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On the Gauss Map of Tubular Surfaces in Pseudo Galilean 3-Space

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Abstract. In this study, we define tubular surfaces in Pseudo Galilean 3-space as type-1 or type-2. Using the $X(s,t)$ position vectors of the surfaces and $G(s,t)$ Gaussian transformations, we obtain equations for the two types of tubular surfaces that satisfy the conditions $\Delta X(s,t) = 0$, $\Delta X(s,t) = AX(s,t)$, $\Delta X(s,t) = \lambda X(s,t)$, $\Delta X(s,t) = \Delta G(s,t)$, $\Delta G(s,t) = 0$, $\Delta G(s,t) = AG(s,t)$ and $\Delta G(s,t) = \lambda G(s,t)$.

1. Introduction

Due to their physical importance in curve and surface theory, Galilean and Pseudo Galilean geometries have been widely studied in recent years. The Cayley Klein geometry with projective signature $(0,0,+,-)$ is an example of a Pseudo Galilean geometry, for detailed information see [5]. The absolute structure of a Pseudo Galilean geometry is represented by an ordered triple $\{w, f, I\}$ consising of its ideal plane w, a line f in w and the fixed hyperbolic involution I of points of f. Pseudo Galilean 3-space, denoted as G_3^1, is equipped with the scalar product g defined by

$$g(X,Y) = \begin{cases} x_1 y_1 & if \quad x_1 \neq 0 \ \vee y_1 \neq 0 \\ x_2 y_2 - x_3 y_3 & if \quad x_1 = 0 \ \wedge y_1 = 0. \end{cases}$$

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for any vectors $X=(x_1,x_2,x_3)$, $Y=(y_1,y_2,y_3)\in G_3^1$. The Pseudo Galilean norm of a vector X defined by

$$\|X\| = \begin{cases} x_1 & if \quad x_1 \neq 0 \\ \sqrt{(x_2)^2-(x_3)^2} & if \quad x_1 = 0\ . \end{cases}$$

A vector $X=(x_1,x_2,x_3)$ in Pseudo Galilean 3-space is called a non-isotropic vector if $x_1\neq 0$, and is otherwise X is called an isotropic vector. The cross product is defined by

$$X\wedge_{G_3^1} Y = \begin{cases} \begin{vmatrix} 0 & -e_2 & e_3 \\ x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{vmatrix} & if \quad x_1\neq 0\ \vee y_1\neq 0 \end{cases}$$

All unit non-isotropic vectors are of the form $(1,x_2,x_3)$. The vector X is called an isotropic space-like vector if $(x_2)^2-(x_3)^2>0$ satisfies and X is called an isotropic time-like vector if $(x_2)^2-(x_3)^2<0$ satisfies. If $(x_2)^2-(x_3)^2=0$ then X is called an isotropic lightlike vector, in this case $x_2=\pm x_3$. If $(x_2)^2-(x_3)^2=\pm 1$ then X is called a non-lightlike isotropic vector [6, 1, 3]. A curve $\gamma: I\subset\mathbb{R}\longrightarrow G_3^1$ defined by $\gamma(s)=(x(s),y(s),z(s))$ is an admissible curve if none of the points are inflection points, all the tangents and the normal vectors are non-isotropic at each points of the curve. If the curve $\gamma(s)$ is an admissible curve with the arc length parameter s then the position vector of $\gamma(s)$ is

$$\gamma(s)=(s,p(s),q(s))\,. \tag{1.1}$$

The curvature $\kappa(s)$ and the torsion $\tau(s)$ are defined by

$$\kappa(s)=\sqrt{\left|(p''(s))^2-(q''(s))^2\right|}\,,\quad \tau(x)=\frac{p''(s)q'''(s)-p'''(s)q''(s)}{\kappa^2(s)}\ . \tag{1.2}$$

An admissible curve has no inflection points, no isotropic tangents or normals whose projections on the absolute plane would be light-like vectors. The Frenet trihedron is given by

$$\begin{aligned} T(s) &= \gamma'(s)=(1,p'(s),q'(s)) \\ N(s) &= \tfrac{1}{\kappa(s)}\,(0,p''(s),q''(s)) \\ B(s) &= \tfrac{1}{\kappa(s)}\,(0,\epsilon q''(s),\epsilon p''(s))\,. \end{aligned} \tag{1.3}$$

where $\epsilon=\mp 1$, under the condition $\det(T,N,B)=1$. This requires that

$$\left|(p''(s))^2-(q''(s))^2\right| = \epsilon\left((p''(s))^2-(q''(s))^2\right).$$

