J. Appl. Math. & Informatics Vol. **40***(2022), No. 5 - 6, pp. 857 - 871*
https://doi.org/10.14317/jami.2022.857

ANALYSIS OF THE MITIGATION STRATEGIES FOR MARRIAGE DIVORCE: FROM MATHEMATICAL MODELING PERSPECTIVE

HAILEYESUS TESSEMA*, YEHUALASHET MENGISTU AND ENDESHAW KASSA

Abstract. In this work, we formulated a mathematical model for divorce in marriage and extended in to an optimal control model. Firstly, we qualitatively established the model positivity and boundedness. Also we saw sensitivity analysis of the model and identified the positive and negative indices parameters. An optimal control model were developed by incorporating three time dependent control strategies (couple relationship education, reducing getting married too young & consulting separators to renew their marriage) on the deterministic model. The Pontryagin's maximum principle were used for the derivation of necessary conditions of the optimal control problem. Finally, with Newton's forward and backward sweep method numerical simulation were performed on optimality system by considering four integrated strategies. So that we reached to a result that using all three strategies simultaneously (the strategy D) is an optimal control in order to effectively control marriage divorce over a specified period of time. From this we conclude that, policymakers and stakeholders should use the indicated control strategy at a time in order to fight against Divorce in a population.

AMS Mathematics Subject Classification : 34A34, 65L05.
Key words and phrases : Marriage divorce, optimal control theory, Pontryagin's maximum principle, numerical simulation.

1. Introduction

In a given society, family is the basic structure that serves the main function to meet the needs and necessities of its member and society and the road to enter family life is marriage [2]. Marriage is union of couples commit to one

Received May 29, 2021. Revised November 19, 2021. Accepted March 23, 2022. *Corresponding author.

another with the expectation of stable and lasting intimate relationship, and this is socially recognized and approved [3].

Within the road of a family, there may exist a conflict due to different reasons that leads to separation and may reach to divorce. In England and Wales, the number of divorces increased from year to year [4] and also it has become a common part of the American life and it affects children of every ethnic background, religion, and socioeconomic status [5]. In Spain, the difference in happiness within the couple, economy has a high impact on divorce [6]. Divorce in Africa is a persistent event which effects into immediate and a continuous results [7, 8]. Divorce cases have increased among young people in South Africa which is amongst the highest in the world [9].

The major causes of divorce are early age of first marriage and childlessness within the first marriage, sexual incompatibility, marital infidelity (cheating), financial stress (poor or negative growth) [9, 10, 11]. Effective ways to fight against divorce in marriage are couple relationship education, reducing getting married too young, Consulting separators to renew their marriage (pre-divorce and post-divorce) [28, 1]. Divorce has negative impact on social, cultural, economic, psychological and political effects [12, 13] and divorce affects all the children [2, 7, 13, 17]. Stable families are creating a prosperous nation and a stable world, while poor families are breeding weak, corrupt and result a country and globe in disarray. This must first be decided out of the family for a country and region or the world at large to be at peace [7]. However, little attention has been given to the causes of divorce with the negative impacts on divorced partners and their children; especially in low income countries [2, 7].

Mathematical modeling and optimal control theory plays an important role in understanding the dynamics of infectious diseases and to investigate the optimal use of intervention strategies in making an appropriate decision regarding the intervention strategies [18]. we propose a social epidemiology model of divorce regarded as a social disorder propagated by divorced ones. Limited research has been done on the mathematical modeling of marriage divorce as a social epidemiology. For example, the study by [19], a non-linear MSD (Married-Separated-Divorced) mathematical model to study the dynamics of divorce epidemic in Ghana was formulated and analyzed. The existence and stability of the divorce free and endemic equilibrium was proved using the computed basic reproduction number. They finally concluded that, reducing the contact rate between the married and divorced, increasing the number of marriage that go into separation and educating separators to refrain from divorce can be useful in combating the divorce epidemic. Similarly, a mathematical modeling of divorce propagation allowing the estimation of the future divorced population was developed and analyzed by [6]. They used a coupled discrete linear-quadratic difference system model. A sensitivity analysis of the growth of the divorced population with respect to the contagion rate is included, and several different economic scenarios.

A discrete model of the marital status of the family dynamics also studied by [21]. They determined two controls: awareness campaign to educate virgin men and women about the benefits of marriage and legal procedures, administrative complications and the heavy financial and social consequences of divorces which allow to reduce the number of virgin, divorced individuals and increase the number of married individuals. A social epidemiology model of divorce regarded as a social disorder propagated by divorced ones were used by researchers [6, 19, 21, 22]. As a transmitted disease a married individual in a population become divorced when they have a contact with divorced individuals or persons who are contacting that individual and feed bad information , which leads unstable to their marriage and grows to divorce. Adapting the concept of social epidemiology on divorce, we proposed a mathematical model of marriage divorce with an optimal control considering divorce as an epidemic disease that disseminate by divorced women/ man over married ones.

