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# FUZZY TRANSPORTATION PROBLEM WITH ADDITIONAL CONSTRAINT IN DIFFERENT ENVIRONMENTS

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Abstract. In this research, we presented the type 2 fuzzy transportation problem with additional constraints and solved by our proposed genetic algorithm model, and the results are verified using the softwares, genetic algorithm tool in Matlab and Lingo. The goal of our approach is to minimize the cost in solving a transportation problem with an additional constraint (TPAC) using the genetic algorithm (GA) based type 2 fuzzy parameter. We reduced the type 2 fuzzy set ($T_2FS$) into a type 1 fuzzy set ($T_1FS$) using a critical value-based reduction method (CVRM). Also, we use the centroid method (CM) to obtain the corresponding crisp value for this reduced fuzzy set. To achieve the best solution, GA is applied to TPAC in type 2 fuzzy parameters. A real-life situation is considered to illustrate the method.

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## 1. Introduction

The Transportation problem (TP) addresses many decision-making problems in our real life situation such as resolving the cost of products, profit for suppliers, decision-making for realistic multiple objective function, etc. Transportation experts are being challenged to meet the goals of achieving effective, safe, and dependable transportation, taking into consideration minimizing environmental and community risks. Availability constraints, environmental degradation, lack of safety measures, unpredictability and wastage of resources are shortlists of these challenges faced by transportation experts. In view of these realities, transport networks turn into complex systems with distinct or even contradictory goals. The Operations Research studies the TP in the initial stage itself to minimize transportation costs from sources to different destination. TPAC has

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primarily focused on problems with crisp data and there is a need to develop an efficient algorithm. In this paper, we reduced the $T_2FS$ into a $T_1FS$ by using a CVRM and CM to obtain the corresponding crisp value for the reduced fuzzy set and under the GA approach, the optimum solution for TPAC is achieved.

## 2. Literature review

The basic TP was initially developed by Hitchcock [8] and then independently progressed by Koopmans [13]. Different researchers have carried out several research papers in different areas and different dimensions of TP, such as those TP are [6, 24]. The commodity varies in certain characteristics according to its source in many real-life circumstances, and the final commodity mixture reaching destinations can then be expected to have known requirements. TPAC is described by Haley [7]. Pandian and Anuradha [21] suggested a floating point method for TPAC. Kumar et al. [14] proposed a mehar approach to solving the dual-hesitant fuzzy TP with restrictions. Dutta and Murthy[5] studied fuzzy TP with additional restrictions. Jana [10] introduced some applications of fuzzy programming techniques to solve solid TPAC. Recent studies used the $T_2FS$ of fuzzy-based membership functions to help deal with challenge uncertainties. Because of the crisp existence of their membership function, conventional $T_1FS$ decline to deal with uncertainties. $T_2FS$ is an extension of $T_1FS$. $T_2FS$ was first suggested by Zadeh, and Mendel & John [19]provide a more comprehensive discussion and more adaptive approach. $T_2FSs$ membership functions are three-dimensional and include a footprint of uncertainty; it is also the third dimension of $T_2FSs$, as well as the footprint of uncertainty, that provide additional degrees of freedom, allowing for direct modelling and handling of uncertainty. Kundu et al.[16] proposed a fixed charge transportation problem with type-2 fuzzy variables.These approaches are not time-consuming to compute, particularly for large-scale circumstances. Approximate algorithms can be used if the problem is too complex to solve accurately. As a result, numerous heuristic and meta-heuristic methods for solving TP were presented in the last couple of decades [17, 2, 1]. Among them, the most effective and commonly used technique is GA, which was first introduced by Holland[9]. GA are adaptive systems in which an evolutionary mechanism based on natural selection seeks solutions to problems. GA has been used extensively in industrial engineering and operations research to solve many challenging problems of combinatorial optimization. GA is one of the most efficient and widely available techniques of stochastic search and optimization and has made great strides in relevant fields of study, such as optimization of a network, multi-objective optimization. Accrued information is exploited by a selection mechanism in GAs, whereas new search space regions are explored by genetic operators [11], [22]. The technique for representing the chromosome is one of the most significant factors in a GA approach. It has been found to have a great impact on the GA's performance and effectiveness. Four fundamental steps are mostly used in GAs: initialization, selection process,

crossover, and mutation. Recently, Kumar et al.[15] developed a real-coded GA approach to the compromised near-optimal solution for the MOTP. Jose and Vijayalakshmi[12] suggested the design and analysis of multi-objective optimization problem using evolutionary algorithms. In congested urban areas, șerban [25] proposed using evolutionary algorithms to optimise public transportation scheduling. Misevičius et al.[20] devised hybrid genetic hierarchical algorithm for the quadratic assignment problem. Lin and Feng-Tse [18] suggested GAs are used to solve the TP using fuzzy coefficients. Das et al.[3] solved solid TP with mixed constraint in different environment that is GA and LINGO software and compared resultant solution.
The remaining part of the paper is sorted out as follows: Preliminaries are discussed in section 3 to review some basic concepts of $T_2FS$, as well as some relevant definitions and methods of de-fuzzification for a triangular type-2 fuzzy variable. The formulation of the mathematical model is presented in section 4. Section 5 explains the pseudocode for the proposed GAA solution for type 2 fuzzy TPAC. The proposed method is demonstrated with a numerical illustration in section 6. We compare our latest contribution to other existing program solver in section 7. Finally, in section 8, we present a conclusion to our new approach as well as an outlook for future study.