Thus the principal normal vector, or simply normal, is space-like if $\epsilon = 1$ and time-like if $\epsilon = -1$. The curve γ given by (1.1) is time-like (resp. space-like) if $N(s)$ is a space-like (resp. time-like) vector. The following Serret-Frenet formulas hold

$$T'(s) = \kappa(s)N(s), \quad N'(s) = \tau(s)B(s), \quad B'(s) = \tau(s)N(s) \tag{1.4}$$

for derivatives of the tangent vector $T(s)$, the normal vector $N(s)$ and the binormal vector $B(s)$, respectively [6, 1, 7, 3]. Karacan and Tunçer studied Weingarten and linear Weingarten type tubular surfaces in Galilean and Pseudo Galilean spaces[4]. They also studied also surfaces in the same spaces[8]. D.W. Yoon, studied the Gauss Map of Tubular Surfaces in Galilean space and classified them in [9]. For an open subset $D \subseteq R^2$ and for a C^r-immersion $X : D \to G_3^1$, the set $\Phi = X(D)$ is called a regular C^r-surface(for $r \geq 2$) in Pseudo Galilean 3-space. If X is a C^r-embedding then the set Φ is called a simple C^r-surface(for $r \geq 2$). If the C^r-surface Φ does not have pseudo-Euclidean tangent planes then Φ is called admissible C^r-surface. Let us denote

$$X = X(x(u_1, u_2), y(u_1, u_2), z(u_1, u_2)),$$

and

$$x_{,i} = \frac{\partial x}{\partial u_i},\ y_{,i} = \frac{\partial y}{\partial u_i},\ z_{,i} = \frac{\partial z}{\partial u_i}$$

then Φ is an admissible surface if and only if $x_{,i} \neq 0$ for some $i = 1, 2$. Assume that $\Phi \subset G_3^1$ is a regular admissible surface. The unit normal vector field of Φ is

$$\eta(u, v) = \frac{(0, x_1z_2 - x_2z_1, x_1y_2 - x_2y_1)}{W(u, v)},$$

where $W(u, v) = \sqrt{\left|(x_1y_2 - x_2y_1)^2 - (x_1z_2 - x_2z_1)^2\right|}$. The function $W(u, v)$ is equal to the Pseudo Galilean norm of the isotropic vector $x_{,1}X_{,2} - x_{,2}X_{,1}$. The vector

$$\rho(u, v) = \frac{(x_{,1}X_{,2} - x_{,2}X_{,1})}{W}$$

is called a side tangential vector. Throughout the study we will consider the surfaces with $W \neq 0$ [8, 10]. Since we have $g(\eta,\ \eta) = \epsilon = \pm 1$, we consider two types of admissible surfaces: space-like surfaces having time-like surface normals ($\epsilon = -1$), and time-like surfaces having space-like normals ($\epsilon = 1$). The first fundamental form (F.F.F.) of a surface in G_3^1 is defined by

$$ds^2 = (g_1du_1 + g_2du_2)^2 + \delta(h_{11}du^2 + 2h_{12}dudv + h_{22}dv^2),$$

where

$$g_i = x_{,i}, \tag{1.5}$$

$$h_{ij} = g\left(\widetilde{X}_{,i}, \widetilde{X}_{,j}\right) \tag{1.6}$$

and

$$\delta = \begin{cases} 0 & ; \text{ if direction } \ du_1 : du_2 \text{ is non-isotropic} \\ 1 & ; \text{ if direction } du_1 : du_2 \text{ is isotropic.} \end{cases}$$

[8, 10]. For a vector x, $\widetilde{x}$ denotes the projection the vector onto the pseudo-Euclidean plane yoz.

In this study, we denote the components of ds^2 by $\widetilde{g}_{ij}$. Furthermore, according to the local coordinate system $\{u_1, u_2\}$ of the surface $X(u, v)$ the Laplacien operator Δ of the F.F.F. is defined by

$$\Delta = \frac{1}{\sqrt{|\det[\widetilde{g}_{ij}]|}} \sum_{i,j=1}^{2} \frac{\partial}{\partial u_i} \left(\sqrt{|\det[\widetilde{g}_{ij}]|} \widetilde{g}^{ij} \frac{\partial}{\partial u_j} \right), \tag{1.7}$$

where $\left[\widetilde{g}^{ij}\right] = [\widetilde{g}_{ij}]^{-1}$ [9, 10].