The paper is organized as follows: In section 2, we present the a baseline model description and formulation. In section 3, we performed the model analysis of the invariant region and positivity of the solution for positive time and the sensitivity of model parameters. In Section 4, we formulated an optimal control model and used Pontryagin's maximum principle to perform the analysis of control strategies and to determine the necessary condition for the optimal control. Numerical simulations are given in 5. and we conclude the paper in Section 6.

2. Baseline Model Formulation

In the introduced model the entire population is divided into four subpopulations: those who reach the age of getting married are single and susceptible individuals, $S(t)$; those who are couple are called married individuals, $M(t)$; those who separate but not divorced are broken marriage individuals, $B(t)$ and those who are separated are called divorced marriage, $D(t)$.

Π is the requirement rate of individuals being single when he/she reaches the age of getting married. These individuals got married at rate of β. The married individuals got broken and move to the broken compartment due to the contact with the divorced individuals and passed the law of marriage at a rate of α. The broken marriage recovered from their conflicts and renew their marriage at a rate of ϵ and live as the previous style. Some of these broken marriage got a permanent divorce at a rate of δ. This divorced people join the single subpopulation at a rate of ρ and some of them will die due to divorce at a rate of σ because they consider that they have lost everything so that they may consciously kill themselves with medicine, gun or any other mince. The whole population has μ as an average death rate. In addition we assume that sex, race and social status do not affect the probability of being divorced and members mix homogeneously (have the same interaction to the same degree). Table 1 shows the description of model parameters.

With regards to the above assumptions, the model is governed by the following system of differential equation:

$$\begin{aligned}\frac{dS}{dt} &= \Pi + \rho D - (\beta + \mu) S \\ \frac{dM}{dt} &= \beta S + \epsilon B - (\alpha D + \mu) M \\ \frac{dB}{dt} &= \alpha M D - (\delta + \epsilon + \mu) B \\ \frac{dD}{dt} &= \delta B - (\sigma + \rho + \mu) D\end{aligned} \tag{1}$$

With the initial condition

$$S(0) = S_0 \geq 0 \ , M(0) = M_0 \geq 0 \ , B(0) = B_0 \geq 0 \ , D(0) = D_0 \geq 0 \tag{2}$$

TABLE 1. Description of parameters of the marriage divorce model (1).

Parameter	**Description**
Π	Requirement rate of individuals to the age of susceptible
β	Average rate of susceptible individuals who got married
ρ	Rate of divorced individuals who become susceptible
σ	Death rate of individuals due to divorce
ϵ	Rate of broken individuals who renew their previous marriage
δ	Rate of broken individuals who got divorced
μ	Natural death rate of individuals
α	The contact rate of divorced individuals with married individuals

3. Model Analysis

3.1. Invariant Region and Positivity of Solutions. For this model the total population is $N(S, M, B, D) = S(t) + M(t) + B(t) + D(t)$. Then, differentiating N with respect to time we obtain:

$$\frac{dN}{dt} = \frac{dS}{dt} + \frac{dM}{dt} + \frac{dB}{dt} + \frac{dD}{dt} = \Pi - \sigma D - \mu N$$

If there is no death due to the divorce, we get

$$\frac{dN}{dt} \leq \Pi - \mu N \tag{3}$$

After solving equation (3) and evaluating it as $t \longrightarrow \infty$, we got

$$\Omega = \{(S, M, B, D) \varepsilon \mathcal{R}^4_+ : N(t) \leq \frac{\Pi}{\mu}\}$$

Which is the feasible solution set for the model (1) and all the solutions of the model are bounded.

3.2. Sensitivity Analysis. To go through sensitivity analysis, we used the normalized sensitivity index definition, defined in [25] and used by [22, 27].
Definition. The Normalized forward sensitivity index of a variable, $\mathcal{R}_0$, that depends differentiably on a parameter, p, is defined as:

$$\Lambda_p^{\mathcal{R}_0} = \frac{\partial \mathcal{R}_0}{\partial p} \times \frac{p}{\mathcal{R}_0}$$

for p represents all the basic parameters and $\mathcal{R}_0 = \frac{\alpha\beta\Pi\delta}{\mu(\mu+\beta)(\mu+\rho+\sigma)(\mu+\epsilon+\delta)}$ For the sensitivity index of $\mathcal{R}_0$ to the parameters:

$$\Lambda_\alpha^{\mathcal{R}_0} = \frac{\partial \mathcal{R}_0}{\partial \alpha} \times \frac{\alpha}{\mathcal{R}_0} = 1 \geq 0$$

$$\Lambda_\sigma^{\mathcal{R}_0} = \frac{\partial \mathcal{R}_0}{\partial \sigma} \times \frac{\sigma}{\mathcal{R}_0} = -\frac{\sigma}{\sigma+\rho+\mu} \leq 0$$

And it is similar with respect to the remaining parameters.