## 3. Preliminaries

The basic concepts of $T_2FS$, regular fuzzy variable (RFV) and their critical values(CVs) can be found in [16, 26, 4]. The definition of CVs of triangular RFVs, reduction methods, and defuzzification process are covered in this section.

**3.1. Critical values of triangular RFVs.** The following theorem introduces the CVs of triangular RFVs.

**Theorem 3.1** (Qin et al. [23]). *Let $\tilde{\beta} = (\theta_1, \theta_2, \theta_3)$ be a triangular RFV. Then we have,*

(1) *The optimistic CV of $\tilde{\beta}$ is* $OCV[\tilde{\beta}] = \dfrac{\theta_3}{1+\theta_3-\theta_2}$

(2) *The pessimistic CV of $\tilde{\beta}$ is* $PCV[\tilde{\beta}] = \dfrac{\theta_2}{1+\theta_2-\theta_1}$

(3) *The CV of $\tilde{\beta}$ is*
$$CV[\tilde{\beta}] = \begin{cases} \dfrac{2\theta_2-\theta_1}{1+2(\theta_2-\theta_1)} & ,\text{if } \theta_2 > \dfrac{1}{2} \\ \dfrac{\theta_3}{1+2(\theta_3-\theta_2)} & ,\text{if } \theta_2 \le \dfrac{1}{2} \end{cases}$$

**3.2. CV-based reduction method for type-2 fuzzy variable.** The membership function in $T_2FS$ is itself a fuzzy set. So, in the calculations, the complexity rises. The $T_1FS$ associated methods cannot be directly applied to the $T_2FS$s. Therefore, a general suggestion is to reduce the $T_2FS$ complexity by converting to a $T_1FS$ in order to adapt the methods for dealing with the converted $T_1FS$. In order to defuzzify a $T_2FS$, different researchers have developed several techniques to defuzzify a $T_2FS$. The CVRM was recently used by Mendal and John [19] to reduce a $T_2FS$ to a $T_1FS$. In this approach the three CVs, namely, optimistic CV($OCV[\tilde{\beta}]$), pessimistic CV($PCV[\tilde{\beta}]$), CV reduction ($CV[\tilde{\beta}]$) are mainly used to obtain a type-1 fuzzy variable ($T_1FV$) from a type-2 fuzzy variable ($T_2FV$).

**3.3. Defuzzification process.** There are two stages in the defuzzification process of a $T_2FV$. In the first stage a $T_2FV$ is converted into its $T_1FV$ and the second stage the crisp value is obtained using defuzzification techniques. In this article we obtain a reduced $T_1FV$ from $T_2FV$ using CVRM. Later, we apply CM ($\frac{\sum_{i=1}^{n} y_i\mu_\beta(y_i)}{\sum_{i=1}^{n}\mu_\beta(y_i)}$ for discrete case and $\frac{\int y_i\mu_\beta(y_i)}{\int \mu_\beta(y_i)}$ for continuous case) to acquire a crisp value from the $T_1FV$.

| ***Nomenclature*** |
|---|
| *m number of Workplaces* |
| *n number of Pharmaceutical intermediaries* |
| $\tilde{c}_{ij}$ *cost per unit of fuzzy transportation from workplaces i to pharmaceutical intermediaries j* |
| $\tilde{t}_i^q$ *fuzzy units of r temperature q =1,2,...,r are contained in each unit of the product, when it is directed from i to j* |
| $\tilde{g}_j^q$ *fuzzy units of temperature q, maximum Celsius requirements.* |
| $\tilde{a}_i$ *fuzzy supply available at work places i* |
| $\tilde{b}_j$ *fuzzy demand requirement at pharmaceutical intermediaries j* |
| ***Variables*** |
| $\tilde{x}_{ij}$ *fuzzy quantity shipped from workplaces i to pharmaceutical* intermediaries j |

## 4. Model formulation and statement of the problem

$$(P) \qquad Minimize \tilde{Z} = \sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{c_{ij}}\tilde{x_{ij}} \tag{1}$$

Subject to the constraint (2)

$$\sum_{j=1}^{n} \tilde{x_{ij}} = \tilde{a_i}, i = 1, 2, ..., m \tag{3}$$

$$\sum_{i=1}^{m} \tilde{x_{ij}} = \tilde{b_j}, j = 1, 2, ..., n \tag{4}$$

$$\sum_{i=1}^{m} \tilde{t_i^q}\tilde{x_{ij}} \le \tilde{g_j^q}, i = 1, 2, ..., m, j = 1, 2, ..., n, q = 1, 2, ..., r \tag{5}$$

$$\tilde{x}_{ij} \ge 0, \ i = 1, 2, ..., m, j = 1, 2, ..., n \tag{6}$$

In problem (P), objective function (1) minimizes the total fuzzy transportation cost involving the additional constraint(AC). Fuzzy supply constraint (2) ensures that the quantity shipped out from a workplace is equal to the available capacity. Fuzzy demand constraint (3) ensures that the total shipment received from a workplace is equal to the demand at each intermediates. Fuzzy additional constraint (4) ensures products with $t_i$units of temperature are acquired which has no more than $g_j$ units of maximum Celsius to the intermediates. Constraint (5) ensures the non-negativity of decision variables.

