph G, denoted by $\gamma(G)$, is the minimum cardinality of a dominating set in G. An independent dominating set of G is a set that is both dominating and independent in G. The independent dominating number of a graph G, denoted by $i(G)$, is the minimum cardinality of an independent dominating set in G. It can be easily checked that an independent dominating set of G is a maximal independent set in G and conversely. It is clear that $\gamma(G) \leq i(G) \leq \alpha(G)$. More details on the independence number, the domination number and the independent dominating number of a graph can be found in [3, 8, 15]. A graph G is called well-covered if all maximal independent sets in G have the same cardinality. Well-covered graphs were defined and first studied by Plummer, see [12]. More details on well-covered graphs can be found in [13]. Mishra and Patra computed the independence number of the graph $\tau(\mathbb{Z}_n)$ in [10], Dhorajia computed the independence number of the graph $\tau(\mathbb{Z}_n \times \mathbb{Z}_m)$ in [7] and Nazzal found the independence number of the graph $\tau(R)$ where R is a finite commutative ring in [11].

We assume through this paper that all rings are finite commutative with nonzero unity. Let R be a ring. In this paper, we again find the independence number of $\tau(R)$ but in a different way. Also, we find the independent dominating number of $\tau(R)$. Finally, we investigate when the graph $\tau(R)$ is well-covered.

2. Independence and independent domination numbers of the graph $\tau(R)$ and when $\tau(R)$ is well-covered

Let R be a ring. Firstly, we determine the independence and independent domination numbers of the graph $\tau(R)$ when R is a local ring. If R is a local ring, then $Z(R)$ is the unique maximal ideal of R.

The following characterization of $\tau(R)$ was given in [4].

Theorem 2.1. *Let R be a local ring where $|Z(R)| = n$ and $|R/Z(R)| = \beta$. Then*

(1) *If $2 \in Z(R)$, then $\tau(R)$ is the union of β disjoint K_n's. The induced subgraph on $Z(R)$ is K_n.*

(2) *If $2 \notin Z(R)$, then $\tau(R)$ is the disjoint union of one copy of K_n and $\frac{\beta-1}{2}$ copies of $K_{n,n}$. The induced subgraph on $Z(R)$ is K_n.*

Nazzal in [11] found the independence number of a local ring in the following theorem.

Theorem 2.2. *Let R be a local ring where $|Z(R)| = n$ and $|R/Z(R)| = \beta$. Then*

(1) *If $2 \in Z(R)$, then $\alpha(\tau(R)) = \beta$.*

(2) *If* $2 \notin Z(R)$, *then* $\alpha(\tau(R)) = n(\frac{\beta-1}{2}) + 1$.

In the following theorem, we find $i(\tau(R))$, where R is a local ring. Also, we get that $\tau(R)$ is well-covered, when R is a local ring.

Theorem 2.3. *Let R be a local ring where $|Z(R)| = n$ and $|R/Z(R)| = \beta$. Then $\tau(R)$ is well-covered and*
(1) *If* $2 \in Z(R)$, *then* $i(\tau(R)) = \alpha(\tau(R)) = \beta$.
(2) *If* $2 \notin Z(R)$, *then* $i(\tau(R)) = \alpha(\tau(R)) = n(\frac{\beta-1}{2}) + 1$.

Proof. If $2 \in Z(R)$, then according to Theorem 2.1 $\tau(R)$ is the disjoint union of β copies of K_n. Hence any maximal independent set must contain exactly one vertex from each copy. Therefore

$$i(\tau(R)) = \alpha(\tau(R)) = \beta.$$

Also, it is clear that $\tau(R)$ is well-covered.

