Nuclear Engineering and Technology 54 (2022) 3488–3493

Contents lists available at ScienceDirect

Nuclear Engineering and Technology

journal homepage: www.elsevier.com/locate/net

Original Article

A complete 3D map of Bell Glasstone spatial correction factors for BRAHMMA subcritical core

Shefali Shukla [a, b], Tushar Roy [b, *], Yogesh Kashyap [a, b], Mayank Shukla [a, b], Prashant Singh [a, b]

[a] Homi Bhabha National Institute, Mumbai, India
[b] Technical Physics Division, Bhabha Atomic Research Centre, Mumbai, India

ARTICLE INFO

Article history:
Received 15 June 2021
Received in revised form
29 March 2022
Accepted 29 March 2022
Available online 12 April 2022

Keywords:
Bell-Glasstone spatial correction factor
BRAHMMA
3D mapping
Area ratio method

ABSTRACT

Accelerator driven subcritical systems have long been discussed as facilities which can be used for solving the nuclear waste problem. The physics of these systems is very different from conventional reactors and new techniques had to be developed for reactivity monitoring. One such technique is the Area Ratio Method which studies the response of a subcritical system upon insertion of a large number of neutron pulses. An issue associated with this technique is the spatial dependence of measured reactivity which is intrinsic to the sub criticality of the system since the reactor does not operate on the fundamental mode and measured reactivity depends on the detector position. This is generally addressed by defining Bell-Glasstone spatial correction factor. This factor upon multiplication with measured reactivity gives the correct reactivity which is independent of detector location. Monte Carlo Methods are used for evaluating these factors. This paper presents a complete three dimensional map of spatial correction factors for BRAHMMA subcritical system. In addition, the dataset obtained also helps in identifying detector locations where the correction factor is close to unity, thereby implying no correction if the detector is used at those locations.

1. Introduction

Accelerator Driven Subcritical systems (ADS) have long been discussed as facilities which can be used for solving the nuclear waste problem, producing fissile materials and generating power as a byproduct. These reactors go for high burn up, use unconventional fuels and are coupled to high power accelerators [1–3]. To ensure the safe operation of accelerator driven subcritical system (ADS) at all times, continuous monitoring of subcritical level of ADS is necessary. Since the dynamics of critical reactors are very different from ADS, the conventional techniques used for critical reactors need to be adapted according to ADS. One of the most commonly used experimental method for measuring the reactivity of subcritical system is the Sjöstrand Area Ratio Method which measures the reaction rate generated from a pulsed neutron source. A drawback of the Sjöstrand method is the spatial dependence of the measured reactivity value on neutron detector location. Several techniques are being devised continuously for eradicating this spatial dependence. One of the most widely used approaches is the one suggested by Bell and Glasstone [4] which involves calculating a spatial correction factor and multiplying it to the measured reactivity to get the corrected or global reactivity

$$\rho_{corrected} = f^{*}\rho_{measured} \tag{1}$$

This method has been widely explored by researchers around the world for making corrections in the experimentally measured reactivity for various accelerator driven sub-critical systems like YALINA (thermal and booster), MUSE, VENUS and BRAHMMA. YALINA is one of the most widely studied system due to the simplicity and flexibility of core design and has been used by researchers for benchmarking their developed codes for correction factors. Talamo et al. [5] determined the spatial correction factors for the YALINA system using Monte Carlo as well as deterministic techniques and results obtained were in good agreement. Talamo et al. [6] also examined the impact of detector choice on the spatial correction factor and showed that the effect of detector material is minimized when measurements are made in thermal zone while its pronounced in fast zone, specifically if the detector is more

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* Corresponding author.
E-mail address: tushar@barc.gov.in (T. Roy).