## 5. Proposed GA Approach

We construct the crisp equivalent problem from the given problem (P) by using CVRM and centroid method [26]. Then the problem (P) split into OCV, PCV, CV and solve these problem using proposed GA model. The GA algorithm starts by creating an initial random population, which is subject to three generic procedures that are repeated multiple times (generations). Every individual in the population is evaluated in the first procedure, based on the results a group of children from the present generation is selected to form the new generation of the population in the second procedure. The GA algorithm then launches a new cycle from the first procedure by joining the parents and newly generated children to form a new generation of the population. The proposed GA is detailed in the simple and self-explanatory pseudocode that follows.

**5.1. Initialization.** A chromosome is often used to represent the encrypted solution. The initial set of solutions, that is population size of k, is created at random, with k/2 being an even number, letting the maximum range of possible solutions to be explored. Using ACs, all impossible allotment cells are identified and removed from the problem (P). All possible feasible solutions are obtained and an initial n-chromosome population is constructed.

### 5.1.1. Pseudocode of the function initialize the population:

**procedure start**
  **Input** → crisp equivalent problem
  **Remove** → impossible allotment cells ($t_i > g_j$)
  **Generate** → all the chromosomes (feasible solutions) in the feasible region as initial population
  **for** → Initial population size times
    **if** → the chromosome is selected, which satisfies the AC
      {
      $t_1x_{11} + t_2x_{21} + t_3x_{31} + \ldots \leq g_1$
      $t_1x_{12} + t_2x_{22} + t_3x_{32} + \ldots \leq g_2$
      ⋮

$t_1 x_{1j} + t_2 x_{2j} + t_3 x_{3j} + \ldots \leq g_j$
}
**then** → move the AC satisfied chromosomes to the Evaluation process
**procedure end**

**5.2. Evaluation.** As the overall transportation costs, including parameter and ACs, should be reduced in this problem. The results with minimum objective functions are better solutions. In the problem, evaluate each chromosome's fitness value using AC. The chromosome with minimum fitness value is considered as the best chromosome.

**5.2.1. Pseudocode of the function Evaluation:**

**procedure start**
**for** → AC satisfied chromosomes ($C_i$)
**Generate** → each chromosome's fitness value
**Eval** ($C_i$) = objective function value ($M_i$)
**procedure end**

**5.3. Selection process.** The roulette wheel selection method [20] is introduced in the selection process, wherein the chromosomes with higher fitness values have more chance of selection. To compute the fittest probability, we use the following formulas,

The fittest values of each chromosome are (7)

$$F_i = \frac{1}{1 + M_i} \tag{8}$$

The total of the fittest values of all the chromosome are (9)

$$Total(T) = \sum_{i=1}^{m} F_i \tag{10}$$

The probability for each chromosome are formulated by (11)

$$P_i = \frac{F_i}{\sum_{i=1}^{m} F_i} \tag{12}$$

The selection procedure is based on the roulette wheel selection procedure, as described in the following pseudocode.

#### 5.3.1. Pseudocode of the Selection:

**procedure start**
  **Input** $\rightarrow M_i$
    **Find** $\rightarrow F_i, \quad T = \sum_{i=1}^{m} F_i$
    **then** $\rightarrow$ probability for each chromosome i
    $P_i = \frac{F_i}{\sum_{i=1}^{m} F_i}$
  **Push** $\rightarrow C_i$ chromosomes to crossover and mutation
**procedure end**

### 5.4. Crossover.

Irrespective of fitness, every single individual in the mating has an equal chance of being a parent. Two parent chromosomes are selected for mating pool. Crossover occurs between these two parents. The crossover points locus is generated at random. Select a pair of parents at random for mating and applying crossover to form two offsprings. A parent is chosen for a single moment of mating. For example, two-point crossover between Parent1 and Parent 2 as shown below.

parent1

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 2 | 2 | 0 | 3 | 2 | 0 | 0 | 1 | 5 |

parent2

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1 | 3 | 0 | 2 | 2 | 1 | 2 | 0 | 4 |

The values between the chosen crossover points are now interchanged, but the rest of the values remain the same. So there's Offspring 1 and Offspring 2 are given below.

