Nuclear Engineering and Technology 54 (2022) 3795-3802

Contents lists available at ScienceDirect

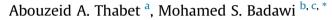
# Nuclear Engineering and Technology

journal homepage: www.elsevier.com/locate/net



**Original Article** 

# Analytical-numerical formula for estimating the characteristics of a cylindrical NaI(Tl) gamma-ray detector with a side-through hole



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## A R T I C L E I N F O

Article history: Received 28 December 2021 Received in revised form 2 April 2022 Accepted 25 April 2022 Available online 28 April 2022

Keywords: Cylindrical Nal(Tl) detector Full side hole γ-ray sources Geometrical and total efficiency And analytical numerical formula (ANF)

#### ABSTRACT

Nal(Tl) scintillation materials are considered to be one of many materials that are used exclusively for  $\gamma$ ray detection and spectroscopy. The gamma-ray spectrometer is not an easy-to-use device, and the accuracy of the numerical values must be carefully checked based on the rules of the calibration technique. Therefore, accurate information about the detection system and its effectiveness is of greater importance. The purpose of this study is to estimate, using an analytical-numerical formula (ANF), the purely geometric solid angle, geometric efficiency, and total efficiency of a cylindrical NaI(Tl) Y-ray detector with a side-through hole. This type of detector is ideal for scanning fuel rods and pipelines, as well as for performing radio-immunoassays. The study included the calculation of the complex solid angle, in combination with the use of various points like gamma sources, located axially and non-axially inside the through detector side hole, which can be applied in a hypothetical method for calibrating the facility. An extended  $\gamma$ -ray energy range, the detector, source dimensions, "source-to-detector" geometry inside the side-through hole, path lengths of  $\gamma$ -quanta photons crossing the facility, besides the photon average path length inside the detector medium itself, were studied and considered. This study is very important for an expanded future article where the radioactive point source can be replaced by a volume source located inside the side-trough hole of the detector, or by a radioactive pipeline passing through the well. The results provide a good and useful approach to a new generation of detectors that can be used for lowlevel radiation that needs to be measured efficiently.

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# 1. Introduction

The measurement of radioactivity requires more knowledge of the spectrum acquired by the gamma spectrometer and excellent accuracy for the detection efficiency system, especially in various fields such as environmental applications, health physics, industry, and energy production, since detection instruments are considered as basic tools of protection against radiation hazard [1]. The Nal(TI) scintillator is widely used as a material for  $\gamma$ -spectrometry since the high efficiency of this material affects its performance [2]. The efficiency of the Nal(TI) scintillator is mainly determined by the source gamma-ray energy range, the shape and volume of both the source and detector and the attenuation of gamma radiation due to

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the layers around the source and detector. In addition, the thickness of the detector material, the cross-section for the interaction of gamma rays during absorption inside the source and detector materials, as well as the "source-to-detector" geometry [3,4].

Cylindrical Nal(Tl) scintillation detector with a side-through hole is considered to be a special design of well-type Nal(Tl) crystals [5–9]. This new detector design with higher counting efficiency has an important advantage when it is necessary to effectively identify low-level radiation, then the source or sample can be placed in the side-through hole perpendicular to the axis of the cylindrical Nal(Tl) scintillation detector. Moreover, the light output can be collected from the optical window of one of the end surfaces of the detector. This type of new design offers better opportunities as a well-type detector for small sample mass and low energies of  $\gamma$ rays, but has some disadvantages due to the shorter average path lengths for high energy gamma-rays crossing the detector crystal [10,11]. Therefore, it is very important to know all the details about the efficiency of various types of detectors depending on the energy

https://doi.org/10.1016/j.net.2022.04.020





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of  $\gamma$ -radiation photons, in addition to the location of various  $\gamma$ -ray sources inside the side-through hole of the detector [12]. Besides, it can improve the limit of detection of various radionuclides by reducing the background level and increasing the efficiency of the detection system.