If $2 \notin Z(R)$, then according to Theorem 2.1 $\tau(R)$ is the disjoint union of one copy of K_n and $\frac{\beta-1}{2}$ copies of $K_{n,n}$. Hence any maximal independent set must contain exactly one vertex from K_n and exactly n vertices from each $K_{n,n}$. So

$$i(\tau(R)) = \alpha(\tau(R)) = n\left(\frac{\beta-1}{2}\right) + 1.$$

Also, it is clear that $\tau(R)$ is well-covered. □

We need some facts from ring theory. An Artinian ring R is either a local ring or a finite direct product of local rings, i.e., $R = R_1 \times R_2 \times \cdots \times R_k$ where each one of the R_i's is a local ring. Since a finite ring is artinian, then R is local or R is a finite direct product of local rings. We have found $\alpha(\tau(R))$ and $i(\tau(R))$ for local rings in Theorem 2.3.

In the rest of the paper, we assume that any ring is of the form $R = R_1 \times R_2 \times \cdots \times R_k$, where R_i is a finite local ring for all $i = 1, 2, \ldots, k$. Since R is finite, then any element of R is a unit or a zero divisor. An element $(x_1, x_2, \ldots, x_k) \in R$ is a unit if and only if $x_i \in R_i^*$ for all $i = 1, 2, \ldots, k$. Thus $(x_1, x_2, \ldots, x_k) \in Z(R)$ if and only if $x_i \in Z(R_i)$ for some $i = 1, 2, \ldots, k$.

We need the following two lemmas. The proof of the first one is easy and we will skip it.

Lemma 2.4. *Let $R = R_1 \times R_2 \times \cdots \times R_k$ be a finite ring. Then $x = (x_1, x_2, \ldots, x_k)$ and $y = (y_1, y_2, \ldots, y_k)$ are adjacent in $\tau(R)$ if and only if one of the following conditions holds*:
(1) x_i *and* y_i *are adjacent in* $\tau(R_i)$ *for some* $i = 1, 2, \ldots, k$.
(2) $x_i = y_i$ *and* $x_i \in Z(R_i)$ *for some* $i = 1, 2, \ldots, k$.
(3) $x_i = y_i$ *and* $2 \in Z(R_i)$ *for some* $i = 1, 2, \ldots, k$.

Lemma 2.5. *Let $R = R_1 \times R_2 \times \cdots \times R_k$ be a finite ring and let S be any maximal independent set in $\tau(R)$. Then there exist $S_1, S_2, \ldots, S_k$ that are maximal independent sets in $\tau(R_1), \tau(R_2), \ldots, \tau(R_k)$, respectively, such that*

$S \subset S_1 \times S_2 \times \cdots \times S_k$. Moreover, if $2 \notin Z(R_i)$ for all $i = 1, 2, \ldots, k$, then $S^* = S_1^* \times S_2^* \times \cdots \times S_k^*$.

Proof. Let M_i be the set of all i^{th} coordinates of S for all $i = 1, 2, \ldots, k$. According to Lemma 2.4 the set M_i is an independent set in $\tau(R_i)$ for all $i = 1, 2, \ldots, k$. Let S_i be a maximal independent set in $\tau(R_i)$ such that $M_i \subset S_i$ for all $i = 1, 2, \ldots, k$. Therefore, $S \subset S_1 \times S_2 \times \cdots \times S_k$. If $2 \notin Z(R_i)$ for all $i = 1, 2, \ldots, k$, let $x = (x_1, x_2, \ldots, x_k)$, $y = (y_1, y_2, \ldots, y_k) \in S_1^* \times S_2^* \times \cdots \times S_k^*$. Since each one of the S_i^*'s is an independent set, then $x+y$ is a unit in R. Thus x and y are not adjacent in $\tau(R)$ and hence $S_1^* \times S_2^* \times \cdots \times S_k^*$ is an independent set in $\tau(R)$. Let $w = (w_1, w_2, \ldots, w_k) \in Z(S)$ and $y = (y_1, y_2, \ldots, y_k) \in S_1^* \times S_2^* \times \cdots \times S_k^*$. Then $w_i, y_i \in S_i$ for all $i = 1, 2, \ldots, k$. So w_i and y_i are not adjacent in $\tau(R_i)$ for all $i = 1, 2, \ldots, k$ and y_i is a unit for all $i = 1, 2, \ldots, k$. Thus according to Lemma 2.4 w and y are not adjacent in $\tau(R)$ and hence $(S_1^* \times S_2^* \times \cdots \times S_k^*) \cup Z(S)$ is an independent set in $\tau(R)$ containing S. Since S is a maximal independent set in $\tau(R)$, then $S = (S_1^* \times S_2^* \times \cdots \times S_k^*) \cup Z(S)$. Therefore $S^* = S_1^* \times S_2^* \times \cdots \times S_k^*$. □