Offspring 1

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 2 | 2 | 0 | 2 | 2 | 1 | 0 | 1 | 5 |

Offspring 2

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1 | 3 | 0 | 3 | 2 | 0 | 2 | 0 | 4 |

#### 5.4.1. Pseudocode of the Crossover:

**procedure start**
  **for** $\rightarrow$ all $C_i$, parents
    **Select** $\rightarrow$ the chromosome with probability $P_i$, which has max1 and max 2
    **Generate** $\rightarrow$ two-point crossover between $(C_i)$ with max1 and max2 probabilities $(P_i)$
    **Print** $\rightarrow$ Resultant off spring $O_i$and $O_{i+1}$
    **Generate** $\rightarrow$ fitness value of the offspring
    **Push** $\rightarrow$ Resultant offspring $O_{ij}$ and $O_{(i+1)}$
    **if** $\rightarrow$ all the Offspring are processed, then
    **Go to** $\rightarrow$ next step(Mutation)
  **Else** $\rightarrow$ go to select the chromosomes with probability $P_i$, which has next max 1 and max 2
**procedure end**

### 5.5. Mutation.

The mutation operator is a vital mechanism in any effective GA because it reorganizes the gene structure so that the algorithm can search in the best local region. It can be considered a basic local search technique as well. Apply the mutation operator to the offspring, and place the resulting offspring with their parents in the new population.

For illustration, it is to Swap mutation for previous example Offspring 1 and Offspring 2.

Offspring 1

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 2 | 2 | 0 | 2 | 2 | 1 | 0 | 1 | 5 |

Offspring 2

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1 | 3 | 0 | 3 | 2 | 0 | 2 | 0 | 4 |

Here we swap the two alleles 2,1 & 2,0 then we get the Offspring 3 & Offspring 4.

Offspring 3

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 2 | 2 | 0 | 2 | 2 | 0 | 0 | 1 | 5 |

Offspring 4

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1 | 3 | 0 | 3 | 2 | 1 | 2 | 0 | 4 |

#### 5.5.1. Pseudocode of the Mutation:

**procedure start**
**for** → $O_i$, $O_{i+1}$ resultant offspring of crossover
**Swap** → mutation between $O_i$ and $O_{i+1}$
**Print** → new offspring $O_{ij}$ and $O_{(i+1)j}$
**Generate** → fitness of the new offspring
**Push** → Resultant new offspring $O_{ij}$ and $O_{(i+1)j}$ to new population
**if** → all the Offspring are processed, then
**Go to** → next step(Termination)
**Else** → go to select for next set of resulting offspring of crossover
**procedure end**

**5.6. Termination.** When the highest fitness value achieved so far does not increase in subsequent generations, we stop generating new generations. The chromosome with the highest fitness overall generations is chosen as the TPAC's solution.

#### 5.6.1. Pseudocode of the Termination:

**procedure start**
**Replace** → the chromosomes of latest population with the new population's chromosome.
**Repeat** → Selection, crossover and mutation for mating of all the parents to get best chromosomes from all the new population's offspring, which satisfy the AC.
**Stop**→ No improvement of minimum value attained so far, in successive generation.
**Else Print** → The best chromosome
**procedure end**

The GA approach for solving a TPAC is shown below using an illustration.

## 6. Numerical Example

A pharmaceutical firm, the medicine manufacturing unit, has numerous types of medicine units in each of the three workplaces ($WP_i$) located in different parts of the country. Workplaces obtain a fixed quantity of medication ($PI_j$) with three distinct pharmaceutical intermediators. The fundamental objective is to evaluate the cost of transportation while meeting the additional requirement that, for the most part, almost all pharmaceutical products need to be kept at a certain temperature during the time of transportation from origin to destination. The following table 1 shows the availabilities, temperatures, requirement, maximum Celsius requirements and cost of transportation is given as triangular type-2 fuzzy numbers.

TABLE 1. Crisp values of type-2 fuzzy unit transportation cost for TPAC (P).

$$\tilde{c_{11}} = \left\{ \begin{array}{c} 2 \; with \; \mu_{\tilde{c_{11}}}(2) = (0.3, 0.4, 0.8) \\ 4 \; with \; \mu_{\tilde{c_{11}}}(4) = (0.4, 0.7, 0.9) \\ 6 \; with \; \mu_{\tilde{c_{11}}}(6) = (0.2, 0.4, 0.6) \end{array} \right\} \quad \tilde{c_{12}} = \left\{ \begin{array}{c} 1 \; with \; \mu_{\tilde{c_{12}}}(1) = (0.3, 0.4, 0.8) \\ 3 \; with \; \mu_{\tilde{c_{12}}}(3) = (0.2, 0.7, 0.9) \\ 5 \; with \; \mu_{\tilde{c_{12}}}(5) = (0.3, 0.7, 0.9) \end{array} \right\}$$