The idea of the current article is to explain in detail the new analytical-numerical formula (ANF) used to estimate the geometrical, and the total efficiencies of the cylindrical NaI(Tl) detector with a full side hole, that joint by a radioactive point source positioned at dissimilar locations inside the detector's full side hole. This type of detector considers being not famous in shape, special design and is more complicated in the calibration process. This is the main reason to build the analytical-numerical formula (ANF) by straightforward physical theory and numerical integrals, which can be solved with a very short and simple program designed by a BASIC language, which makes the calibration technique faster. This method can be a useful tool for calibrating a  $\gamma$ -ray spectrometry system and determining radioactivity directly or estimating several correction factors indirect. In general, most of these formulas come from the hypothesis of certain approximations and conditions that are different from one source to another. Analytical formulas can be obtained for each source-to-detector geometry inside the sidetrough hole, where the geometric efficiency can be described as the probability that  $\gamma$ -quanta from sources will hit the detector [13-16].

The most difficult task in constructing an analytical-numerical formula (ANF) is, firstly, determining the geometric solid angle between a point radioactive source of  $\gamma$ -quanta located somewhere inside the side trough hole and the detector surface [17-21]. The results of this work will be useful for extended studies when a volume source is placed inside the side through-hole of the detector or a radioactive pipeline passes through it. This can be done after some mathematical modifications to consider the radioactive point move in the X-Z plane to cover the whole volumetric source. This paper will open up a new way for researchers to determine the total energy-dependent efficiency of gamma-ray energy absorption in the case of a cylindrical NaI(Tl) scintillation detector with a side through the hole, considered a specially designed NaI(Tl) well-type crystal detector. The organization of this manuscript after the introduction is as follows: Section 2 presents analytical-numerical formulas for the geometric solid angle, the geometric efficiency of the detector, the total efficiency, and the average path lengths of the gamma-rays from various sources of different sizes and shapes located axially or coaxially inside the side-through hole of the detector. Section 3 includes the results of calculations obtained using the formulas given in Section 2. Conclusions from this work are drawn in Section 4.

## 2. Mathematical model

The next section justifies the new analytic-numerical formula (ANF) used to estimate the geometric solid angle in addition to the geometric and the total efficiency of a cylindrical NaI (Tl) detector with a side-trough hole based on previous work [9] in the case of using axial and non-axial isotropic point sources; in addition, calculations are given for the mean free path of photons inside the detector material itself. The new technique will help find a solution for the complicated detector design used in several special industrial procedures. The investigated gamma detector has the following dimensions: outer radius R and outer length L, and the cylindrical side-trough hole has a radius r and the same length as the detector itself, as shown in Fig. 1.

The polar and azimuth angles  $\theta$  and  $\varphi$  determine the path of the incident photons; the geometric solid angle  $\Omega_G$  formed by a point source and the cylindrical NaI(Tl) detector with a lateral through a

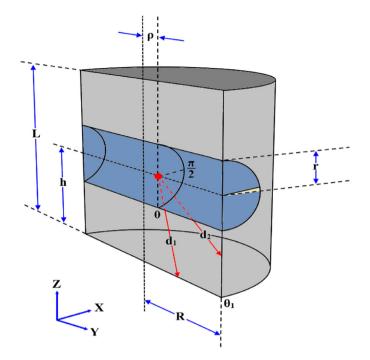


Fig. 1. A point source is located on the main Y-axis at a distance  $\rho$  from the main Z-axis and at possible path lengths d<sub>1</sub> and d<sub>2</sub>.

hole in a spherical coordinate system can be expressed by the formula:

$$\Omega_G = \iint_{\varphi} \iint_{\theta} \sin \theta \ d\theta \ d\varphi \tag{1}$$

The geometrical efficiency,  $\varepsilon_G$ , for that source placed inside the detector side hole can be given by:

$$\varepsilon_{G} = \frac{1}{4\pi} \int_{\Omega_{C}} d\Omega_{G}, \text{ where } : d\Omega_{G} = \sin \theta \ d\theta \ d\varphi \tag{2}$$

which can be rewritten in detail as:

$$\varepsilon_{G} = \frac{1}{\pi} \left[ \int_{0}^{\frac{\pi}{2}} \left( \int_{0}^{\varphi_{\max}-1} \sin \theta d\varphi + \int_{0}^{\varphi_{\max}-2} \sin \theta d\varphi \right) d\theta \right]$$
(3)