Theorem 2.6. *Let $R = R_1 \times R_2 \times \cdots \times R_k$ be a finite ring with $2 \notin Z(R_i)$ for all $i = 1, 2, \ldots, k$. Then*

$$\alpha(\tau(R)) = \prod_{i=1}^{k}(\alpha(\tau(R_i)) - 1) + k \text{ and } i(\tau(R)) = \prod_{i=1}^{k}(\alpha(\tau(R_i)) - 1) + 1.$$

Moreover $\tau(R)$ is not well-covered for all $k \geq 2$.

Proof. Let S be any maximal independent set in $\tau(R)$. Then according to Lemma 2.5 there exist $S_1, S_2, \ldots, S_k$ that are maximal independent sets in $\tau(R_1)$, $\tau(R_2), \ldots, \tau(R_k)$, respectively, such that $S \subset S_1 \times S_2 \times \cdots \times S_k$. According to Theorem 2.1 S_i contains exactly one zero divisor from R_i for all $i = 1, 2, \ldots, k$. Also, using Lemma 2.5

$$|S^*| = \prod_{i=1}^{k} |S_i^*| = \prod_{i=1}^{k}(\alpha(\tau(R_i)) - 1).$$

Let $u = (u_1, u_2, \ldots, u_k)$ and $v = (v_1, v_2, \ldots, v_k) \in Z(S)$. Since S is an independent set in $\tau(R)$, then $u_i \notin Z(R_i)$ or $v_i \notin Z(R_i)$ for all $i = 1, 2, \ldots, k$. Thus S has at most k zero divisors in R. Indeed S must contain at least one zero divisor in R to show that let w_i be the zero divisor of S_i in R_i for all $i = 1, 2, \ldots, k$. Then $(w_1, w_2, \ldots, w_k)$ is not adjacent to $x = (x_1, x_2, \ldots, x_k)$ in $\tau(R)$ for all $x \in S^*$. Therefore

$$\prod_{i=1}^{k}(\alpha(\tau(R_i)) - 1) + 1 \leq |S| \leq \prod_{i=1}^{k}(\alpha(\tau(R_i)) - 1) + k.$$

To show that $\alpha(\tau(R)) = \prod_{i=1}^{k}(\alpha(\tau(R_i)) - 1) + k$, let $(x_1, x_2, \ldots, x_k) \in S_1^* \times S_2^* \times \cdots \times S_k^*$ and $S' = \{(w_1, x_2, \ldots, x_k), (x_1, w_2, \ldots, x_k), \ldots, (x_1, x_2, \ldots, w_k)\}$. Then S' is an independent set in $\tau(R)$ with k zero divisors in R. Take $S_\alpha = (S_1^* \times S_2^* \times \cdots \times S_k^*) \cup S'$. Observe that S_α is an independent set in $\tau(R)$ with $|S_\alpha| = \prod_{i=1}^{k}(\alpha(\tau(R_i))-1)+k$. Hence S_α is a maximum independent set in $\tau(R)$ with