The sensitivity indices of the basic reproductive number with respect to main parameters are found in Table 2. Those parameters that have positive indices $(\alpha, \beta,\ and\ \delta)$ show that they have great impact on expanding the divorce in the community if their values are increased. Also those parameters in which their sensitivity indices are negative $(\epsilon, \rho, \sigma,\ and\ \mu)$ have an effect of minimizing the burden of the divorce in the community as their values increased. Therefore, policy makers, stakeholders should work on decreasing the positive indices and increasing negative indices parameters, which leads to an optimal control model.

TABLE 2. Sensitivity indecies table.

Parameter symbol	**Sensitivity indecies**
α	+ve
δ	+ve
ϵ	-ve
ρ	-ve
μ	-ve
σ	-ve

4. An Optimal control model

In this section, we used optimal control strategies to find an optimal intervention strategies for eradicating divorce in the in the specified time. The basic model of marriage divorce is generalized by incorporating three control interventions defined as:

(i) u_1 Couple relationship education.
(ii) u_2 Reducing getting married too young.

(iii) u_3 Consulting separators to renew their marriage (pre-divorce and post-divorce).

Incorporating the controls u_1, u_2 and u_3 in the original model, the optimal control model, is given by:

$$\begin{array}{rcl} \frac{dS}{dt} & = & \pi + (\rho + u_1)D - (\beta + u_1)\,S - \mu S \\ \frac{dM}{dt} & = & (\beta + u_1)S + (\epsilon + u_3)B - (1 - u_2)\alpha DM + \mu M \\ \frac{dB}{dt} & = & (1 - u_2)\alpha MD - (\delta + \epsilon + u_1 + u_3 + \mu)\,B \\ \frac{dD}{dt} & = & (1 - u_3)\delta B - (\sigma + \rho + u_1 + \mu)\,D \end{array} \tag{4}$$

The ultimate goal of introducing controls in the model is to find the optimal level intervention strategy preferred to minimize marriage divorce. Thus, we want to find the optimal values u_1, u_2 and u_3 that minimizes the objective functional given below subject to the optimal control model in equation (4)

$$J = \int_{t_0}^{t_f} \left[a_1 B + a_2 D + \frac{1}{2}(w_1 u_1^2 + w_2 u_2^2 + w_3 u_3^2) \right] dt \tag{5}$$

where t_f is the final time, a_1 and a_2 are coefficients of the broken marriage and divorced marriage respectively while w_1, w_2 and w_3 are coefficients for each individual control measure. We assume that there is no linear relation between the coverage of these interventions and their associated costs, so we use a quadratic cost for the controls, which is consistent with prior literature on epidemic cost control [16, 20]. Optimal control function (u_1^*,u_2^*, u_3^*) need to be found such that.

$$J(u_1^*, u_2^*, u_3^*) = min\{J(u_1, u_2, u_3)|(u_1, u_2, u_3) \in U\} \tag{6}$$

where $U = \{(u_1, u_2, u_3) \mid u_i(t)$ is measurable on $[0, t_f]$, $0 \le u_i(t) \le 1, i = 1, 2, 3\}$ is the closed set.

4.1. Characterization of optimal controls. To derive the conditions of optimality for the control problem, we apply the Pontryagin's Maximum Principle [29]. To obtain the optimality conditions, we formulate a problem of minimizing point-wise a Hamiltonian ($\mathcal{H}$) from the cost functional equation (5) and the governing dynamics equation (4). Converts the system in and into , with respect to $u_1(t), u_2(t)$ and $u_3(t)$ as

$$\begin{aligned} \mathcal{H} = {} & a_1 B + a_2 D \\ & + \frac{1}{2}(w_1 u_1^2 + w_2 u_2^2 + w_3 u_3^2) \\ & + \lambda_1[\pi + (\rho + u_1)D - (\beta + u_1 + \mu)\,S] \\ & + \lambda_2[(1 - u_1)\beta S + (\epsilon + u_1)B - (1 - u_2)\alpha DM + (u_1 + \mu)M] \\ & + \lambda_3[(1 - u_2)\alpha MD - (\delta + \epsilon + u_1 + u_3 + \mu)\,B] \\ & + \lambda_4[(1 - u_3)\delta B - (\sigma + \rho + u_1 + \mu)\,D] \end{aligned}$$

Where $\lambda_i, i = 1, .., 4$ are the adjoint variable functions to be obtained properly by applying Pontryagin's maximal principle [29].