2. Tubular Surfaces in Pseudo Galilean 3-Space

In this section, we will classify the admissible tubular surfaces in G_3^1 satisfying the equations $\Delta X = 0$, $\Delta X = AX$, $\Delta X = \lambda X$, $\Delta X = \Delta G$, $\Delta G = 0$, $\Delta G = AG$ and $\Delta G = \lambda X$ where X is the position vector of tubular surface, G is the Gauss map of tubular surface, λ is nonzero constant, $A \in Mat(3, \mathbb{R})$ and Δ is the Laplacien operator of the surface. Y.Tunçer and M.K.Karacan defined the canal surfaces in Pseudo Galilean 3-Spaces in [8]. Generalising this, we definition tubular surfaces in pseudo galilean 3-space. Let $\gamma : (a, b) \to G_3^1$ be an admissible curve satisfying (1.1), and let M be a tubular surface with the centered curve $\gamma(s)$. There are two types non-isotropic tubular surfaces in G_3^1.

Type-1: If M is space-like (time-like) tubular surface and $\gamma(s)$ is space-like (time-like) curve then M is parametrized by

$$X^{\mu}(s, t) = \gamma(s) + r\cosh(t) N(s) + r\sinh(t) B(s), \tag{2.1}$$

$$\mu = \begin{cases} +1 & \text{if } M \text{ is a space-like canal surface with space-like centered curve} \\ -1 & \text{if } M \text{ is a time-like canal surface with time-like centered curve.} \end{cases}$$

Type-2: If M is space-like (time-like) tubular surface and $\gamma(s)$ is time-like (space-like) curve then M is parametrized by

$$X^{\sigma}(s, t) = \gamma(s) + r\sinh(t) N(s) + r\cosh(t) B(s), \tag{2.2}$$

$$\sigma = \begin{cases} +1 & \text{if } M \text{ is a space-like canal surface with time-like centered curve} \\ -1 & \text{if } M \text{ is a time-like canal surface with space-like centered curve.} \end{cases}$$

Let M be a type-1 tubular surface in G_3^1 is parametrized by (2.1), then we have the natural frame $\{X_s^{\mu}, X_t^{\mu}\}$ of M given by

$$\begin{aligned} X_s^{\mu}(s, t) &= T(s) + r\tau(s)\sinh(t) N(s) + r\tau(s)\cosh(t) B(s) \\ X_t^{\mu}(s, t) &= r\sinh(t) N(s) + r\cosh(t) B(s) \end{aligned}$$

and from (1.4), (1.5) and (1.6), we have

$$g_1 = 1 \ , \ g_2 = 0 \ , \ h_{11} = \mu r^2 \tau(s)^2 \ , \ h_{21} = h_{12} = \mu r^2 \tau(s) \ , \ h_{22} = \mu r^2$$

which are the components of F.F.F., so we obtain the $\widetilde{g}_{ij}$ as

$$\widetilde{g}_{11} = 1 + \mu r^2 \tau(s)^2 \ , \widetilde{g}_{12} = \widetilde{g}_{21} = \mu r^2 \tau(s) \ , \widetilde{g}_{22} = \mu r^2.$$

By a direct computation using the equation (1.7), the Laplacian operator Δ on M is

$$\Delta = \frac{\left(\mu r^2 \tau(s)^2 + 1\right)}{\mu r^2} \frac{\partial^2}{\partial t^2} - \tau'(s) \frac{\partial}{\partial t} - 2\tau(s) \frac{\partial^2}{\partial t \partial s} + \frac{\partial^2}{\partial s^2}. \tag{2.3}$$

Suppose that M satisfies $\Delta X^\mu(s,t) = AX^\mu(s,t)$, with the matrix $A \in Mat(3,\mathbb{R})$, then from (2.1) and (2.3) we obtain the equality

$$\frac{1}{\mu r}\left\{(\mu r \kappa(s) + \cosh(t)) N(s) + \sinh(t) B(s)\right\} = A\gamma(s) + r\cosh(t) AN(s) + r\sinh(t) AB(s), \tag{2.4}$$

so it is easy to see that the equality $\Delta X^\mu(s,t) = 0$ is not satisfied for type-1 tubular surface. Hence we give the following theorem.

Theorem 2.1. *There is not any harmonic type-1 tubular surface given by (2.1) in G_3^1.*

For the other cases, we give the following theorem.