https://doi.org/10.1016/j.net.2022.03.042

sensitive for fast neutrons. Gazda [7] compared the different methods available for correcting the measured reactivity for the Yalina system. Cao and Gohar [8] studied the impact of long and short detectors on reactivity measurements and inferred that reactivity values from short detector are more accurate as the long detectors are susceptible to dead time issues. Becares [9] developed a new hypothesis assuming that it is always possible to find one or more parameters of the subcritical system, which present a univocal dependence with the reactivity for small variations in the geometry, composition and cross sections. Extensive studies on the MUSE subcritical system have been carried out by Cao [10], Gabrielei [11] and Carta [12]. Correction factors for area ratio method as well as source jerk method were calculated separately. Gazda [13] was the first one to carry out measurement of spatial correction factors for a lead accelerator driven subcritical system VENUS. Cao and Gohar [14] carried out extensive simulations for the KIPT neutron source facility and evaluated correction factors for the different fuel loading configurations at various detector locations for the flux to current ratio method. Apart from the static and dynamic methods being previously used recently Talamo [15] developed a new method of spatial correction factor evaluation which allowed pulse superimposition without a time-dependent simulation. Bajpai [16] carried out measurements of spatial correction factors for few locations in BRAHMMA system. This work is an extension of our previous publication. Here we give a complete 3D map of the correction factors also giving us the locations where almost no correction is required.

2. Bell and Glasstone spatial correction factor

Reactivity obtained experimentally using area ratio method is highly dependent on the detector location. This dependence can be removed by using a straightforward method developed by Bell and Glasstone. Bell and Glasstone did a theoretical treatment of area ratio method and derived an expression for correction factor [4]. They suggested multiplying the measured reactivity with this spatial correction factor which takes into account the source importance. This approach has been used by several researchers to account for the spatial dependence of reactivity measured by Area ratio method [5–16].

The Bell and Glasstone spatial correction factor for Area Ratio Method is given by Eq. 2

$$f=\left(\frac{\rho_{cri}}{\rho_{src}}\right)=\left(\frac{A_d}{A_p}\right)\left(\frac{1}{\beta_{eff}}\right)\left(\frac{1-k_{eff}}{k_{eff}}\right) \tag{2}$$

where ρ_{cri} is the reactivity calculated by computer codes in criticality mode, ρ_{src} is the reactivity calculated by computer codes in source mode (with the external and fission neutron sources), A_p is the prompt area and A_d is the delayed area. In Area Ratio method, ρ_{src} (in $) is calculated as the ratio of prompt area to delayed area. Fig. 1 shows a typical detector response to a source neutron pulse and the corresponding prompt and delayed areas.

All the factors in eqn (2) can be derived by numerical methods. Reactivity obtained by computer codes in source mode (ρ_{src}) can be calculated using static and dynamic methods. In Static method (time independent), two separate simulations are done with steady state external neutron source - one with total neutrons (prompt and delayed) and other with delayed neutrons suppressed. Detector reaction rates determine the total area and prompt area respectively. In Dynamic method (time dependent), a single neutron pulse is simulated and the detector reaction rate is counted till the pulse vanishes. This detector reaction rate is superimposed several times till the delayed neutron contribution becomes constant. The detector reaction rate is now integrated to find out the total and delayed area. The prompt area is the difference between the total and delayed area. For our calculations we have used the Static method similar to our previous work [16].

From equation (2) it is observed that correction factor is inversely proportional to prompt area. Prompt area has contributions from both source and fission neutrons while delayed area has contribution from just fission neutrons. In source zone prompt area is enhanced due to source neutrons, so correction factor is less than unity. For measurements made away from source where the contribution of source neutrons decreases the correction factor becomes more than unity. The positions where the correction factor is very close to unity are ideal for reactivity measurements. For other locations correction factor has to be taken into account.

Similar to our previous work the statistical error in spatial correction factors was determined assuming that total area (A_t), prompt area (A_p), prompt neutron multiplication factor (k_p) and the multiplication factor (k_{eff}) calculated from simulations are independent of each other [8]. The associated error in f due to statistical nature of simulation is estimated using standard error propagation method [8,15].

$$\frac{\sigma_f^2}{f^2}=\frac{(k_p-1)^2k^2}{(k-k_p)^2(k-1)^2}\left[\frac{\sigma_k^2}{k^2}\right]+\frac{k_p^2}{(k-k_p)^2}\left[\frac{\sigma_k^2}{k_p^2}\right]+\left[\frac{A_t/A_p}{\left(1-\frac{A_t}{A_p}\right)}\right]^2\left[\frac{\sigma_{A_t}^2}{A_t^2}+\frac{\sigma_{A_p}^2}{A_p^2}\right]^2 \tag{3}$$

3. BRAHMMA subcritical assembly

BRAHMMA (**B**eO **R**eflected **A**nd **H**DPe **M**oderated **M**ultiplying **A**ssembly) [17–19] is a subcritical thermal core that can be coupled to a D-D or D-T neutron generator.