$$\tilde{c_{13}} = \left\{ \begin{array}{c} 1 \; with \; \mu_{\tilde{c_{13}}}(1) = (0.3, 0.4, 0.8) \\ 2 \; with \; \mu_{\tilde{c_{13}}}(2) = (0.6, 0.7, 0.9) \\ 3 \; with \; \mu_{\tilde{c_{13}}}(3) = (0.4, 0.7, 0.8) \end{array} \right\} \quad \tilde{c_{21}} = \left\{ \begin{array}{c} 2 \; with \; \mu_{\tilde{c_{21}}}(2) = (0.6, 0.7, 0.9) \\ 3 \; with \; \mu_{\tilde{c_{21}}}(3) = (0.4, 0.7, 0.8) \\ 7 \; with \; \mu_{\tilde{c_{21}}}(7) = (0.2, 0.7, 0.8) \end{array} \right\}$$

$$\tilde{c_{22}} = \left\{ \begin{array}{c} 3 \; with \; \mu_{\tilde{c_{22}}}(3) = (0.4, 0.6, 0.9) \\ 6 \; with \; \mu_{\tilde{c_{22}}}(6) = (0.2, 0.7, 0.9) \\ 9 \; with \; \mu_{\tilde{c_{22}}}(9) = (0.1, 0.5, 0.8) \end{array} \right\} \quad \tilde{c_{23}} = \left\{ \begin{array}{c} 5 \; with \; \mu_{\tilde{c_{23}}}(5) = (0.1, 0.5, 0.7) \\ 7 \; with \; \mu_{\tilde{c_{23}}}(7) = (0.1, 0.4, 0.5) \\ 9 \; with \; \mu_{\tilde{c_{23}}}(9) = (0.7, 0.8, 0.9) \end{array} \right\}$$

$$\tilde{c_{31}} = \left\{ \begin{array}{c} 5 \; with \; \mu_{\tilde{c_{31}}}(5) = (0.1, 0.7, 0.9) \\ 6 \; with \; \mu_{\tilde{c_{31}}}(6) = (0.2, 0.4, 0.6) \\ 10 \; with \; \mu_{\tilde{c_{31}}}(10) = (0.2, 0.8, 1.0) \end{array} \right\} \quad \tilde{c_{32}} = \left\{ \begin{array}{c} 2 \; with \; \mu_{\tilde{c_{32}}}(2) = (0.6, 0.7, 0.9) \\ 4 \; with \; \mu_{\tilde{c_{32}}}(4) = (0.3, 0.4, 0.7) \\ 6 \; with \; \mu_{\tilde{c_{32}}}(6) = (0.2, 0.7, 0.9) \end{array} \right\}$$

$$\tilde{c_{33}} = \left\{ \begin{array}{c} 3 \; with \; \mu_{\tilde{c_{33}}}(3) = (0.4, 0.7, 0.8) \\ 7 \; with \; \mu_{\tilde{c_{33}}}(7) = (0.2, 0.7, 0.8) \\ 8 \; with \; \mu_{\tilde{c_{33}}}(8) = (0.3, 0.8, 0.9) \end{array} \right\} \quad \tilde{a_1} = \left\{ \begin{array}{c} 2 \; with \; \mu_{\tilde{a_1}}(2) = (0.6, 0.7, 0.9) \\ 4 \; with \; \mu_{\tilde{a_1}}(4) = (0.3, 0.4, 0.7) \\ 6 \; with \; \mu_{\tilde{a_1}}(6) = (0.2, 0.7, 0.9) \end{array} \right\}$$

$$\tilde{a_2} = \left\{ \begin{array}{c} 3 \; with \; \mu_{\tilde{a_2}}(3) = (0.4, 0.6, 0.8) \\ 5 \; with \; \mu_{\tilde{a_2}}(5) = (0.4, 0.4, 0.6) \\ 7 \; with \; \mu_{\tilde{a_2}}(7) = (0.3, 0.6, 0.8) \end{array} \right\} \quad \tilde{a_3} = \left\{ \begin{array}{c} 3 \; with \; \mu_{\tilde{a_3}}(3) = (0.4, 0.5, 0.6) \\ 6 \; with \; \mu_{\tilde{a_3}}(6) = (0.2, 0.7, 0.8) \\ 8 \; with \; \mu_{\tilde{a_3}}(8) = (0.3, 0.8, 0.9) \end{array} \right\}$$

$$\tilde{t_1} = \left\{ \begin{array}{c} 1 \; with \; \mu_{\tilde{t_1}}(1) = (0.3, 0.6, 1.0) \\ 2 \; with \; \mu_{\tilde{t_1}}(2) = (0.5, 0.6, 1.0) \\ 3 \; with \; \mu_{\tilde{t_1}}(3) = (0.4, 0.7, 0.9) \end{array} \right\} \quad \tilde{t_2} = \left\{ \begin{array}{c} 0 \; with \; \mu_{\tilde{t_2}}(0) = (0.3, 0.6, 0.8) \\ 1 \; with \; \mu_{\tilde{t_2}}(1) = (0.5, 0.6, 0.7) \\ 2 \; with \; \mu_{\tilde{t_2}}(2) = (0.4, 0.7, 0.9) \end{array} \right\}$$

