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COMMUTATIVITY OF PRIME GAMMA NEAR RINGS WITH GENERALIZED DERIVATIONS

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Abstract. The purpose of the present paper is to obtain commutativity of prime Γ-near-ring N with generalized derivations F and G with associated derivations d and h respectively satisfying one of the following conditions:(i) $G([x,y]_\alpha = \pm f(y)\alpha(xoy)_\beta\gamma g(y)$, (ii) $F(x)\beta G(y) = G(y)\beta F(x)$, for all $x, y \in N, \beta \in \Gamma$ (iii) $F(u)\beta G(v) = G(v)\beta F(u)$, for all $u \in U, v \in V, \beta \in \Gamma$,$(iv)$ if $0 \neq F(a) \in Z(N)$ for some $a \in V$ such that $F(x)\alpha G(y) = G(y)\alpha F(x)$ for all $x \in V$ and $y \in U, \alpha \in \Gamma$.

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1. Introduction

A Gamma near ring is a triple $(N, +, \Gamma)$, where $(N, +)$ is a group (not necessarily abelian); Γ is a non-empty set of binary operations on N such that for each $\gamma \in \Gamma$, $(N, +, \gamma)$ is a near ring and; $(x\gamma y)\mu z = x\gamma(y\mu z)$ for all $x, y, z \in N$ and $\gamma, \mu \in \Gamma$ [7]. Throughout the paper, N represents zero-symmetric left Γ-near-ring. For a gamma near ring N, the set $N_o = \{x \in N : 0\alpha x = 0, \alpha \in \Gamma\}$ is called the zero-symmetric part of N. A gamma near-ring N is said to be zero-symmetric if $N = N_o$ for all $x \in N$. A gamma near-ring N is called prime if N has the property that for all $x, y \in N$, $x\Gamma N\Gamma y = \{0\}$ implies $x = 0$ or $y = 0$. A gamma near ring N is called a semi prime if N has the property that for $x \in N$, $x\Gamma N\Gamma x = \{0\}$ implies that $x = 0$. A non empty subset U of N is called a semigroup right ideal (resp. semigroup left ideal) if $U\Gamma N \subseteq U$ $(N\Gamma U \subseteq U)$. U is called semigroup ideal if it is both right and left semi group ideal. A gamma near ring N is called 2-torsion free if $x \in N$ and $2x = 0$, then $x = 0$. For any $x, y \in N$,$(x, y) = x + y - x - y$ is called

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additive commutator. The multiplicative center of a gamma near ring N is defined as $Z(N) = \{n \in N : x\alpha n = n\alpha x, x \in N\}$. For all $x, y \in N$, $\alpha \in \Gamma$, the notation $[x, y]_\alpha$ and$(xoy)_\alpha$ will denote $x\alpha y - y\alpha x$ and $x\alpha y + y\alpha x$, respectively. For all $x, y, z \in N$ and $\alpha, \beta \in \Gamma$ the following basic commutator identities hold.(i) $[x\beta y, z]_\alpha = x\beta[y, z]_\alpha + [x, z]_\alpha\beta y + x\beta z\alpha y - x\alpha z\beta y$,$(ii)$ $[x, y\beta z]_\alpha = y\beta[x, z]_\alpha + [x, y]_\alpha\beta z + y\alpha x\beta z - y\alpha x\beta z$,$(iii)$ $(x\beta yoz)_\alpha = x\beta(yoz)_\alpha - [x, z]_\alpha\beta y + x\beta z\alpha y - x\alpha z\beta y$ equivalently $(x\beta yoz)_\alpha = (xoz)_\alpha\beta y + x\beta[y, z]_\alpha + x\alpha z\beta y - x\beta z\alpha y$ and (iv) $(xoy\beta z)_\alpha = (xoy)_\alpha\beta z - y\beta[x, z]_\alpha + y\alpha x\beta z - y\beta x\alpha z$ [8].