Theorem 4.1. *For an optimal control set u_1, u_2, u_3 that minimizes J over U, there is an adjoint variables, $\lambda_1, ..., \lambda_4$ such that:*

$$\frac{d\lambda_1}{dt} = \lambda_1 (\beta + u_1 + \mu) - \lambda_2 (\beta + u_1)$$
$$\frac{d\lambda_2}{dt} = \lambda_2 ((1 - u_2) \alpha D + \mu) - \lambda_3 (1 - u_2) \alpha D$$
$$\frac{d\lambda_3}{dt} = -a_1 - \lambda_2 (\epsilon + u_3) + \lambda_3 (\delta + \epsilon + u_3 + u_1 + \mu) - \lambda_4 (1 - u_3) \delta$$
$$\frac{d\lambda_4}{dt} = -a_2 - \lambda_1 (\rho + u_1) + \lambda_2 (1 - u_2) \alpha M - \lambda_3 (1 - u_2) \alpha M + \lambda_4 (\rho + u_1 + \sigma + \mu)$$

With transversality conditions, $\lambda_i(t_f) = 0, i = 1, ..., 4$. Furthermore, we obtain the control set (u_1^*, u_2^*, u_3^*) characterized by

$$u_1^* = max\{0, min(1, \psi_1)\}, u_2^* = max\{0, min(1, \psi_2)\}, u_3^* = max\{0, min(1, \psi_3)\}$$

Where

$$\psi_1 = \frac{D (\lambda_4 - \lambda_1) + \lambda_3 B + S (\lambda_1 - \lambda_2)}{w_1}$$
$$\psi_2 = \frac{\alpha DM (\lambda_3 - \lambda_2)}{w_2}$$
$$\psi_3 = \frac{B (\lambda_3 + \delta\lambda_4 - \lambda_2)}{w_3}$$

Proof. The existence of an optimal control is guaranteed using the result by Fleming and Rishel [15]. The Pontryagin's Maximum Principle [29] establishes that the Co-state system is obtained by differentiating the Hamiltonian partially with respect to the State variables. Thus, we have :

$$\frac{d\lambda_1}{dt} = -\frac{\partial \mathcal{H}}{\partial S} = \lambda_1 (\beta + u_1 + \mu) - \lambda_2 (\beta + u_1)$$
$$\frac{d\lambda_2}{dt} = -\frac{\partial \mathcal{H}}{\partial M} = \lambda_2 ((1 - u_2) \alpha D + \mu) - \lambda_3 (1 - u_2) \alpha D$$
$$\frac{d\lambda_3}{dt} = -\frac{\partial \mathcal{H}}{\partial B} = -a_1 - \lambda_2 (\epsilon + u_3) + \lambda_3 (\delta + \epsilon + u_3 + u_1 + \mu) - \lambda_4 (1 - u_3) \delta$$
$$\frac{d\lambda_4}{dt} = -\frac{\partial \mathcal{H}}{\partial D} = a_2 - \lambda_1 (\rho + u_1) + \lambda_2 (1 - u_2) \alpha M - \lambda_3 (1 - u_2) \alpha M + \lambda_4 (\rho + u_1 + \sigma + \mu)$$

With transversality conditions, $\lambda_i(t_f) = 0, i = 1, ..., 4$. Similarly to get the controls, we solved the equation, $\frac{\partial \mathcal{H}}{\partial u_i} = 0, i = 1, ..., 3$ and obtained:

$$\frac{\partial \mathcal{H}}{\partial u_1} = w_1 u_1 + \lambda_1 (D - S) + \lambda_2 S - \lambda_3 B - \lambda_4 D$$

$$\frac{\partial \mathcal{H}}{\partial u_2} = w_2 u_2 + \lambda_2 \alpha\, DM - \lambda_3 \alpha\, DM$$
$$\frac{\partial \mathcal{H}}{\partial u_3} = w_3 u_3 + \lambda_2 B - \lambda_3 B - \lambda_4 \delta B$$

If we set $\frac{\partial \mathcal{H}}{\partial u_i} = 0$ at u_i^*, we get

$$u^*{}_1 = \frac{D\,(\lambda_4 - \lambda_1) + \lambda_3 B + S\,(\lambda_1 - \lambda_2)}{w_1}$$
$$u_2^* = \frac{\alpha\, DM\,(\lambda_3 - \lambda_2)}{w_2}$$
$$u_3^* = \frac{B\,(\lambda_3 + \delta\lambda_4 - \lambda_2)}{w_3}$$