The core (Fig. 2) consists of natural uranium metallic fuel rods embedded in a high density polyethylene moderator, a radial beryllium oxide reflector, and a shielding of borated polyethylene and cadmium. The core consists of 160 fuel rods arranged in a 13 × 13 square lattice with a central cavity for inserting the external source. This system is very compact and modular and has theoretical k_{eff} value of 0.890 and is currently being used for various ADS experiments. There are 3 axial and 4 radial channels provided in the core for various measurements. Three of the radial channels run only halfway i.e. till the cavity while one of them runs through the entire core. For creating the radial channels the beryllium oxide at few locations had been replaced by graphite which creates dissimilarities in an otherwise symmetrical system. Details of the position and dimension of these channels can be found in our previous reported work [17].

4. Evaluation of spatial correction factor for full core

Monte Carlo simulations have been performed to map the spatial correction factor for complete core. This work is in continuation of our previous work where we evaluated correction factors only for experimental channels [16]. Calculation for the entire core was performed as data was required for development of a theoretical model on modal contamination and source importance.

Two step static calculation method has been used in which prompt and total area were evaluated in two different simulations, as discussed in Section 2. This data along with beta effective and k_{eff}

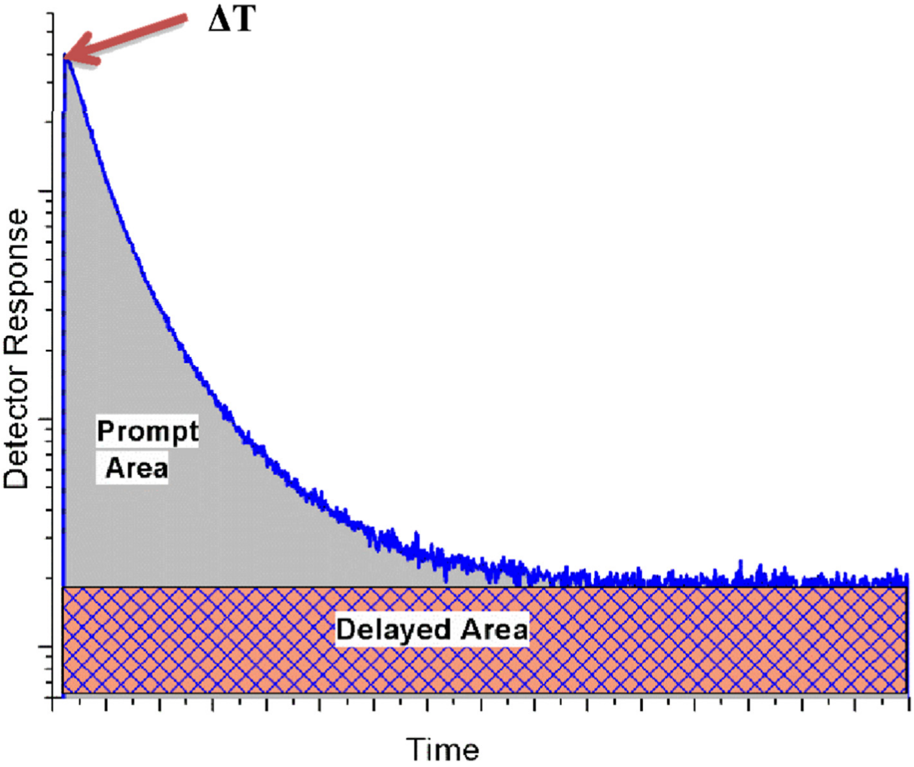

Fig. 1. A typical detector response to a source neutron pulse and the corresponding prompt and delayed areas.