$$\tilde{t_3} = \left\{ \begin{array}{c} 0 \; with \; \mu_{\tilde{t_3}}(0) = (0, 0, 0) \\ 0 \; with \; \mu_{\tilde{t_3}}(0) = (0, 0, 0) \\ 0 \; with \; \mu_{\tilde{t_3}}(0) = (0, 0, 0) \end{array} \right\} \quad \tilde{b_1} = \left\{ \begin{array}{c} 3 \; with \; \mu_{\tilde{b_1}}(3) = (0.4, 0.4, 0.9) \\ 5 \; with \; \mu_{\tilde{b_1}}(5) = (0.4, 0.4, 0.6) \\ 7 \; with \; \mu_{\tilde{b_1}}(7) = (0.3, 0.6, 0.6) \end{array} \right\}$$

$$\tilde{b_2} = \left\{ \begin{array}{c} 3 \; with \; \mu_{\tilde{b_2}}(3) = (0.4, 0.6, 0.9) \\ 5 \; with \; \mu_{\tilde{b_2}}(5) = (0.4, 0.4, 0.6) \\ 7 \; with \; \mu_{\tilde{b_2}}(7) = (0.3, 0.6, 0.6) \end{array} \right\} \quad \tilde{b_3} = \left\{ \begin{array}{c} 3 \; with \; \mu_{\tilde{b_3}}(3) = (0.4, 0.4, 0.9) \\ 5 \; with \; \mu_{\tilde{b_3}}(5) = (0.4, 0.4, 0.6) \\ 7 \; with \; \mu_{\tilde{b_3}}(7) = (0.3, 0.6, 0.6) \end{array} \right\}$$

$$\tilde{g_1} = \left\{ \begin{array}{c} 2 \; with \; \mu_{\tilde{g_1}}(2) = (0.6, 0.7, 0.9) \\ 4 \; with \; \mu_{\tilde{g_1}}(4) = (0.3, 0.4, 0.7) \\ 6 \; with \; \mu_{\tilde{g_1}}(6) = (0.2, 0.7, 0.9) \end{array} \right\} \quad \tilde{g_2} = \left\{ \begin{array}{c} 0 \; with \; \mu_{\tilde{g_2}}(0) = (0.3, 0.6, 0.8) \\ 1 \; with \; \mu_{\tilde{g_2}}(1) = (0.5, 0.6, 0.7) \\ 2 \; with \; \mu_{\tilde{g_2}}(2) = (0.4, 0.7, 0.9) \end{array} \right\}$$

$$\tilde{g_3} = \left\{ \begin{array}{c} 7 \; with \; \mu_{\tilde{g_3}}(7) = (0.1, 0.1, 0.2) \\ 8 \; with \; \mu_{\tilde{g_3}}(8) = (0.2, 0.5, 0.6) \\ 11 \; with \; \mu_{\tilde{g_3}}(11) = (0.4, 0.4, 0.5) \end{array} \right\}$$

Since problem(P) in type-2 fuzzy number, it is defuzzified using proposed method by Sobana and Anuradha [26]. The conversion of triangular type-2 fuzzy numbers into crisp form are shown in Table 2.

TABLE 2. Crisp values of type-2 fuzzy unit transportation cost for TPAC.

| | OCV | | | | | PCV | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | PI1 | PI2 | PI3 | $a_i$ | $t_i$ | PI1 | PI2 | PI3 | ai | ti |
| WP1 | 3.92 | 3.17 | 2.08 | 4 | 2 | 3.95 | 3.21 | 2.11 | 3.8 | 2 |
| WP2 | 3.98 | 5.89 | 7.25 | 5 | 1 | 3.75 | 5.68 | 7.53 | 4.9 | 1 |
| WP3 | 7.24 | 4 | 6.08 | 6 | 0 | 7.23 | 3.77 | 5.95 | 5.8 | 0 |
| $b_j$ | 5 | 5 | 5 | | | 5.1 | 5.1 | 5.2 | | |
| $g_j$ | 4 | 1 | 9 | | | 3.8 | 1 | 9.2 | | |

| | CV | | | | |
|---|---|---|---|---|---|
| | PI1 | PI2 | PI3 | $a_i$ | $t_i$ |
| WP1 | 4.02 | 3.24 | 2.13 | 3.92 | 2.04 |
| WP2 | 4.02 | 5.87 | 7.3 | 4.99 | 1.04 |
| WP3 | 7.11 | 3.92 | 6.05 | 5.89 | 0 |
| $b_j$ | 5.23 | 5.23 | 5.21 | | |
| $g_j$ | 3.92 | 1.04 | 9.1 | | |