Theorem 2.2. *Let M be a type-1 tubular surface given by (2.1) in G_3^1. M satisfies $\Delta X^\mu(s,t) = AX^\mu(s,t)$, $A \in Mat(3,\mathbb{R})$ if*

$$2\kappa'(s)\tau(s) + \kappa(s)\tau'(s) = 0 \ , \ \frac{\kappa''(s)}{\kappa(s)} + \tau(s)^2 = \frac{1}{\mu r^2}. \tag{2.5}$$

Proof. Differentiating (2.1) with respect to t we get

$$(\Delta X^\mu(s,t))_t = \frac{1}{\mu r}\left\{(\sinh(t)) N(s) + \cosh(t) B(s)\right\} = r\sinh(t) AN(s) + r\cosh(t) AB(s). \tag{2.6}$$

Taking the derivative (2.6) with respect to t, we have

$$(\Delta X^\mu(s,t))_{tt} = \frac{1}{\mu r}\left\{\cosh(t) N(s) + \sinh(t) B(s)\right\} = r\cosh(t) AN(s) + r\sinh(t) AB(s). \tag{2.7}$$

Combining (2.6) and (2.7) we can obtain the following two equation

$$AN(s) = \frac{1}{\mu r^2} N(s), \tag{2.8}$$

$$AB(s) = \frac{1}{\mu r^2} B(s). \tag{2.9}$$

On the other hand, from (2.4), (2.7) and (2.8)

$$A\gamma(s) = \kappa(s) N(s) \tag{2.10}$$

and differentiating (2.9) with respect to s and by using (2.7) and (2.8), we have

$$AT(s) = \kappa'(s) N(s) + \kappa(s)\tau(s) B(s). \tag{2.11}$$

By taking the derivative (2.11) with respect to s , we get

$$(AT(s))' = \left(\kappa''(s) + \kappa(s)\tau(s)^2\right) N(s) + \left(\kappa'(s)\tau(s) + (\kappa(s)\tau(s))'\right) B(s). \tag{2.12}$$

By considering $(AT(s))' = AT'(s) = \kappa(s) AN(s)$ in (2.12) and from (2.8), we obtain

$$2\kappa'(s)\tau(s) + \kappa(s)\tau'(s) = 0 \quad , \quad \frac{\kappa''(s)}{\kappa(s)} + \tau(s)^2 = \frac{1}{\mu r^2}. \tag{2.13}$$

Thus, this completes the proof. □

From the first equation of (2.5), we can obtain

$$\kappa^2(s) = \frac{a}{\tau(s)} \tag{2.14}$$

and by using the second equation of (2.5), we have

$$\frac{\kappa''(s)}{\kappa(s)} + \frac{a}{\kappa^4(s)} = \frac{1}{\mu r^2}. \tag{2.15}$$

Equation (2.15) has complex solutions but in the case of $\kappa(s)$ and $\tau(s)$ are constant, then (2.15) has the real solution. Thus we can give following corollary as a remark of theorem 2.2.

Corollary 2.3. *Let M be a type-1 tubular surface given by (2.1) in G_3^1. If M satisfies*

$$\Delta X^\mu(s,t) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{\mu r^2} & 0 \\ 0 & 0 & \frac{1}{\mu r^2} \end{bmatrix} X^\mu(s,t)$$

then M is one of the following.

i. *M is a type-1 surface determined by*

$$X^\mu(s,t) = \big(s, c_1 s^2 + c_2 s + c_3 + 2rc_1\cosh(t) + 2rd_1\sinh(t)$$
$$, d_1 s^2 + d_2 s + d_3 + 2rd_1\cosh(t) + 2rc_1\sinh(t)\big)$$

where $d_1 \neq 0, c_1 \neq 0, c_2, c_3, d_2, d_3$ are constants (for time-like centered curve $c_1 > d_1$ and for space-like centered curve $c_1 < d_1$, see Figures 1 and 2).

ii. *M is a type-1 surface determined by*

$$X^{\mu}(s,t) = \left(s, c_1 s^2 + c_2 s + c_3 + 2rc_1\cosh(t), d_1 s + d_2 + 2rc_1\sinh(t)\right)$$

or

$$X^{\mu}(s,t) = \left(s, c_1 s + c_2 + 2d_1 r\sinh(t), d_1 s^2 + d_2 s + d_3 + 2d_1 r\cosh(t)\right)$$

where $d_1 \neq 0, c_1 \neq 0, c_2, c_3, d_2, d_3$ are constants (see Figures 3 and 4).