When we write using standard control arguments involving the bounds on the controls, we get

$$u_1^* = \begin{cases} \psi_1, & \text{if } 0 < \psi_1 < 1 \\ 0, & \text{if } \psi_1 \le 0 \\ 1, & \text{if } \psi_1 \ge 1 \end{cases}, u_2^* = \begin{cases} \psi_2, & \text{if } 0 < \psi_2 < 1 \\ 0, & \text{if } \psi_2 \le 0 \\ 1, & \text{if } \psi_2 \ge 1 \end{cases}, u_3^* = \begin{cases} \psi_3, & \text{if } 0 < \psi_3 < 1 \\ 0, & \text{if } \psi_3 \le 0 \\ 1, & \text{if } \psi_3 \ge 1 \end{cases}$$

In compact notation

$$\begin{aligned} u_1^* &= max\{0, min(1, \psi_1)\} \\ u_2^* &= max\{0, min(1, \psi_2)\} \\ u_3^* &= max\{0, min(1, \psi_3)\} \end{aligned} \tag{7}$$

The optimal control system and the adjoint variable system, by incorporating the characterized control set and initial and transversal condition, forms the optimality system of the model and it is given by:

$$\begin{cases} \frac{dS}{dt} & = \pi + (\rho + u_1^*)D - (\beta + u_1^*)\,S - \mu S \\ \frac{dM}{dt} & = (\beta + u_1^*)S + (\epsilon + u_3^*)B - (1 - u_2^*)\alpha DM + \mu M \\ \frac{dB}{dt} & = (1 - u_2^*)\alpha MD - (\delta + \epsilon + u_1^* + u_3^* + \mu)\,B \\ \frac{dD}{dt} & = (1 - u_3^*)\delta B - (\sigma + \rho + u_1^* + \mu)\,D \\ \frac{d\lambda_1}{dt} & = \lambda_1\,(\beta + u_1^* + \mu) - \lambda_2\,(\beta + u_1^*) \\ \frac{d\lambda_2}{dt} & = \lambda_2\,((1 - u_2^*)\,\alpha D + \mu) - \lambda_3\,(1 - u_2^*)\,\alpha D \\ \frac{d\lambda_3}{dt} & = -a_1 - \lambda_2\,(\epsilon + u_3^*) + \lambda_3\,(\delta + \epsilon + u_3^* + u_1^* + \mu) - \lambda_4\,(1 - u_3^*)\,\delta \\ \frac{d\lambda_4}{dt} & = -a_2 - \lambda_1\,(\rho + u_1^*) + \lambda_2\,(1 - u_2^*)\,\alpha\, M - \lambda_3\,(1 - u_2^*)\,\alpha\, M \\ & \quad +\lambda_4\,(\rho + u_1^* + \sigma + \mu) \end{cases} \tag{8}$$

$\lambda_i(t_f) = 0, i = 1, ..., 4\; S(0) = S_0\,, M(0) = M_0\,, B(0) = B_0\,, D(0) = D_0,\; P(0) = P_0$

□

4.2. Uniqueness of the Optimality System. Due to the a priori boundedness of the solutions of both the state and adjoint equations and the resulting Lipschitz structure of these equations, we obtain the uniqueness of the optimality system (8) for small t_f. Hence the following theorem:

Theorem 4.2. *For $t \in [0, t_f]$, the bounded solutions to the optimality system are unique. Look* [14, 1, **?**]*, for the proof of the theorem.*

5. Numerical Simulations

In this section, we perform some numerical experimentation on the basic model 1 and the resulting optimality system (8) consisting of the state equations and the adjoint system. We make use of the parameter values given in Table 3 for the simulation.

An iterative scheme is used to find the optimal solution of the optimality system. Since the state system have initial conditions and the adjoint systems have final conditions, we solve the state system using a forward fourth-order Runge-kutta method and solve the adjoint system using a backward fourth-order Runge-Kutta method. The solution iterative scheme involves making a guess of the controls and using that guess to solve the state system. The initial guess of the controls together with the solution of the state systems is used to solve the adjoint systems. The controls are then updated using a convex combination of the previous controls and the values obtained using the characterizations. The updated controls are then used to repeat the solution of the state and adjoint systems. This process is repeated until the values in the current iteration are close enough to the previous iteration values [22, 30].

Table 3. Parameter values for the Marriage divorse model.

Parameter symbol	**Value**	**Source**
Π	15	Assumed
β	0.04	[21]
α	0.03	[17]
σ	0.02	[6]
δ	0.04	[6]
μ	0.016	Assumed
ρ	0.003	[6, 19]
ϵ	0.3	Assumed

Next, we investigate numerically the effort of the following optimal control strategies on the spread of marriage divorce in a population by considering strategies which incorporate more than one intervention are ordered below and compared pairwise:

- **Strategy A:** Couple relationship education (u_1) and reducing getting married too young (u_2).
- **Strategy B:** Couple relationship education (u_1) and consulting separators to renew their marriage (u_3).
- **Strategy C:** Reducing getting married too young (u_2) and consulting separators to renew their marriage (u_3).
- **Strategy D:** Using all the three controls (u_1), (u_2) & (u_3)

The parameters $a_1 = 1$, $a_2 = 5$, $w_1 = 10$, $w_2 = 20$ and $w_3 = 30$ are used for simulation the optimality system 8. In addition to this parameters, we used $S(0) = 1000$, $M(0) = 150$, $B(0) = 20$, $D(0) = 10$ as initial values.