The following step is to eliminate the allotments that are impossible to fulfill the AC. Analyze the condition that pharmaceutical intermediates j acquire no more than $g_j$ units of temperature-controlled products with maximum temperature restrictions. The TPAC has been reduced by removing the impossible allotment cells using AC. In Table 2, impossible allotment cell value in OCV is 3.17,CV is 3.21 and CV is 3.24. Now, we remove the impossible allotment cell value and compute the feasible solution by using existing method. The feasible allocations of OCV is $x_{11} = 0$,$x_{12} = 0$,$x_{13} = 4$,$x_{21} = 5$,$x_{22} = 0$,$x_{23} = 0$,$x_{31} = 0$,$x_{32} = 5$,$x_{33} = 1$ . It is termed as (0 0 4 5 0 0 0 5 1) and the objective values is 54.3. The initialization is performed with feasible allocation of OCV. A chromosome is a feasible solution and evaluation of the chromosome is an objective function in the TP. Gene of the chromosome is a component part ($x_{ij}$) of the TP. Length of each chromosome in this problem is 9. An initial population of size 8 is randomly generated from the feasible region. Let the initial population chromosomes be $C_1$= (0 0 4 5 0 0 0 5 1), $C_2$= (4 0 0 1 0 4 0 5 1), $C_3$= (0 4 0 4 1 0 1 0 5), $C_4$= (2 2 0 3 2 0 0 1 5), $C_5$= (1 3 0 2 2 1 2 0 4), $C_6$= (0 0 4 4 1 0 1 4 1), $C_7$= (0 0 4 4 0 1 1 5 0), $C_8$= (0 0 4 1 4 0 4 1 1) for OCV without considering any AC. Checking j=1 the AC (4), on the first chromosome $C_1$ of the OCV problem is not satisfied, whereas j=2,3, AC is satisfied. As a result, $C_1$ does not satisfy the AC and discarded from the initial population. In general, discard any chromosome that fails to satisfy all of the ACs, such as products with $t_i$ units of temperature which has no more than $g_j$ units of maximum Celsius to the pharmaceutical intermediates. Repeat the process for the remaining chromosomes of OCV. Table 3 shows the satisfaction of ACs for each chromosome of OCV.

Table 3. Satisfaction table of the chromosomes of OCV.

| Chromosomes | Satisfaction |
|---|---|
| $C_1$ | NO |
| $C_2$ | NO |
| $C_3$ | NO |
| $C_4$ | NO |
| $C_5$ | NO |
| $C_6$ | YES |
| $C_7$ | YES |
| $C_8$ | NO |

Table 3, demonstrates that chromosomes $C_6$ and $C_7$ satisfy all further restrictions, and we get the objective function values of OCV as $M_6$ =59.45 and $M_7$=58.73. The next procedure makes the process of the roulette wheel selection. To compute fittest probability, we must compute the fittest of each chromosome. The fittest values and total values in equation (6) and (7) are calculated for the satisfied chromosomes $C_6$ and $C_7$ . To avoid the divide by zero problem in the calculation of fittest values, the objective value of $C_6$ and $C_7$ is increased by 1. The fittest chromosomes with higher probability are selected for further processing. The probability for each chromosome is formulated by using (8), which is $P_1$ = 0.497 and $P_2$ =0.503. Select the higher probability chromosomes and mate them. In this problem, only 2 chromosomes are available for mating. In the chosen parents, $C_6$ and $C_7$, a random cut point is performed to exchange the gene, using two-point crossover, as below

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $C_6$ | 0 | 0 | 4 | 4 | 1 | 0 | 1 | 4 | 1 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $C_7$ | 0 | 0 | 4 | 4 | 0 | 1 | 1 | 5 | 0 |

The values between the chosen crossover points are now interchanged, but the rest of the values remain the same. The resulting offspring are

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $O_1$ | 0 | 0 | 4 | 4 | 0 | 1 | 1 | 4 | 1 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $O_2$ | 0 | 0 | 4 | 4 | 1 | 0 | 1 | 5 | 0 |

The objective values of the resultant offsprings are $O_1$=60.81, $O_2$= 57.37. Mutation is a genetic operator that is used to maintain genetic diversity in a population of genetic algorithm chromosomes from one generation to the next. A mutation changes the value of one or more genes in a chromosome from its original state,when we use swap mutation to obtain a viable solution. To swap the gene, we perform a random cut point on $O_1$ and $O_2$, and the results are as follows.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $O_1$ | 0 | 0 | 4 | 4 | 0 | 1 | 1 | 4 | 1 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $O_2$ | 0 | 0 | 4 | 4 | 1 | 0 | 1 | 5 | 0 |

The values between the chosen mutation points are now interchanged, but the rest of the values remain the same. The following are the resultant offsprings

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $O_{11}$ | 0 | 0 | 4 | 4 | 1 | 0 | 1 | 4 | 1 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $O_{21}$ | 0 | 0 | 4 | 4 | 0 | 1 | 1 | 5 | 0 |

The objective values of the resultant offspring are $O_{11}$ =59.45, $O_{21}$= 58.73. Replace the best chromosome arrived so far to the new population. Each new population needs chromosomes to satisfy all the constraints. Repeat the proposed process for the next generation of population. When there is no improvement of the minimum value attained so far it stops creating new generation. The objective value of a chromosome in each generation should be minimum. Here, the offspring $O_{21}$= (0 0 4 4 0 1 1 5 0) is evaluated as minimum objective value. Therefore, an optimal solution to the given problem OCV is calculated as $x_{13} = 4$,$x_{21} = 4$,$x_{23} = 1$,$x_{31} = 1$,$x_{32} = 5$ and the minimum transportation cost as 58.73. Repeat this process for PCV and CV. We observed near optimal solution for OCV, PCV and CV with minimum costs are shown in below Table 4.