The polar angle  $\theta$  takes values from 0 to  $\pi/2$ , the azimuthal angle,  $\varphi$ , takes values started from 0 to  $\varphi_{max-1}$  and from 0 to  $\varphi_{max-2}$ . Under all these circumstances, the effective solid angle,  $\mathcal{Q}_{Eff(Point-Hole)}$ , for a non-axial radioactive isotropic point source that is placed inside the detector side-trough hole along the main Y-axis at a lateral distance  $\rho$ , as shown in Fig. 2, can be determined from the following relations:

$$\Omega_{Eff(Point-Hole)} = 4[I_1 + I_2] \tag{4}$$

where  $I_1$  and  $I_2$  can be defined as:

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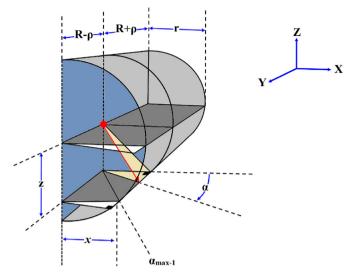


Fig. 2. Side-through hole with radius r and azimuth angle  $\varphi$ .

$$I_{1} = \int_{0}^{\frac{\pi}{2}} \left( \int_{0}^{\varphi_{\max}-1} f_{att} f_{i} \sin \theta d\varphi \right) d\theta \qquad \& \qquad I_{2}$$
$$= \int_{0}^{\frac{\pi}{2}} \left( \int_{0}^{\varphi_{\max}-2} f_{att} f_{i} \sin \theta d\varphi \right) d\theta \qquad (5)$$

where 
$$f_i = (1 - e^{-\mu \cdot d_i})$$
, for  $i = 1$  and 2 &  $f_{att}$   
=  $e^{-\mu_{Al} \cdot d_{Al}}$ 

where  $\mu$  is the total attenuation coefficient without a coherent scattering of the detector material, where the contribution of scattered photons is negligible compared to an expected uncertainty of measurement in the real efficiency calibration, while  $d_1$  and  $d_2$  are the path lengths from end to end through the Nal(Tl) detector material and can be determined using the following expression:

$$d_{1} = \frac{h}{\cos(\theta_{\varphi})} - \frac{x}{\sin(\theta_{\varphi})\cos(\varphi)}$$

$$d_{2} = \frac{\rho\cos(\psi) + \sqrt{R^{2} - \rho^{2}\sin^{2}(\psi)}}{\sin(\theta_{\varphi})} - \frac{x}{\sin(\theta_{\varphi})\cos(\varphi)}$$
(6)

Where 
$$\psi = \frac{\pi}{2} + \varphi$$
 in case of  $I_1$  and  $\psi = \frac{\pi}{2} - \varphi$  in case of  $I_2$ 

The attenuation factor  $f_{att}$  represents the attenuation due to the Aluminum end cap layer of attenuation coefficient  $\mu_{Al}$  and thickness  $d_{Al}$ .

If  $\theta < \theta_1$ , then the photon will pass through the bottom of the detector at a distance  $d_1$ , and if  $\theta > \theta_1$ , then the photon will pass through the side of the detector at a distance  $d_2$ . The polar angle  $\theta_1$  is the maximum angle of the passage photon through the base of the detector, as shown in Fig. 3, and can be defined as:

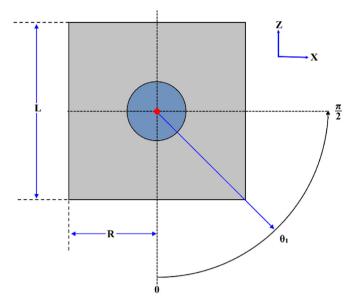


Fig. 3. Vertical section of the detector in the X-Z plane.

$$\theta_1 = \tan^{-1} \left( \frac{\rho \cos(\psi) + \sqrt{R^2 - \rho^2 \sin^2(\psi)}}{h} \right)$$
(7)

where h represents the vertical separation distance between the source and the detector base which is equal to L/2 as shown in Fig. 1. The polar angle  $\theta_{\phi}$  can be expressed as a function of the azimuth angle  $\phi$ , therefore The polar angle  $\theta_{\phi}$  (0) represents the polar angle at zero azimuth angle,  $\theta_{\phi max-1}$  corresponds to the azimuth angle  $\phi_{max-1}$ , and  $\theta_{\phi max-2}$  corresponds to  $\phi_{max-2}$ , as shown in Fig. 4 and Fig. 5, where x and z as functions of  $\theta_{\phi}$  (0), can be calculated using the formulas below:

$$x = r \sin(\theta_{\varphi(0)})$$
 and  $z = \frac{x}{\tan(\theta_{\varphi(0)})}$  (8)