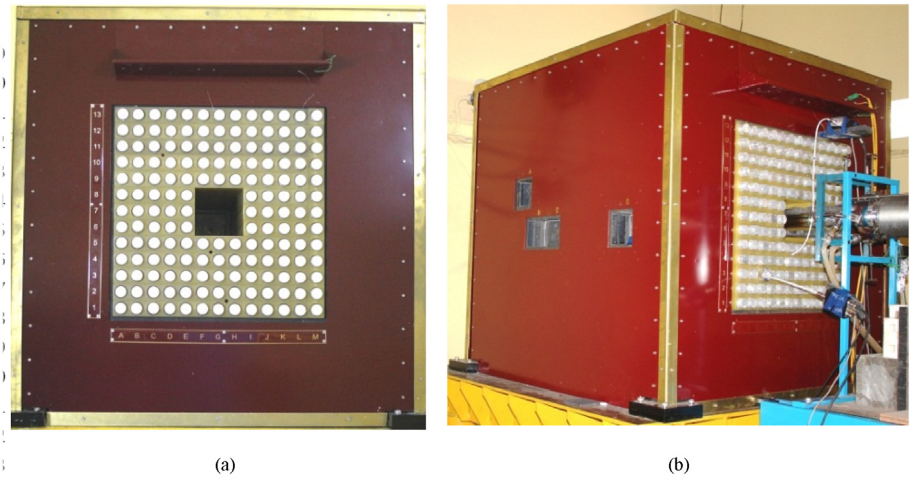

Fig. 2. BRAHMMA subcritical assembly showing 160 lattice locations (a). Assembly coupled to DD-DT neutron generator for experiments (b).

values (theoretical values β_{eff}: 704 ± 10 pcm and k_{eff}: 0.890 ±0 .001) is used for the calculation of correction factors using equation (2). We have simulated a detector of diameter 25 mm and active length 100 mm wherein detector material has not been considered and has been replaced by air BRAHMMA core is a 13 × 13 lattice having a 3 × 3 cavity in the center so total fuel rod positions is 160. We have considered 11 axial positions in each channel (1100 mm axial length and 100 mm detector so total 11 positions) for simulations, therefore a total of 1760 positions (160*11 = 1760) have been considered while evaluating correction factors for the entire core. Calculations were done for all the locations and the results have been shown in Figs. 3 and 4. A 14.1 MeV mono-energetic neutron source has been considered for all simulations. Fig. 3(a) shows the schematic of BRAHMMA core with detector position marked for a single measurement.

Fig. 3(b) shows the spatial correction factors at various locations inside the core. The box represents the subcritical core and the points shown are the mid-points of detector locations in the core. The colour indicates the relative value of correction factor at any location. It is quite evident that the values of correction factor are less than one near the source located at the center of the assembly and become more than one as we move away from the center. This is due to the decrease in source neutron contribution to prompt area (refer equation (2)) when we measure at locations far away from the source.