$$\alpha(\tau(R)) = |S_\alpha| = \prod_{i=1}^{k}(\alpha(\tau(R_i)) - 1) + k.$$

To show that $i(\tau(R)) = \prod_{i=1}^{k}(\alpha(\tau(R_i)) - 1) + 1$, take $S_\beta = (S_1^* \times S_2^* \times \cdots \times S_k^*) \cup \{(w_1, w_2, \ldots, w_k)\}$ and let $x = (x_1, x_2, \ldots, x_k) \notin S_\beta$. If $x_i \in Z(R_i)$ for some $i = 1, 2, \ldots, k$, then x_i and w_i are adjacent in $\tau(R_i)$ and hence x and w are adjacent in $\tau(R)$ and if $x_i \in R_i^*$ for all $i = 1, 2, \ldots, k$, then x is a unit and $x = (x_1, x_2, \ldots, x_k) \notin S_1^* \times S_2^* \times \cdots \times S_k^*$. So $x_i \notin S_i$ for some $i = 1, 2, \ldots, k$. But, since S_i is a maximal independent set in $\tau(R_i)$, then x_i is adjacent to y_i in $\tau(R_i)$ for some $y_i \in S_i$. Hence x must be adjacent to y in $\tau(R)$ for some $y \in S_\beta^*$ such that y_i is the i^{th} component of y. Thus S_β is a smallest maximal independent set in $\tau(R)$ and hence S_β is a smallest independent dominating set in $\tau(R)$. So

$$i(\tau(R)) = |S_\beta| = \prod_{i=1}^{k}(\alpha(\tau(R_i)) - 1) + 1.$$

Thus $\tau(R)$ is not well-covered for all $k \geq 2$. $\square$

Example 2.7. Let $R = \mathbb{Z}_{45}$. Then $R = \mathbb{Z}_9 \times \mathbb{Z}_5$, $2 \notin Z(\mathbb{Z}_9)$ and $2 \notin Z(\mathbb{Z}_5)$ with $|Z(\mathbb{Z}_9)| = 3$ and $|Z(\mathbb{Z}_5)| = 1$. So $|\mathbb{Z}_9/Z(\mathbb{Z}_9)| = 3$ and $|\mathbb{Z}_5/Z(\mathbb{Z}_5)| = 5$. By Theorem 2.3 $\alpha(\tau(\mathbb{Z}_9)) = 4$ and $\alpha(\tau(\mathbb{Z}_5)) = 3$. Thus by Theorem 2.6

$$\alpha(\tau(R)) = (4-1)(3-1) + 2 = 8 \text{ and } i(\tau(R)) = (4-1)(3-1) + 1 = 7.$$

Consider $S_1 = \{0, 1, 4, 7\}$ and $S_2 = \{0, 1, 2\}$. Then S_1 and S_2 are maximal independent sets in $\tau(\mathbb{Z}_9)$ and $\tau(\mathbb{Z}_5)$, respectively, and

$$S_1^* \times S_2^* = \{(1,1), (1,2), (4,1), (4,2), (7,1), (7,2)\}.$$

Take $S_\alpha = (S_1^* \times S_2^*) \cup \{(0,1), (1,0)\}$ and $S_\beta = (S_1^* \times S_2^*) \cup \{(0,0)\}$. Then (according to the proof of Theorem 2.6) S_α is a maximum independent set in $\tau(R)$ and S_β is a smallest independent dominating set in $\tau(R)$ and with $|S_\alpha| = 8$ and $|S_\beta| = 7$.

Theorem 2.8. *Let $R = R_1 \times \cdots \times R_k \times R'_1 \times \cdots \times R'_m$ be a finite ring with $2 \in Z(R_i)$ for all $i = 1, 2, \ldots, k$ and $2 \notin Z(R'_j)$ for all $j = 1, 2, \ldots, m$. Then $i(\tau(R)) = \alpha(\tau(R)) = \min\{\alpha(\tau(R_i)) : i = 1, 2, \ldots, k\}$. Moreover, $\tau(R)$ is well-covered.*