The values of all of these polar and azimuthal angles can be obtained by using the following expressions presented in equations (9) and (10):

$$\theta_{\varphi(0)} = \tan^{-1}\left(\frac{x}{z}\right) \text{ and } \theta_{\varphi} = \tan^{-1}\left(\frac{x}{z\cos(\varphi)}\right)$$
$$\theta_{\varphi_{\max-1}} = \tan^{-1}\left(\frac{\sqrt{x^2 + y_1^2}}{z}\right) \text{ and } \theta_{\varphi_{\max-2}} = \tan^{-1}\left(\frac{\sqrt{x^2 + y_2^2}}{z}\right)$$
(9)

 $\varphi_{\max-1} = \tan^{-1}\left(\frac{y_1}{x}\right)$  & &  $\varphi_{\max-2} = \tan^{-1}\left(\frac{y_2}{x}\right)$ 

Where;

$$y_{1} = \beta \cos(\pi - \delta) + \sqrt{\beta^{2} \left[ \cos^{2}(\pi - \delta) - 1 \right] + R^{2}}$$

$$y_{2} = \beta \cos(\delta) + \sqrt{\beta^{2} \left[ \cos^{2}(\delta) - 1 \right] + R^{2}}$$
wehere  $\beta = \sqrt{\rho^{2} + x^{2}} \qquad \& \qquad \delta = \tan^{-1} \left( \frac{x}{\rho} \right)$ 
(10)

The total efficiency  $\varepsilon_{T(Point-Hole)}$  of the NaI(Tl) detector with a

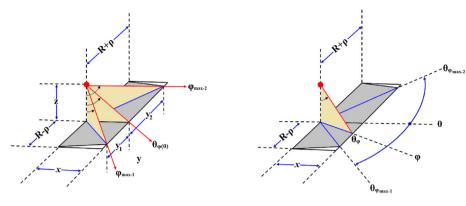


Fig. 4. Polar and azimuth angles inside a side-through hole.

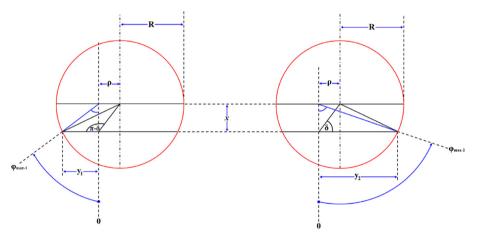


Fig. 5. Cross-sectional view for the detector.

side-through hole can be expressed as:

$$\epsilon_{\mathrm{T}(\mathrm{Point-Hole})} = \frac{\mathcal{Q}_{Eff(Point-Hole)}}{4\pi}$$
(11)

which can be rewritten in detail as:

$$\varepsilon_{\mathrm{T}(\mathrm{Point-Hole})} = \frac{1}{\pi} \left[ \int_{0}^{\frac{\pi}{2}} \left( \int_{0}^{\varphi_{\mathrm{max}-1}} f_{att} f_{i} \sin \theta d\varphi + \int_{0}^{\varphi_{\mathrm{max}-2}} f_{att} f_{i} \sin \theta d\varphi \right) d\theta \right]$$
(12)

The average path length  $d_{(Point-Hole)}$  traveled by photons in the active medium of a NaI(Tl) scintillation detector with a side-through hole depends on the location where the photons hit the detector and can be defined as:

$$\bar{d}_{(Point-Hole)} = \frac{\int_{\Omega_G} d(\theta, \varphi) d\Omega_G}{\int_{\Omega_G} d\Omega_G} = \frac{\int_{\Omega} \int_{\theta} d(\theta, \varphi) \sin \theta d\theta d\varphi}{\Omega_G}$$
(13)

The average path length  $d_{(Point-Hole)}$  for a non-axial point source can be determined by the following formula:

$$\bar{d}_{(Point-Hole)} = \frac{1}{2} \begin{bmatrix} \int_{0}^{\frac{\pi}{2}\varphi_{\max}-1} d_{i} \sin \theta d\varphi d\theta & \int_{0}^{\frac{\pi}{2}\varphi_{\max}-2} d_{i} \sin \theta d\varphi d\theta \\ \int_{0}^{\frac{\pi}{2}\varphi_{\max}-1} \int_{0}^{\frac{\pi}{2}\varphi_{\max}-1} \sin \theta d\varphi d\theta & \int_{0}^{\frac{\pi}{2}\varphi_{\max}-2} \int_{0}^{\frac{\pi}{2}\varphi_{\max}-2} \sin \theta d\varphi d\theta \end{bmatrix}$$
(14)