5.1. Strategy A: Couple relationship education & reducing getting married too young. Under this strategy, we used controls couple relationship education and reducing getting married too young to minimize the objective function without consulting separators to renew their marriage. The experiment in Figure 1, shows that due to the control strategies there is a significant decrease in the number of divorced marriage. But the broken marriage grows up even if it seams deceased after four to five years.

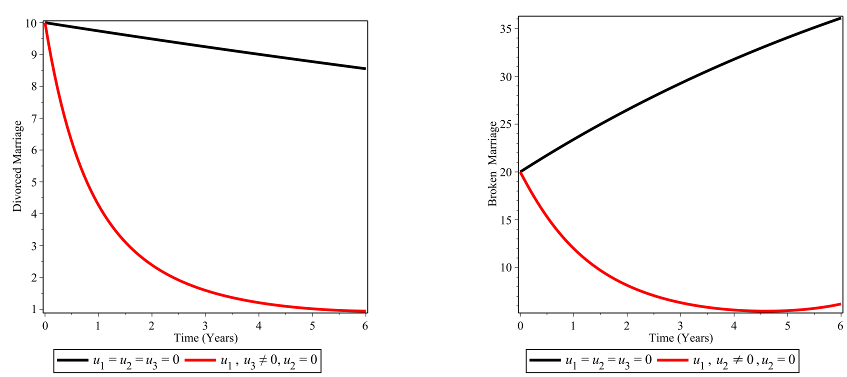

FIGURE 1. Simulation of the model with couple relationship education & reducing getting married too young.

5.2. Strategy B: Couple relationship education & consulting separators to renew their marriage. Here we optimized the objective function by couple relationship education & consulting separators to renew their marriage. The simulation results in the Figure 2, showed that there is a significant drop in the number of divorced marriage and becomes zero after 6 years. The broken marriage also decreased to zero after 5 years when there is control compared the situation without control. Therefore the strategy is effective to fight even if it takes time.

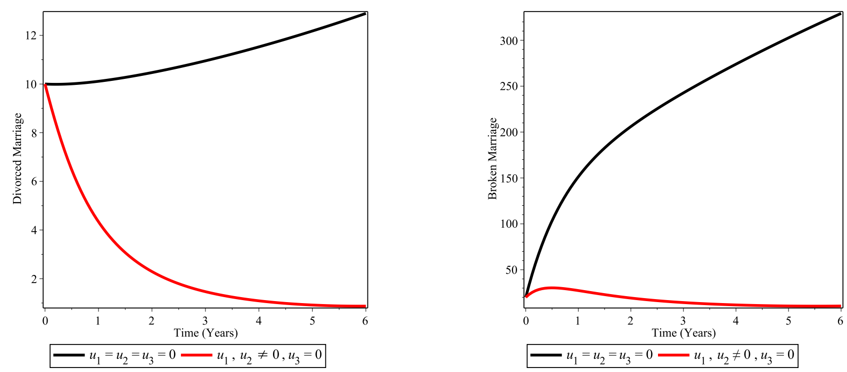

FIGURE 2. Simulation of the model with couple relationship education & consulting separators to renew their marriage.

5.3. Strategy C: Reducing getting married too young & consulting separators to renew their marriage. In utilizing the strategies, reducing getting married too young & consulting separators to renew their marriage while setting Couple relationship education zero. We observed from the Figure 3, that there is a significant drop in the number of divorced marriage after 6 years. The broken marriage grows up after the 5^{th} year even if it seams decreasing, when there is control compared to that without control.

5.4. Strategy D: Using all control strategies. In this section, we used all strategies to optimize the objective function. The results from Figure 4 shows that the strategy brings down the divorced and broken population in short period of time as compared to all the above three control technique. Therefore, government decision makers, and all stakeholders must consider in applying all strategies to combat divorce in the specified time.