Proof. Without loss of generality we will assume that

$$\alpha(\tau(R_1)) = \min\{\alpha(\tau(R_i)) : i = 1, 2, \dots, k\} = n.$$

Let S be any maximal independent set in $\tau(R)$. According to Lemma 2.5 $S \subset S_1 \times \cdots \times S_k \times S'_1 \times \cdots \times S'_m$, where $S_1, \dots, S_k, S'_1, \dots, S'_m$ are maximal independent sets in $\tau(R_1), \dots, \tau(R_k), \tau(R'_1), \dots, \tau(R'_m)$, respectively. Let $x = (x_1, \dots, x_k, x'_1, \dots, x'_m)$ and $y = (y_1, \dots, y_k, y'_1, \dots, y'_m) \in S$. Since S is an independent set in $\tau(R)$, then $x = (x_1, \dots, x_k, x'_1, \dots, x'_m)$ and $y = (y_1, \dots, y_k, y'_1, \dots, y'_m)$ are not adjacent in $\tau(R)$. Therefore $x_1 \neq y_1$. Because if $x_1 = y_1$, then $x_1 + y_1 = 2x_1 \in Z(R_1)$ and so $x + y \in Z(R)$ which is a contradiction. Thus $|S| \leq |S_1| = n$. Suppose $|S| < n \leq |S_i|$ for all $i = 1, 2, \dots, k$. Thus there exists $u_i \in S_i$ such that u_i does not belong to the set of all i^{th} coordinates of S for all $i = 1, 2, \dots, k$. Take $u = (u_1, \dots, u_k, u'_1, \dots, u'_m)$, where u'_j is a unit in S'_j for all $j = 1, 2, \dots, m$. Then $u \notin S$ and u is not adjacent to $x = (x_1, \dots, x_k, x'_1, \dots, x'_m)$ for all $x \in S$. Thus $S \cup \{u\}$ is an independent set in $\tau(R)$ which is a contradiction. Thus $|S| = n$. Thus all maximal independent sets have the same cardinality and we get $i(\tau(R)) = \alpha(\tau(R)) = \min\{\alpha(\tau(R_i)) : i = 1, 2, \dots, k\}$. Therefore $\tau(R)$ is well-covered. $\square$

Corollary 2.9. *A finite ring R is well-covered if and only if R is a local ring or $R = R_1 \times R_2 \times \cdots \times R_k$ is a ring with $2 \in Z(R_i)$ for some $i = 1, 2, \dots, k$.*

Example 2.10. Let $R = \mathbb{Z}_8 \times R_2 \times \mathbb{Z}_3$, where $R_2 = \mathbb{Z}_2[x] / \left(x^2 + 1\right) = \{0, 1, x, 1 + x\}$. Then $2 \in Z(\mathbb{Z}_8)$, $2 \in Z(R_2)$ and $2 \notin Z(\mathbb{Z}_3)$ with $|Z(\mathbb{Z}_8)| = 4$, $|Z(R_2)| = 1$ and $|Z(\mathbb{Z}_3)| = 1$. So $|\mathbb{Z}_8/Z(\mathbb{Z}_8)| = 2$, $|R_2/Z(R_2)| = 4$ and $|\mathbb{Z}_3/Z(\mathbb{Z}_3)| = 3$. By Theorem 2.3, $\alpha(\tau(\mathbb{Z}_8)) = 2$, $\alpha(\tau(R_2)) = 4$ and $\alpha(\tau(\mathbb{Z}_3)) = 2$. Using Theorem 2.8, we get

$$\alpha(\tau(R)) = \min\{\alpha(\tau(\mathbb{Z}_8)), \alpha(\tau(R_2))\} = 2.$$

Consider $S_1 = \{0, 1\}$. Then S_1 is a maximal independent set in $\tau(\mathbb{Z}_8)$. Take $S = \{(0, 0, 1), (1, 1, 1)\}$. Then (according to the proof of Theorem 2.8) S is a maximal independent set in $\tau(R)$ with $|S| = 2$.

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