In the case of an axial location of a gamma-ray source inside the Nal(Tl) detector with a side-through hole, to calculate the effective axial solid angle  $\Omega_{Eff(Axial-Hole)}$ , geometric and total efficiencies  $\varepsilon_{T(Axial-Hole)}$  and determine the average path length d (Axial hole) of photons inside the hole, the lateral distance  $\rho$  should be the same as at zero in all the above equations that depend on it. All analytical-numerical formulas were solved by the trapezoidal method.

# 3. Results and Discussion

Currently, gamma-ray detectors are used much more often due to their greater importance for quantitative gamma-ray spectroscopy in various fields, especially in industry. It was interesting and useful to carry out intensive research using an analytical-numerical formula (ANF) to estimate the characteristics of a cylindrical Nal(Tl)  $\gamma$ -ray detector with a side-through hole in the energy range from 0.05 to 3.00 MeV. This can help in studying the detection capability of the new detector design at various locations within the detector cavity. In the present work, it was challenging to develop a

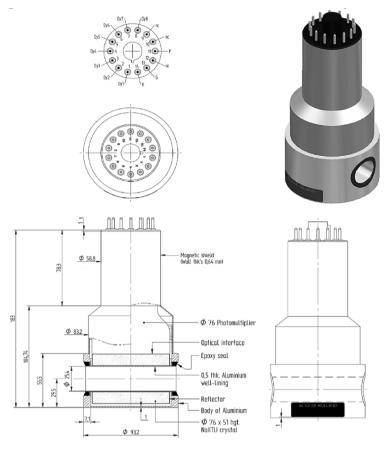
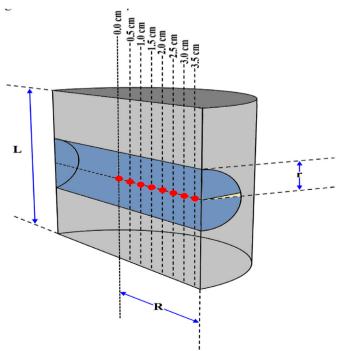


Fig. 6. 3D model of Nal(Tl) probe with PMT and its drawing, drawing of PMT socket, and method of connecting its dynodes.



calibration process and obtain the characteristics of the gamma-ray detector due to the complex source-detector geometry. Great

**Fig. 7.** Lateral distances  $\rho$  of different locations of gamma-ray sources on the main axis

of the side-through hole.

difficulties were associated with finding the correct and accurate solid angle, in addition to estimating the length of the photon path through the active medium of the detector. It is difficult to determine the efficiency of gamma-ray detectors over a wide range of energies for two reasons. Firstly, there are a limited number of calibrated standard radioactive sources, and none of them emit  $\gamma$ -quanta with energies above about 3 MeV. The Analytical-Numerical, Formula considers the total attenuation coefficient is

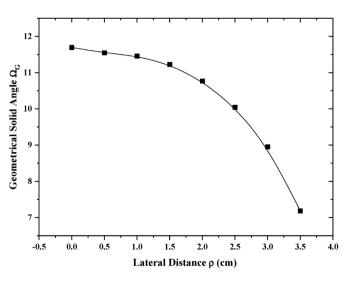


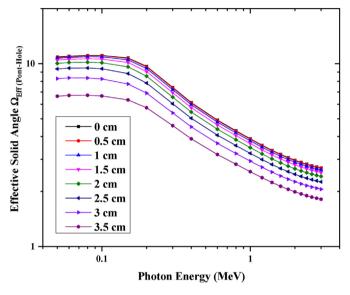
Fig. 8. Geometric solid angle  $\Omega_G$  as a function of the lateral distance  $\rho$ .