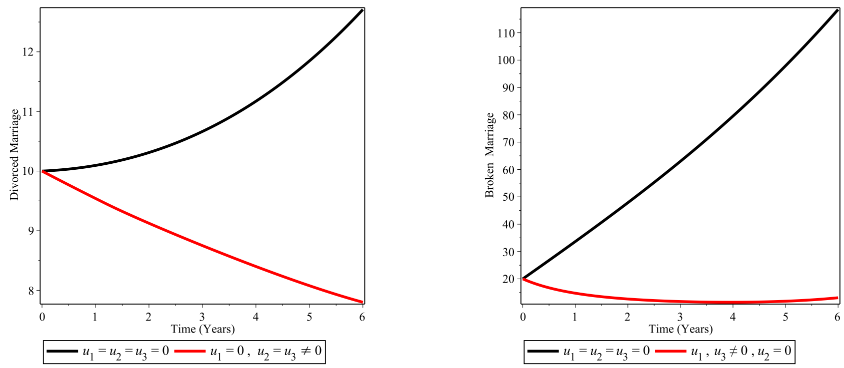

FIGURE 3. Simulation of the model with reducing getting married too young & consulting separators to renew their marriage
.

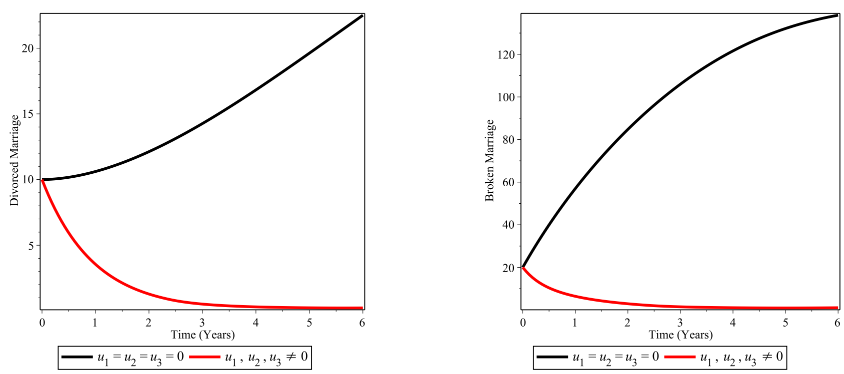

FIGURE 4. Simulation of the model with all control strategies.

6. Conclusion

We formulated a deterministic mathematical model for marriage divorce and extended it to an optimal control model. From the model solutions positivity and boundedness, were showed so that the model is well-posed mathematically and epidemiologically meaningful. Sensitivity analysis of the model is performed and identified positive and negative indices parameters. Then, the optimal control model was formulated by adding three times-dependent controls (couple relationship education , reducing getting married too young & consulting separators to renew their marriage). Optimal control theory was used to establish conditions under which divorce in marriage can be stopped and to examine the impact of a possible combination of these controls on reducing divorce in the population. The characterization of the optimal control was obtained by the application of the Pontryagin's maximum principle. It is evident from the results of the numerical simulations that, using all the integrated control strategies simultaneously (Strategy D), we can fight against marriage divorce. As a result government representatives and stakeholders should see the strategy while working on divorce with in their community.

Conflicts of Interest : The authors declare that there are no conflicts of interest.

Data Availability : The data used in this study are included within the article.

Acknowledgments : The authors acknowledge the anonymous reviewers for the insightful comments.

References

1. S. Abuse, *Effects of Divorce on Children's Behavior*, Mankind Quarterly **44** (2004), 1-12.
2. S. Bekele, *Divorce: Its Cause and Impact on The Lives of Divorced Women and Their Children A Comparative Study between Divorced and Intact Families*, Ph.D. thesis, Addis Ababa University, 2006.
3. Y. Mekonnen, K. Kassa, and M. Ayalew, *Prevalence, causes and consequences of divorce in bahir dar city, ethiopia*, African J. of Social Work **9** (2019), 73-78.
4. J. Mikolai and H. Kulu, *Short-and long-term effects of divorce and separation on housing tenure in england and wales*, Population Studies **72** (2018), 17–39.
5. C. Kleinsorge and L.M. Covitz, *Impact of divorce on children: developmental considerations*, Pediatrics in review **33** (2012), 147–54.
6. R. Duato and L.J. Odar, *Mathematical modeling of the spread of divorce in spain*, Math. and Comp. Modelling **57** (2013), 1732–1737.
7. A.A. Olaniyi, *Analytical study of the causal factors of divorce in African homes*, Research on Hum. and Soci Scie. **5** (2015), 18–29.
8. S. Clark and S.B.-Otto, *Divorce in sub-saharan africa: Are unions becoming less stable?*, Pop. and Dev. Review **41** (2015), 583–605.
9. N.E. Mohlatlole, S. Sithole, and M.L. Shirindi, *Factors contributing to divorce among young couples in lebowakgomo*, Social Work **54** (2018), 256–274.
10. M. Samuel, *Divorce under the revised family code of federal government:-it's impact on women*, Ph.D. Thesis, St. Mary's University, 2008.
11. A. Adamu and M. Temesgen, *Divorce in east gojjam zone: rates, causes and consequences*, Wudpecker J. of Socio. and Anthro. **2** (2013), 8-16.
12. L.F. Ganerre, *Factors predicting emotional and behavioural resilience on families going through a divorce*, Australian Institute of Family Studies Conference, Australia, Melbourne Convention and Exhibition Centre, 2016.
13. P.F. Fagan and A. Churchill, *The effects of divorce on children*, Marri Research **1** (2012), 1–48.
14. K.R. Fister, S. Lenhart, and J.S. McNally, *Optimizing chemotherapy in an hiv model*, Elect. J. of Diff. Equa. **32** (1998), 1–12.