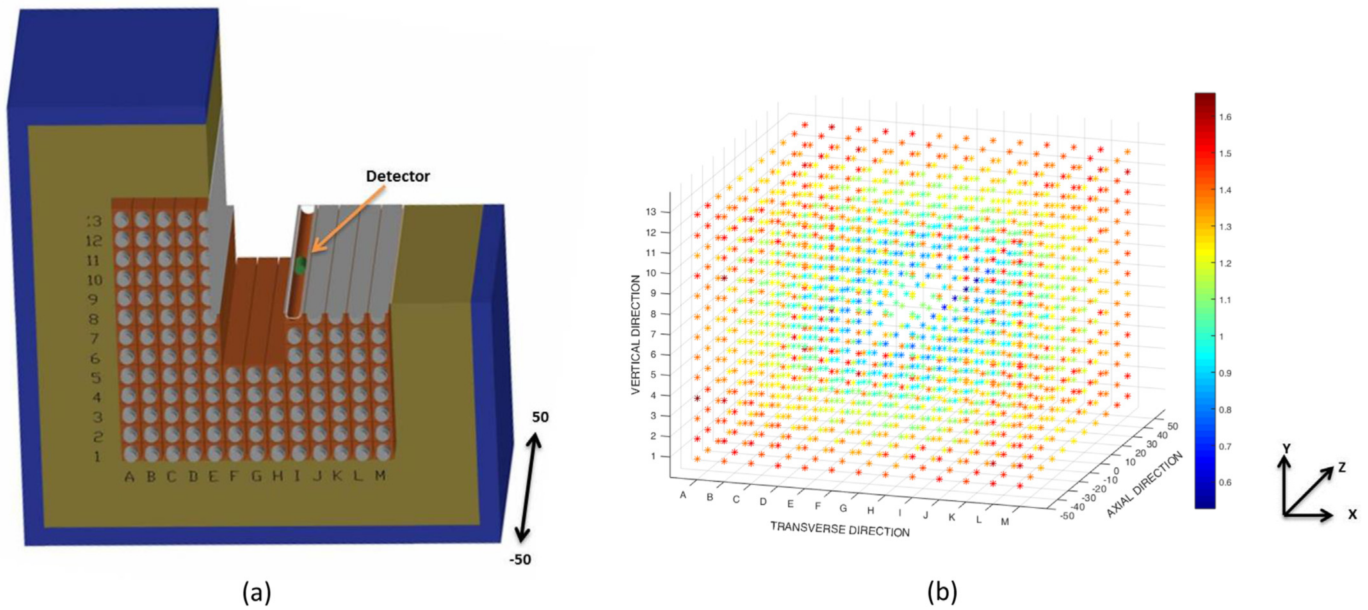

Fig. 3. A 3 dimensional map of spatial correction factors (a). Schematic of core showing detector location for a single measurement (b) All 1760 values have been depicted. Box represents the Core.

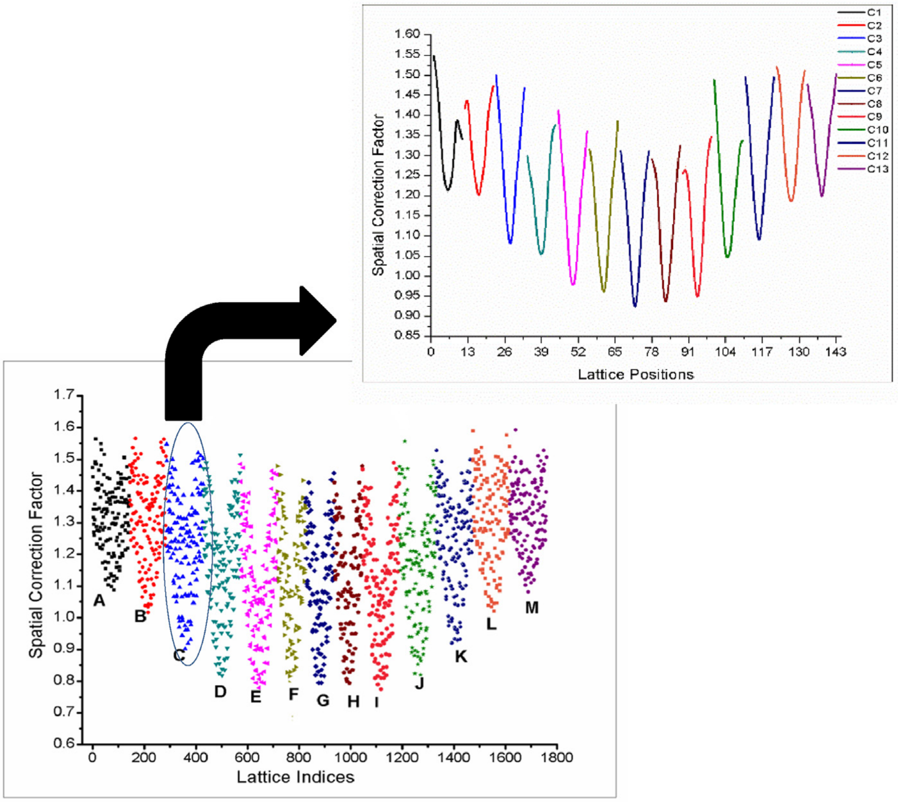

Fig. 4. Spatial correction factors for full core: **(inset)** Spatial correction factor values for a single plane C.

5. Experimental validation

Fig. 4 shows the correction factor values at all 1760 locations. We can see layer by layer values as we move radially through the core. Layers F, G and H are closest to the source hence report many correction factor values quite less than one. Inset shows the enlarged view of values for a single layer showing all values in transverse direction. It can be seen that for a single layer C, C6–C7–C8, have many correction factor values less than one as they are close to source.

The main objective of full 3D mapping is to find out locations where the spatial correction factor is very close to unity. These

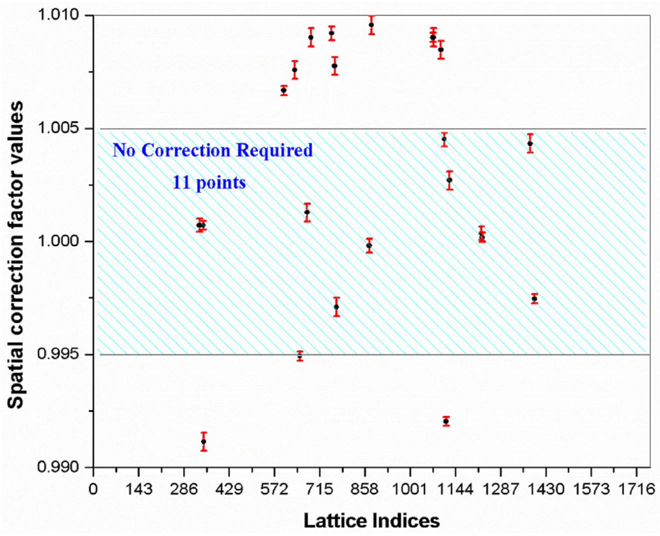

Fig. 5. Spatial correction factors for locations in lattice where no or minimal correction is required.