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Table 1

ncy $\varepsilon_G$ depending on the lateral distance $\rho$ .
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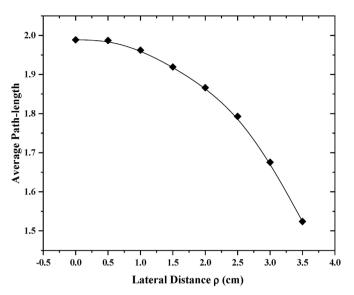
Lateral Distance, $\rho$ (cm)	Geometrical Efficiency, $\epsilon_G$
0	0.931
0.5	0.919
1	0.912
1.5	0.893
2	0.857
2.5	0.799
3	0.712
3.5	0.572



**Fig. 9.** Effective solid angle  $\Omega_{Eff(Point-Hole)}$  of a radioactive point source located at a lateral distance  $\rho$  as a function of the energy of gamma-rays.

used for describing a decrease in the number of photons in a narrow beam, where the photons are emitted in the  $4\pi$  direction. In





**Fig. 10.** Average path-length  $\overline{d}_{(Point-Hole)}$  as a function of lateral distance  $\rho$ .

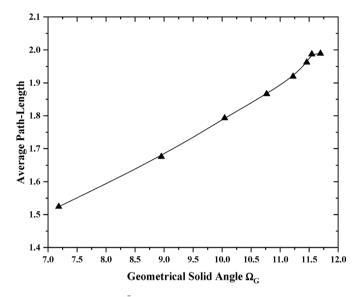
the current method, the radiation interacting within the detector materials is based on the integration technique that covers the whole volume of the detector crystal, therefore the varied problems of the light collection efficiency were covered and negligible.

Therefore, direct measurement of detector efficiency at high energies is not possible with calibrated isotopic gamma-radiation sources. Secondly, the mathematical functions describing the energy dependence of the efficiency for different distances from the sources to the detector, have not been established, which makes the interpolation between the measured experimental data points and extrapolation behind them unclear mainly at low, as well as intermediate and high energies. In addition, the number of measured data points may not be sufficient to determine the shape of the efficiency curve in areas where it is difficult to obtain its correct energy dependence. To overcome these problems, this paper has proposed an analytic-numerical formula to determine the solid

#### Table 2

The total efficiency  $e_{T(Point-Hole)}$  of a cylindrical NaI (TI)  $\gamma$ -ray detector with a lateral side-through hole at a lateral distance  $\rho$ .

		Total Efficiency	, <i>e</i> <sub>T(Point-Hole)</sub>						
	Lateral Distance, $\rho$ (cm)	Axial $\rho = 0$	Non-Axia	l ρ≠0					
			0	0.5	1	1.5	2	2.5	3
Photon Energy (MeV)	0.05	0.865	0.859	0.851	0.833	0.799	0.745	0.659	0.528
	0.06	0.875	0.870	0.860	0.842	0.807	0.752	0.665	0.534
	0.08	0.882	0.877	0.867	0.846	0.810	0.753	0.664	0.535
	0.10	0.882	0.878	0.866	0.844	0.806	0.746	0.656	0.530
	0.15	0.855	0.850	0.835	0.807	0.764	0.702	0.616	0.504
	0.20	0.769	0.764	0.748	0.720	0.680	0.623	0.549	0.457
	0.30	0.590	0.586	0.573	0.552	0.522	0.481	0.428	0.365
	0.40	0.488	0.485	0.475	0.458	0.433	0.401	0.359	0.309
	0.60	0.391	0.389	0.381	0.368	0.349	0.324	0.292	0.254
	0.80	0.342	0.340	0.333	0.322	0.306	0.284	0.257	0.224
	1.00	0.309	0.307	0.301	0.291	0.277	0.258	0.233	0.204
	1.20	0.285	0.283	0.278	0.269	0.256	0.238	0.216	0.189
	1.40	0.268	0.266	0.261	0.252	0.240	0.224	0.203	0.178
	1.60	0.254	0.253	0.248	0.240	0.228	0.213	0.193	0.170
	1.80	0.244	0.242	0.238	0.230	0.219	0.204	0.186	0.164
	2.00	0.236	0.235	0.230	0.223	0.212	0.198	0.180	0.158
	2.20	0.230	0.228	0.224	0.217	0.206	0.193	0.175	0.154
	2.40	0.224	0.223	0.219	0.212	0.202	0.188	0.171	0.151
	2.60	0.220	0.219	0.215	0.208	0.198	0.185	0.168	0.148
	2.80	0.217	0.216	0.212	0.205	0.195	0.182	0.166	0.146
	3.00	0.214	0.213	0.209	0.202	0.192	0.180	0.164	0.144