iii. *M is a type-1 space-like surface determined by*

$$X^{\mu}(s,t) = \left(s, r\sqrt{a^2-1}\cosh(t) + ra\sinh(t), ra\cosh(t) + r\sqrt{a^2-1}\sinh(t)\right)$$

where $a > 1$ is constant (see Figure 3). The Gauss map G of type-1 tubular surface M is

$$G(s,t) = \cosh(t)N(s) + \sinh(t)B(s)$$

and from (2.3), we find

$$\Delta G(s,t) = \frac{1}{\mu r^2}\left\{\cosh(t)N(s) + \sinh(t)B(s)\right\}. \tag{2.16}$$

Thus, it is easy to see that, following theorem holds.

Theorem 2.4. *Let M be a type-1 tubular surface given by (2.1) in G_3^1 then followings are true.*

i. *There are no type-1 tubular surface given by (2.1) in G_3^1 with the Gauss map G being harmonic.*

ii. *All type-1 tubular surfaces satisfy $\Delta G(s,t) = \lambda G(s,t)$, $\lambda \neq 0$.*

iii. *All type-1 tubular surface satisfy $\Delta G(s,t) = AG(s,t)$ where $A = \frac{1}{\mu r^2} I_3$.*

As a result of Theorem 2.4., we can say M has a type-1 Gauss map $G(s,t)$ in the sense of Chen [2].

Assume that M satisfy $\Delta X^{\mu}(s,t) = \Delta G(s,t)$. From (2.3) and (2.16)

$$\frac{1}{\mu r^2}\left\{(\mu r\kappa(s) + \cosh(t))N(s) + \sinh(t)B(s)\right\} = \frac{1}{\mu r^2}\left\{\cosh(t)N(s) + \sinh(t)B(s)\right\}, \tag{2.17}$$

and so $\kappa = 0$. Thus we get following theorem.

Theorem 2.5. *Let M be a type-1 space-like tubular surface given by (2.1) in G_3^1, then M satisfies $\Delta X^{\mu}(s,t) = \Delta G(s,t)$ if its position vector is*

$$X^{\mu}(s,t) = \left(s, r\sqrt{a^2-1}\cosh(t) + ra\sinh(t), ra\cosh(t) + r\sqrt{a^2-1}\sinh(t)\right)$$

where $a > 1 \in \mathbb{R}$ and $r > 0$.

Let M be a type-2 tubular surface G_3^1 parametrized by (2.2), then we have the natural frame $\{X_s^\sigma(s,t), X_t^\sigma(s,t)\}$ of M given by

$$X^\sigma(s,t) = \gamma(s) + r(s)\sinh(t)N(s) + r(s)\cosh(t)B(s) \tag{2.18}$$

$$\begin{aligned} X_s^\sigma(s,t) &= T(s) + r\tau(s)\cosh(t)N(s) + r\tau(s)\sinh(t)B(s) \\ X_t^\sigma(s,t) &= r\cosh(t)N(s) + r\sinh(t)B(s) \end{aligned}$$

and we have

$$g_1 = 1\ ,\ g_2 = 0\ ,\ h_{11} = \sigma r^2(s)\tau(s)^2\ \ ,\ h_{12} = \sigma r^2(s)\tau(s)\ \ ,\ h_{12} = \sigma r^2(s)$$

which are the components of F.F.F.

$$\widetilde{g}_{11} = 1 + \sigma r^2(s)\tau(s)^2\ \ ,\ \widetilde{g}_{12} = \widetilde{g}_{21} = \sigma r^2(s)\tau(s)\ \ ,\ \widetilde{g}_{11} = \sigma r^2(s)$$

and the Laplacian operator Δ on M is obtained as

$$\Delta = \left\{ \frac{\left(\sigma r^2\tau(s)^2 + 1\right)}{\sigma r^2}\frac{\partial^2}{\partial t^2} - \tau'(s)\frac{\partial}{\partial t} - 2\tau(s)\frac{\partial^2}{\partial t\partial s} + \frac{\partial^2}{\partial s^2} \right\}. \tag{2.19}$$

The Gauss map $G(s,t)$ of type-1 tubular surface M is

$$G(s,t) = \sinh(t)N(s) + \cosh(t)B(s) \tag{2.20}$$

and Laplacians of $X^\sigma(s,t)$ and $G(s,t)$ are

$$\Delta X^\sigma(s,t) = \frac{1}{\sigma r}\{(\sigma r\kappa(s) + \sinh(t))N(s) + \cosh(t)B(s)\} \tag{2.21}$$

and

$$\Delta G(s,t) = \frac{1}{\sigma r^2}\{\sinh(t)N(s) + \cosh(t)B(s)\} \tag{2.22}$$

respectively. We can also obtain similar results for type-2 surfaces in G_3^1 by using (2.18), (2.21) and (2.22).