Madhuchelvi [12] An additive mapping $d : N \to N$ is called a derivation if $d(x\alpha y) = x\alpha d(y) + d(x)\alpha y$ for all $x, y \in N$ and $\alpha \in \Gamma$. An additive mapping $F : N \to N$ is said to be a right generalized derivation if there exists a derivation d on N such that $F(x\alpha y) = F(x)\alpha y + x\alpha d(y)$, for all $x, y \in N$ and $\alpha \in \Gamma$ And F is said to be a left generalized derivation if there exists a derivation d on N such that $F(x\alpha y) = d(x)\alpha y + x\alpha F(y)$,for all $x, y \in N, \alpha \in \Gamma$. Finally, F is said to be a generalized derivation associated with d if it is both right as well as left generalized derivation on N. Ibraheem [8] investigate the conditions for a prime gamma near-ring N to be a commutative gamma ring if N admitting a generalized derivation F with associated derivation d satisfying some properties of a non-zero invariant subset U of N. Madhuchelvi [11] proved some identities involving generalized derivations of a gamma near ring which facilitates the commutativity of a gamma near ring. Thereafter, Madhuchelvi [12] extended some results generalized derivations in gamma near ring in the setting of semigroup ideals which shows that a gamma near ring is a commutative gamma ring. The purpose of the present paper is to obtain some commutativity condition of prime gamma near ring N with generalized derivations F and G with associated derivations d and h respectively satisfying one of the following conditions:(i) $G([x, y]_\alpha) = \pm f(y)\alpha(xoy)_\beta\gamma g(y)$, (ii) $F(x)\beta G(y) = G(y)\beta F(x)$, for all $x, y \in N, \beta \in \Gamma$ (iii) $F(u)\beta G(v) = G(v)\beta F(u)$, for all $u \in U, v \in V, \beta \in \Gamma$,$(iv)$ if $0 \neq F(a) \in Z(N)$ for some $a \in V$ such that $F(x)\alpha G(y) = G(y)\alpha F(x)$ for all $x \in V$ and $y \in U, \alpha \in \Gamma$.

2. Generalized Derivations on Gamma Near Rings

In this section, we define polynomial with integral coefficients and we recall some known results which are essential to develop the proof of our main results.

Definition 2.1. Let N be a prime gamma near ring. For ever $x, y \in N$, $\alpha \in \Gamma$. Define a polynomials with integral coefficients as $x\alpha f(x) = x\alpha \sum_{i=1}^{n} x_i = f(x)\alpha x, x\alpha g(x) = x\alpha \sum_{j=1}^{m} x_j = g(x)\alpha x, y\alpha f(y) = y\alpha \sum_{k=1}^{p} y_k = f(y)\alpha y. y\alpha g(y) = y\alpha \sum_{n=1}^{q} y_r = g(y)\alpha y.$

Lemma 2.2. *(Madhuchelvi [12]) Let N be a prime gamma near ring and U be a non-zero semigroup ideal of N. Let d be a non-zero derivation on N. Then*
(i) *If $x, y \in N$ and $x\Gamma U\Gamma y = \{0\}$,then $x = 0$ or $y = 0$*
(ii) *If $x \in N$ and $x\Gamma U = \{0\}$ or $U\Gamma x = \{0\}$, then $x = 0$*
(iii) *If $x \in N$ and $d(U)\Gamma x = \{0\}$ or $x\Gamma d(U) = \{0\}$, then $x = 0$*

Lemma 2.3. *(Madhuchelvi[12]) Let N be a prime gamma near ring and d be a non zero derivation on N and Let $a\alpha b\beta c = a\beta b\alpha c$, for all $a, b, c \in N$ and $\alpha, \beta \in \Gamma$. Then*
(i) *If U is a non zero semigroup right ideal or a non zero semigroup left ideal of N, then $d(U) \neq \{0\}$.*
(ii) *If U is a non zero semigroup right ideal of N and x is an element of N which centralizes U, then $x \in Z(N)$, a multiplicative center of N.*

Lemma 2.4. *(Madhuchelvi [12]) Let N be a prime gamma near ring and U a non-zero semigroup ideal of N. If F is a non zero generalized derivation of N with associated derivation d, then $F(U) \neq \{0\}$*

Lemma 2.5. *(Mustafa [13]) If a prime Γ near-ring N admits a non-trival derivation d for which $d(N) \subseteq Z(N)$,then $(N, +)$ is abelian. Moreover, if N is 2-torsion free, then N is a commutative Γ-ring.*

Lemma 2.6. *(Madhuchelvi [12]) LetN be a 3-prime 2-torsion free Γ-near ring and $a\alpha b\beta c = a\beta b\alpha c$ for all $\alpha, \beta \in \Gamma$, $a, b, c \in N$. If N admits a non-zero generalized derivation F such that $F(N) \subseteq Z(N)$, then N is commutative gamma ring.*