TABLE 4. The solution to the problem (P)obtained from the proposed GAA model.

| Proposed GA method | OCV | PCV | CV |
|---|---|---|---|
| Near optimal allocations | $x_{13} = 4$,$x_{21} = 4$, $x_{23} = 1$,$x_{31} = 1$, $x_{32} = 5$ | $x_{13} = 3.8$,$x_{21} = 3.8$, $x_{23} = 1.1$,$x_{31} = 0.4$, $x_{32} = 5.1$,$x_{33} = 0.3$, $x_{41} = 0.9$ | $x_{13} = 3.92$,$x_{21} = 3.77$, $x_{22} = 0.16$,$x_{23} = 1.06$, $x_{31} = 0.59$,$x_{32} = 5.07$, $x_{33} = 0.23$,$x_{41} = 0.87$ |
| Minimum cost | 58.73 | 56.12 | 57.844 |

## 7. Discussion

We study the same problem for different generation using genetic algorithm solver of MATLAB and run on a PC with an Intel Core i3 −2.60 GHz and 4 GB of RAM as the implementation environment.

*Figure 1 Fitness value and the Best individuals for OCV*

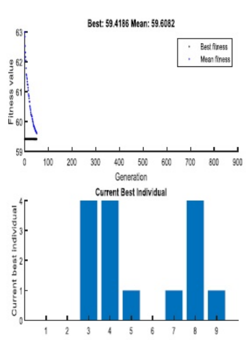



*Figure 2 :Fitness value and the Best individuals for PCV*

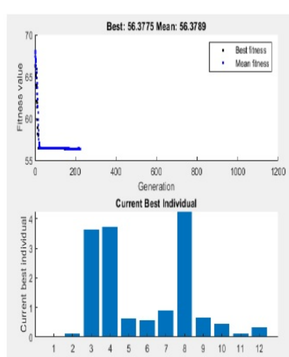
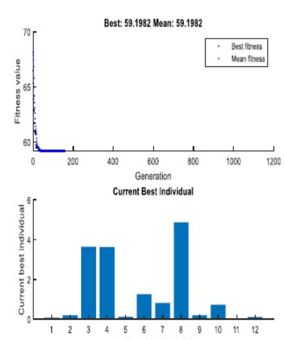



*Figure 3: Fitness value and the Best individuals for CV*



The resultant transportation cost value is nearer to our proposed method as shown in the Figure 1,2 and 3.

For the (OCV, PCV, CV) problem, the curve of the best values acquired in 100 to 200 generations. The dotted lines lead to variation between the best and mean fitness levels. After 20 generations, the curve is stabilised. As a result, the best fitness value is (59.4186, 56.3775, 59.1982), while the average value is (59.6082, 56.3789, 59.1982), which is nearer to our proposed GA model objective values. In addition, the proposed method is compared to Lingo 10.0 software, produce the almost the same minimum objective value. The results of the proposed GA model were compared to the GA tool and lingo software, as shown in table 5.

Table 5. Comparison on results obtained using GA and LINGO.

| Model | Proposed GA | GA tool(Matlab) | Lingo software |
|---|---|---|---|
| OCV | 58.73 | 59.4186 | 58.73 |
| PCV | 56.12 | 56.3775 | 56.12 |
| CV | 57.844 | 59.1982 | 57.65 |

Based on these analyses in Table 5, the decision maker (DM) can select the OCV, PCV, and CV based on the situation of the objective function. We can be concluded that the proposed method is the best for solving similar type of TPAC decision-making optimization problem.

## 8. Conclusions and Future work

The TPAC is an important network optimization problem. This paper views the cost, supply, demand, and ACs as $T_2FV$s to describe the ambiguity in the realistic decision environment. To provide a modelling context for optimization problems with multi-fold uncertainty, CVRM is used to transform a type-2 into a $T_1FV$ and CV-based reduction method and centroid method used to transform a $T_1FV$ to crisp variable for OCV, PCV, and CV problems. It is easy to understand the proposed method used in this article. We have numerical tests to assess how simple it is to execute the suggested method and solution. The approach is the same as compared to Lingo 10.0 software and GA Tool. Using type-2 fuzzy parameters, the proposed approach in this paper can be extended to decision-making problems in different areas. The knowledge in the paper can be utilized to solve specific problems in the areas of manufacturing, supply chain systems, communications technology and so on, because the TP can be seen as a resource allocation problem. We believe it will be useful to investigate the efficiency of a genetic algorithm in the future, specifically its impact on the population size, dimensions, crossover, and mutation rates on the final solution and the use of various methods of selection (Tournament, ranking system, etc.).

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