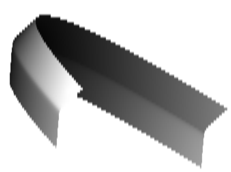
Example 2.2. *Timelike tube with time-like centered curve satisfying* $\Delta X^\mu(s,t) = AX^\mu(s,t)$ *where* $A = \frac{1}{-4}I_3$

$$X^\mu(s,t) = \left(s, 2s^2 + 2s + \frac{2\cosh(t) + \sinh(t)}{3}, s^2 + s + 1 + \frac{\cosh(t) + 2\sinh(t)}{3}\right)$$

(a) Figure 1

Spacelike tube with space-like centered curve satisfying $\Delta X^{\mu}(s,t) = AX^{\mu}(s,t)$ *where* $A = \frac{1}{4} I_3$

$$X^{\mu}(s,t) = \left(s, s^2 + 2s + \frac{\cosh(t) - 2\sinh(t)}{3}, 2s^2 + s + 1 + \frac{2\cosh(t) - \sinh(t)}{3} \right)$$

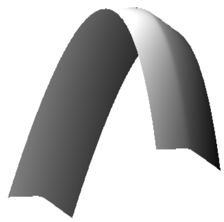


(b) Figure 2

Timelike tube with time-like centered curve satisfying $\Delta X^{\mu}(s,t) = AX^{\mu}(s,t)$ *where* $A = \frac{1}{-4} I_3$

$$X^{\mu}(s,t) = \left(s, 2s^2 + 2s + 1 + \frac{1}{2}\cosh(t), s + 1 + \frac{1}{2}\sinh(t) \right)$$

Spacelike tube with space-like centered curve satisfying $\Delta X^{\mu}(s,t) = AX^{\mu}(s,t)$



(c) Figure 3

where $A = \frac{1}{4}I_3$

$$X^{\mu}(s,t) = \left(s, 2s+1-\frac{1}{2}\sinh(t), 2s^2+s+1+\frac{1}{2}\cosh(t)\right)$$

(d) Figure 4

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References

[1] M. Akyigit and A. Z. Azak, *Admissible Mannheim Curves in Pseudo Galilean Space* G_3^1, Afr. Diaspora J. Math., **10(2)**(2010), 58–65.

[2] B. Y. Chen, *A report on submanifold of finite type*, Soochow J. Math., **22**(1996), 117–337

[3] B. Divjak, *Curves in Pseudo Galilean Geometry*, Ann. Univ. Sci. Budapest. Eötvös Sect. Math., **41**(1998), 117–128.

[4] M. K. Karacan and Y. Tunçer, *Tubular Surfaces of Weingarten Types in Galilean and Pseudo Galilean Spaces*, Bull. Math. Anal. Appl., **5(2)**(2013), 87–100.

[5] E. Molnar, *The projective interpretation of the eight 3-dimensional homogeneous geometries*, Beiträge Algebra Geom., **38**(1997), 261–288.

[6] H. B. Oztekin, *Weakened Bertrand curves in The Galilean Space* G_3^1, J. Adv. Math. Stud., **2(2)**(2009), 69–76.

[7] H. Öztekin, H. Bozok, *Position vectors of admissible curves in 3-dimensional Pseudo Galilean space* G_3^1, Int. Electron. J. Geom., **8(1)**(2015), 21–32.

[8] Y. Tunçer and M. K. Karacan, *Canal Surfaces in Pseudo-Galilean 3-Spaces*, Kyungpook Math. J. **60(2)**(2020), 361–373.

[9] D. W. Yoon, *On the Gauss Map of Tubular Surfaces in Galilean 3-space*, Int. J. Math. Anal., **8(45)**(2014), 2229–2238.

[10] D. W. Yoon, *Surfaces of revolution in the three dimensional Pseudo Galilean space*, Glas. Mat. Ser. III, **48**(2013), 415–428.