**Fig. 11.** Average path-length  $\overline{d}_{(Point-Hole)}$  as a function of the geometric solid angle  $\Omega_{G}$ .

angle and detection efficiency at different energies, in addition to the average gamma-ray path lengths inside the detector materials. Thus, it will be possible to determine the efficiency curve mathematically, but the mathematical procedure should be more accurate when comparing its results with the measured values. A drawing of a cylindrical Nal(Tl)  $\gamma$ -ray detector with a side-through hole was provided by Scionix Holland BV for scintillation detectors and radiation detection instruments. The 3D model of the Nal(Tl) 76BAP51/3M-X probe with a PMT and its drawing VS-0172, as well as the diagram of the PMT socket and the connection way is shown in Fig. 6.

The XCOM program was used to calculate the attenuation of gamma radiation in the active medium of the detector and its Al end cap [22]. It is assumed that the point radioactive source is located in different places on the main axis of the side-through hole, as shown in Fig. 7, and its lateral distance  $\rho$  along the axis varies from 0 to 3.5 cm with a step of 0.5 cm. The geometric solid angle  $\Omega_{\rm G}$  was calculated for each lateral distance  $\rho$ , and it was found that its value decreased as the lateral distance  $\rho$  increased, as shown in Fig. 8. The geometric efficiency  $e_{\rm G}$  of a detector with a side-through hole for a source placed inside it is based on equation (3) and given in Table 1 also decreased with increasing transverse distance  $\rho$ .

For each axial and non-axial position of a point gamma-ray source located inside the side-through hole of the detector at a lateral distance  $\rho$ , the effective solid angle  $\Omega_{Eff(Point-Hole)}$  was calculated. The obtained data were plotted as a function of photon energy as shown in Fig. 9. It has been observed that the effective solid angle in the low energy region is higher than in the high energy region. This is because photoelectric interactions dominate at lower energies. The total efficiency  $\varepsilon_{T(Point-Hole)}$  of the cylindrical Nal(Tl) gamma-ray detector with a side-through hole gamma-rays with energies from 0.05 to 3.00 MeV was calculated as a function of the lateral distance  $\rho$  and is presented in Table 2. The total efficiency  $\varepsilon_{T(Point-Hole)}$  decreased with an increase in both the gamma-ray energy and the lateral distance  $\rho$ .

The average path length  $d_{(Point-Hole)}$  of gamma-quanta from a radioactive point source was calculated using equation (14). The average path length of gamma rays through the detector material was determined as a function of the lateral distance  $\rho$  and is shown

in Fig. 10. In addition, it was plotted as a function of the geometric solid angle  $\Omega_G$ , as shown in Fig. 11. The average path length has the maximum value as lateral distance,  $\rho$ , equal to zero and was decreased as the lateral distance,  $\rho$ , was increased, which means the source moved away from the center of the cavity. Moreover, it was noticed that the average path length has the maximum value as the geometrical solid angle has the maximum value at a lateral distance,  $\rho$ , equal to zero as well, Therefore, this indicates that the center of the cavity has the maximum interaction probability in the calibration process.

# 4. Conclusions

This article proposes an analytic-numerical formula (ANF) for calculating the geometric solid angle, geometric efficiency, and total efficiency of a cylindrical NaI(Tl)  $\gamma$ -ray detector with a sidethrough hole. This type of detector is ideal for scanning radioactive fuel rods and pipelines, as well as for performing radioimmunoassay. This method is considered to be uncomplicated for not well-known detector shape, special design, and more difficult in the calibration process, plus it's a very quick method for the calibration computation procedures. In addition to determining the average path lengths of  $\gamma$ -quanta inside the detector medium, their path lengths through the entire detector were also considered and studied. With minor modifications, this technique can also be used for estimating the efficiency of gamma-ray total energy absorption in the case of a cylindrical radioactive source located inside the side-through hole of the detector. This will be demonstrated and experimentally confirmed in a future paper. The results show the best potential of this useful approach also for the development of new detectors for efficient detection of low-level radiation.

#### **Declaration of competing interest**

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

# Acknowledgment

The authors, as specialists in the field of radiation measurements and calibration methods, would like to thank SCIONIX Holland BV in the Netherlands, specialized in the design and manufacture of devices for detecting nuclear radiation based on the scintillation principle, for providing a geometric drawing of the detector, without which this study could not have been successfully carried out.

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