Lemma 2.7. *(Madhuchelvi [12]) Let N be a prime gamma near ring and U be a non zero semigroup ideal of N and let $a\alpha b\beta c = a\beta b\alpha c$ for all $\alpha, \beta \in \Gamma$, $a, b, c \in N$. Let F be a non-zero generalized derivation with associated derivation d. If $F(U) \subseteq Z(N)$, then $(N, +)$ isabelian.Moreover, if N is 2-torsion free, then N is a commutative gamma ring.*

3. Commutativity of Prime Gamma Near Rings with Generalized Derivations

This section is devoted to study the commutativity of prime gamma near ring admitting a non-zero generalized derivation F of N with associated zero derivation d and a non-zero generalized derivation G with associated derivation h satisfies $G[x, y]_\alpha = f(y)\alpha(xoy)_\beta\gamma g(y)$, $F(u)\beta G(v) = G(v)\beta F(u)$, for all $u \in U, v \in V$ and $\beta \in \Gamma$, $F(x)\beta G(y) = G(y)\beta F(x)$,for all $x, y \in N, \beta \in \Gamma$, $F(x)\alpha G(y) = G(y)\alpha F(x)$ for all $x \in V$ and $y \in U, \alpha \in \Gamma$, then N is a commutative Γ- ring. Theorem 3.1 is a generalization of result proved by Shang [18] and, also we have generalized results of Kamal and Al-Shaalan[9] by Theorem 3.2, Theorem 3.3 and Theorem 3.4.

Theorem 3.1. *Let N be a 2-torsion free zero symmetric prime Γ-near ring. For ever $x, y \in N$ there exist polynomials with integral coefficients $f(x), g(x) \in Z[x]$*

such that a non-zero generalized derivation G of N with associated non-zero derivation d satisfies
(i) $G[x,y]_\alpha = f(y)\alpha(xoy)_\beta\gamma g(y)$
(ii) $G[x,y]_\alpha = -f(x)\alpha(xoy)_\beta\gamma g(x)$,*for all* $x, y \in N$ *then* N *is a commutative* Γ-*ring*

Proof. Assume that condition (i) holds

$$G[x,y]_\alpha = f(y)\alpha(xoy)_\beta\gamma g(y), forall\ x,y \in N \tag{3.1}$$

Replacing x by $y\delta x$ in (3.1), we get

$$G[y\delta x,y]_\alpha = f(y)\alpha y\delta(xoy)_\beta\gamma g(y)$$

$$G(y\delta[x,y]_\alpha) = f(y)\alpha y\delta(xoy)_\beta\gamma g(y)$$

By definition of generalized derivation

$$d(y)\delta[x,y]_\alpha + y\delta G[x,y]_\alpha = f(y)\alpha y\delta(xoy)_\beta\gamma g(y)$$

$$d(y)\delta[x,y]_\alpha + y\delta f(y)\alpha(xoy)_\beta\gamma g(y) = f(y)\alpha y\delta(xoy)_\beta\gamma g(y$$

$$d(y)\delta[x,y]_\alpha = 0, forall\ x,y \in N, \alpha,\delta \in \Gamma \tag{3.2}$$

Putting x by $z\beta x$ in (3.2), we find that

$$d(y)\delta[z\beta x,y]_\alpha = 0, forall\ x,y,z \in N, \alpha,\beta,\delta \in \Gamma$$

$$d(y)\delta z\beta[x,y]_\alpha = 0, forall\ x,y,z \in N, \alpha,\beta,\delta \in \Gamma$$

$$d(y)\Gamma N\Gamma[x,y]_\alpha = \{0\}$$

Since N is prime, we have $d(y) = 0$,for any $y \in N$ or $[x,y]_\alpha = 0$,for all $x, y \in N, \alpha \in \Gamma$
But $d \neq 0$, we have