15. W.H. Fleming, and R.W. Rishel, , *Deterministic and stochastic optimal control*, Bulletin of the Ameri. Math. Soci. **82** (1976), 869-870.
16. H.T. Alemneh, O.D. Makinde, and M.T. David, *Ecoepidemiological model and analysis of msv disease transmission dynamics in maize plant*, I.J. Math. and Math. scie. **2018**, 1-14.
17. A. Shibeshi, *Causes of Divorce and its Effects on Children's Wellbeing in Yeka Sub-City, Addis Ababa*, Ph.D. Thesis, Addis Ababa University, 2015.
18. J.D. Murray, *Mathematical biology: I. An Introduction (interdisciplinary applied mathematics)(Pt. 1)*, Springer, New York, 2007.
19. P.P. Gambrah and Y. Adzadu, *Mathematical model of divorce epidemic in ghana*, I. J. of Stat. and A. Math. **3** (2018), 395–401.
20. H.T. Alemneh and N.Y. Alemu, *Mathematical modeling with optimal control analysis of social media addiction*, Infectious Disease Modeling **6** (2021), 405–419.
21. M. Lhous, M. Rachik, H. Laarabi, and A. Abdelhak, *Discrete mathematical modeling and optimal control of the marital status: the monogamous marriage case*, Adv. in Diff. Equa. **2017** (2017), 339.
22. H. Tessema, I. Haruna, S. Osman, and E. Kassa, *A mathematical model analysis of marriage divorce*, Commun. Math. Biol. Neurosci. **2022** (2022).
23. P.V.D. Driessche and J. Watmough, *Reproduction numbers and sub-threshold endemic equilibria for compartmental models of disease transmission*, Math. biosciences **180** (2002), 29–48.
24. C.C.-Chavez and B. Song, *Dynamical models of tuberculosis and their applications*, Math. biosciences and engineering **1** (2004), 361–404.
25. S.M. Blower and H. Dowlatabadi, *Sensitivity and uncertainty analysis of complex models of disease transmission: an hiv model, as an example*, I. Statistical Review/Revue **1** (1994), 229–243.
26. H.T. Alemneh, O.D. Makinde, and M.T. David, *Mathematical modelling of msv pathogen interaction with pest invasion on maize plant*, G. J. of Pure and App. Math. **15** (2019), 55–79.
27. N.O. Mokaya, H.T. Alemmeh, C.G. Ngari, G.G. Muthuri, *Mathematical Modelling and Analysis of Corruption of Morals amongst Adolescents with Control Measures in Kenya*, Disc. Dyn. in Nat. & Soci. **2021** (2021), 1-16.
28. S.M. Stanley, H.J. Markman, M.St. Peters, and B.D. Leber, *Strengthening marriages and preventing divorce: New directions in prevention research*, Family Relations **44** (1995), 392-401.
29. L.S. Pontryagin, *Mathematical theory of optimal processes*, CRC press, 1987.
30. L. Suzanne, and J.T. Workman, *Optimal control applied to biological models*, Chapman and Hall/CRC, 2007.

Haileyesus Tessema is currently working as an Assistant Professor in University of Gondar, Ethiopia. He received his Ph.D. from Pan Africa University of Basic Sciences Technology and Innovation, Kenya and M.Sc. from Arba Minch University, Ethiopia. His research interests are in the areas of Infectious disease modeling. He has published 14 papers in reputable and peer reviewed journals.

Departement of Mathematics, University of Gondar, Gondar, Ethiopia.
e-mail: haila.tessema@gmail.com

Yehualashet Mengistu is currently working as Lecturer in University of Gondar, Ethiopia. He received his M.Sc. from Arba Minch University, Ethiopia in Numerical Analysis. His research interests are in the areas of Infectious disease modeling. He has published No published paper.

Departement of Mathematics, University of Gondar, Gondar, Ethiopia.

e-mail: yehualashetmeng@gmail.com

Endeshaw Kassa is currently working as Lecturer in Debre Markose University, Ethiopia. He received his M.Sc. from University of Gondar, Ethiopia in Differential Equation. His research interests are in the areas of Infectious disease modeling. He has no published paper.

Departement of Mathematics, Debre Markose University, Debre Markose, Ethiopia.
e-mail: endkassa21@gmail.com