locations signify the detector positions where no correction is required or correction has very little influence on experimentally measured reactivity. The criterion for no correction has been taken as a difference of less than 50 pcm between measured corrected reactivity values which corresponds to correction factor values in the range of 0.996–1.004. For simplicity we take the interval to 0.995–1.005 (a 1% total spread around 1). Fig. 5 shows the lattice positions where no correction is required. This whole exercise helped in identifying 11 detector locations where no correction is required. All the values presented here have been reported with less than 0.4% error. As shown in Fig. 5 only the positions inside the margin of 0.995–1.005 have been considered as positions of zero corrections. The point lying at the boundary has been omitted.

In order to validate the simulations, experimental measurement of reactivity using Area Ratio method (Sjöstrand) were carried out at 5 of the 11 locations identified. For experimental studies the subcritical core BRAHMMA has been coupled to a neutron generator which emits a neutron beam of energy 14.1 MeV via D-T reaction. For the experiment, the fuel rod from the particular lattice location under investigation is removed and replaced with the detector. Helium-3 gas filled detector (diameter 25 mm, active length 100 mm, pressure 4 atm, operating voltage 1100–1250 V) is used for the experiment. Fig. 6 shows the time histogram for the reactivity measurements. The experimental results have been tabulated in Table 1. From the table, it is quite evident that values of reactivity obtained are very close to theoretical value of −17.73$ corresponding to k_{eff} value of 0.889 (corresponding to 159 fuel rods). The advantage of this exercise is that now experiments can be done at these detector positions without worrying about the spatial dependence arising because of the presence of source at center.

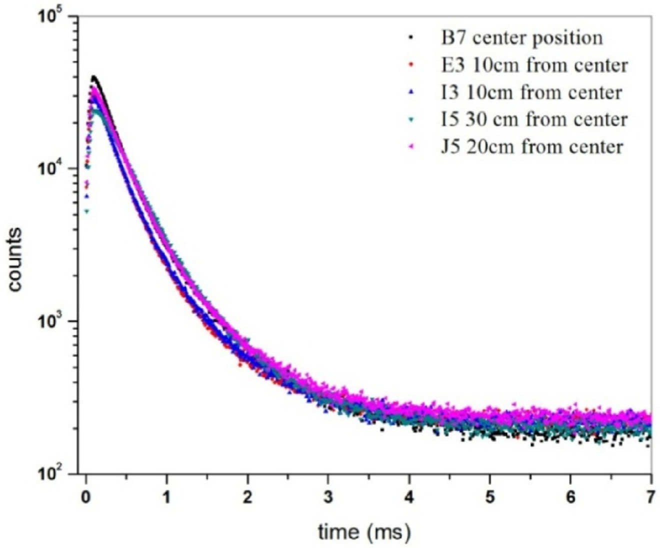

Fig. 6. Time histogram for some of the detector locations where no correction is required.

Table 1
Experimental Measurements on few detector locations where no correction is required.

Lattice Position	Axial Distance from Centre (cm)	Correction Factor Value	Experimentally Measured Reactivity Value using Area Ratio Method
B7	0	1.003	−17.74
E3	10	1.000	−17.72
I3	10	0.997	−17.69
J5	20	1.003	−17.72
I5	30	0.996	−17.75

6. Conclusion

Sjöstrand Area Ratio Method is one of the most popular techniques for reactivity measurement in accelerator driven subcritical systems. However, the measured reactivity suffers from spatial dependence and must be corrected using Bell and Glasstonne spatial correction factor. In this paper, a full 3D map of spatial correction factors was evaluated for the BRAHMMA core. We were able to locate positions where no correction is required. To validate the simulations, we performed experiments to measure reactivity at few of the identified locations. The results obtained are in good agreement with the theoretical values.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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