$$[x,y]_\alpha = 0, forall\ x,y \in N, \alpha \in \Gamma$$

$$x\alpha y = y\alpha x,\ \ forall\ x,y \in N, \alpha \in \Gamma \tag{3.3}$$

Taking derivation both sides in (3.3), we get

$$d(x\alpha y) = d(y\alpha x)$$

$$d(x)\alpha y + x\alpha d(y) = d(y)\alpha x + y\alpha d(x)$$

Since derivation is additive commutative

$$d(x)\alpha y + x\alpha d(y) = y\alpha d(x) + d(y)\alpha x$$

But $x \in Z(N)$, then $x\alpha d(y) = d(y)\alpha x$

$$d(x)\alpha y + x\alpha d(y) = y\alpha d(x) + x\alpha d(y)$$

$$d(x)\alpha y = y\alpha d(x),\ \ for\ all\ y \in N, \alpha \in \Gamma$$

$$d(x) \in Z(N),\ \ for\ all\ x \in N$$

$$d(N) \subseteq Z(N)$$

Therefore, by Lemma 2.5, we obtain N is a commutative Γ-ring. □

(ii) Assume that condition (ii) holds

$$G([x,y]_\alpha) = -f(y)\alpha(xoy)_\beta\gamma g(y) \tag{3.4}$$

Replacing x by $y\delta x$ in (3.2), we get

$$G[y\delta x,y]_\alpha = -f(y)\alpha(y\delta xoy)_\beta\gamma g(y)$$

$$G(y\delta[x,y]_\alpha) = -f(y)\alpha y\delta(xoy)_\beta\gamma g(y)$$

By definition of generalized derivation

$$d(y)\delta[x,y]_\alpha + y\delta G[x,y]_\alpha = -f(y)\alpha y\delta(xoy)_\beta\gamma g(y)$$

$$d(y)\delta[x,y]_\alpha + y\delta(-f(y)\alpha(xoy)_\beta\gamma g(y)) = -f(y)\alpha y\delta(xoy)_\beta\gamma g(y) \tag{3.5}$$

Taking $f(y)$ instead of $-f(y)$ in (3.5), we have

$$d(y)\delta[x,y]_\alpha + y\delta(f(y)\alpha(xoy)_\beta\gamma g(y)) = f(y)\alpha y\delta(xoy)_\beta\gamma g(y)$$

$$d(y)\delta[x,y]_\alpha + y\delta f(y)\alpha(xoy)_\beta\gamma g(y) = f(y)\alpha y\delta(xoy)_\beta\gamma g(y)$$

$$d(y)\delta[x,y]_\alpha = 0,\ for\ all\ x,y\in N,\alpha,\delta\in\Gamma \tag{3.6}$$

The rest of the proof follows from (3.2)

Theorem 3.2. *Let N be a 2-torsion free zero symmetric prime Γ-near ring and U,V are non-zero semi-group idels of N and let $a\alpha b\beta c = a\beta b\alpha c$, for all $a,b,c\in N,\alpha,\beta\in\Gamma$. If a non-zero generalized derivation F of N with associated zero derivation d and a non-zero generalized derivation G with associated derivation h satisfies $F(u)\beta G(v) = G(v)\beta F(u)$, For all $u\in U, v\in V$ and $\beta\in\Gamma$. Then N is a commutative Γ- ring.*

Proof. For all $s,u\in U, v\in V$ and $\alpha,\beta\in\Gamma$ we have

$$F(u\alpha s)\beta G(v) = G(v)\beta F(u\alpha s)$$

$$F(u)\alpha s\beta G(v) = G(v)\beta F(u)\alpha s \tag{3.7}$$

Replacing s by $s\mu r$ in (3.7), where $r\in N,\mu\in\Gamma$, we get

$$F(u)\alpha s\mu r\beta G(v) = G(v)\beta F(u)\alpha s\mu r$$

By using (3.7), we get

$$F(u)\alpha s\mu r\beta G(v) = F(u)\alpha s\beta G(v)\mu r$$

$$F(u)\alpha s\mu r\beta G(v) = F(u)\alpha s\mu G(v)\beta r$$

$$F(u)\alpha s\mu(r\beta G(v) - G(v)\beta r) = 0,\ for\ all\ s\in U,\alpha,\mu\in\Gamma$$

$$F(u)\Gamma U\Gamma(r\beta G(v) - G(v)\beta r) = \{0\}$$

By Lemma 2.2, we have
$F(u) = 0$, for any $u\in U$ or $r\beta G(v) - G(v)\beta r = 0$, for any $r\in N,\beta\in\Gamma$ Since U is a non-zero semigroup ideal by Lemma 2.4,$F(U)\neq 0$, we get

$$r\beta G(v) - G(v)\beta r = 0, for\ any\ r\in N,\beta\in\Gamma$$

$$r\beta G(v) = G(v)\beta r, for\ any\ r\in N,\beta\in\Gamma$$

$$G(v)\in Z(N), for\ all\ v\in V$$

$$G(V) \subseteq Z(N)$$

By Lemma 2.7, N is a commutative Γ ring. □

The following example proves that the primeness hypothesis in the Theorem 3.2, is not superfluous.

Example 3.1 Let $N = \left\{ \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} : a, b \in \mathbb{R} \right\}$, where R is a commutative ring and $\Gamma = \left\{ \begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix} : a \in \mathbb{R} \right\}$. Then N is a Γ-near ring. Let $U = \left\{ \begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix} : a \in \mathbb{R} \right\}$ and $V = \left\{ \begin{pmatrix} 0 & 0 \\ 0 & b \end{pmatrix} : b \in \mathbb{R} \right\}$. Clearly U and V are non-zero semigroup ideals of N.
A map $G : N \to N$ is defined by $G \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & b \end{pmatrix}$ with associated derivation $h : N \to N$ is defined by $h \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & b \end{pmatrix}$ and $F : N \to N$ is defined by $F \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} = \begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix}$ with associated derivation $d : N \to N$ is defined by $d \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$. Then it is easy check that F and G are generalized derivations with associated derivations d and h respectively. However F and G satisfies the property of Theorem 3.2, i.e., $F(u)\beta G(v) = G(v)\beta F(u)$, for all $u \in U, v \in V$ and $\beta \in \Gamma$. But N is not prime.

Theorem 3.3. *Let N be a 2-torsion free zero symmetric prime Γ- near ring and let $a\alpha b\beta c = a\beta b\alpha c$,for all $a, b, c \in N, \alpha, \beta \in \Gamma$. If a non-zero generalized derivation F of N with associated derivation d and a non-zero generalized derivation G of N with associated derivation h such that $F(x)\beta G(y) = G(y)\beta F(x)$, For all $x, y \in N, \beta \in \Gamma$.Then N is a commutative Γ- ring.*

Proof. For all $x, y, z \in N$ and $\alpha, \beta \in \Gamma$ we have

$$F(x\alpha y)\beta G(z) = G(z)\beta F(x\alpha y)$$

$$F(x)\alpha y\beta G(z) = G(z)\beta F(x)\alpha y \tag{3.8}$$

Replacing y by $y\mu r$ in (3.8), where $y, r \in N, \mu \in \Gamma$, we get

$$F(x)\alpha y\mu r\beta G(z) = G(z)\beta F(x)\alpha y\mu r \tag{3.9}$$

Using (3.8) in (3.9), we get

$$F(x)\alpha y\mu r\beta G(z) = F(x)\alpha y\beta G(z)\mu r$$

$$F(x)\alpha y\mu r\beta G(z) = F(x)\alpha y\mu G(z)\beta r$$

$$F(x)\alpha y\mu(r\beta G(z) - G(z)\beta r) = 0, \ for\ all y \in N, \alpha, \beta \in \Gamma$$

$$F(x)\Gamma N\Gamma(r\beta G(z) - G(z)\beta r) = \{0\}$$

Since N is prime, we have

$$F(x) = 0, \ for\ all\ x \in N\ or\ r\beta G(z) = G(z)\beta r, \ r \in N, \beta \in \Gamma$$

But $F(N) \neq 0$, we have

$$r\beta G(z) = G(z)\beta r,\ for\ all\ r \in N, \beta \in \Gamma$$

$$G(z) \in Z(N),\ for\ all z \in N$$

$$G(N) \subseteq Z(N)$$

Therefore, by Lemma 2.6 ,N is a commutative Γ-ring. □

Theorem 3.4. *Let N be a 2-torsion free zero symmetric prime gamma near ring with two non-zero generalized derivation (F, d) and (G, h) such that $0 \neq F(a) \in Z(N)$ for some $a \in V$ and let $a\alpha b\beta c = a\beta b\alpha c$,for all $a, b, c \in N, \alpha, \beta \in \Gamma$. If $F(x)\alpha G(y) = G(y)\alpha F(x)$ for all $x \in V$ and $y \in U, \alpha \in \Gamma$, where U is a non-zero semigroup ideal of N, then N is a commutative Γ ring.*

Proof. If $h = 0$ or $d = 0$, then N is a commutative Γ-ring, by Theorem 3.2 Suppose that $h \neq 0 \neq d$, then for all $x, y \in U, z \in V$, we have

$$F(z)\alpha G(x\beta G(y)) = G(x\beta G(y))\alpha F(z)$$

$$F(z)\alpha G(x)\beta G(y) + F(z)\alpha x\beta h(G(y)) = G(x)\beta G(y)\alpha F(z) + x\beta h(G(y)\alpha F(z)$$

Since $F(z) \in Z(N)$, for some $z \in V$, we have

$$F(z)\alpha G(x)\beta G(y) + F(z)\alpha x\beta h(G(y)) = F(z)\alpha G(x)\beta G(y) + x\beta h(G(y)\alpha F(z)$$

So,

$$F(z)\alpha x\beta h(G(y)) = x\beta h(G(y)\alpha F(z) \tag{3.10}$$

Replacing x by $r\delta x$, for any $r \in N, \delta \in \Gamma$ it yields

$$F(z)\alpha r\delta x\beta h(G(y)) = r\delta x\beta h(G(y))\alpha F(z) \tag{3.11}$$

Using (3.10) in (3.11), we get

$$F(z)\alpha r\delta x\beta h(G(y)) = r\alpha F(z)\delta x\beta h(G(y))$$

$$(f(z)\alpha r - r\alpha f(z))\delta x\beta h(G(y)) = 0,\ for\ all\ x \in U, \beta, \delta \in \Gamma$$

$$(F(z)\alpha r - r\alpha F(z))\Gamma U\Gamma h(G(y)) = \{0\}$$

By using Lemma 2.2 , we have

$$F(z)\alpha r - r\alpha F(z) = 0\ or\ h(G(y)) = 0$$

But we assume that $h \neq 0$, we get

$$F(z)\alpha r = r\alpha F(z),\ for\ all\ r \in N, \alpha \in \Gamma$$

$$F(z) \in Z(N),\ for\ all\ z \in V$$

$$F(V) \subseteq Z(N)$$

Therefore, by Lemma 2.7, N is commutative Γ-ring. □

References

1. A. Ali, H.E. Bell and P. Miyan, *Generalized derivations on prime near rings*, Inter. J. Math. and Math.Sci. **2013** (2013), Article ID 170749, 5 pages.
2. A. Ali, H.E. Bell and P. Miyan, *Generalized derivations on prime near rings II*, Afrika Matematika **26** (2015), 275-282.
3. H.E. Bell and G. Mason, *On derivations in near-rings*, in: Near-rings and Near-Fields, North-Holland Mathematics Studies, **137** (1987), 31-35.
4. H.E. Bell, *On derivations in near-rings II*, in: Near-rings, Near-Fields and K-Loops, Kluwer Academic Publishers, Amsterdam, 1997, 191-197.
5. Y.U. Cho, and Y.B. Jun, *Gamma-derivations in prime and semiprime gamma-near-rings*, Indian Journal of Pure and Applied Mathematics **33** (2002), 1489-1494.
6. J.R. Clay, *Nearrings*, Geneses and applications, Oxford University Press, 1992.
7. K.K. Dey, A.C. Paul, and I.S. Rakhimov, *On Prime-Gamma-Near-Rings with Generalized Derivations*, International Journal of Mathematics and Mathematical Sciences **2012** (2012), Article ID 625968, 14 pages.
8. M.A. Ibraheem, *On commutativity of prime gamma near rings*, IOSR Journal of Mathematics **10** (2014), 68-71.
9. A.A. Kamal, and K.H. Al-Shaalan, *Commutativity of rings and near rings with generalized derivations*, Indian Journal of Pure and Applied Mathematics **44** (2013), 473-496.
10. M. Kazaz, and A. Alkan, *Two-sided* Γ-α *derivations in prime and semiprime* Γ*-near-rings*, Communications of the Korean Mathematical Society **23** (2008), 469-477.
11. L. Madhuchelvi, *On Generalized derivations in* Γ*-near rings*, International Journal of mathematical sciences and Applications **6** (2016), 89-100.
12. L. Madhuchelvi, *On Generalized derivations in* 3*-prime* Γ*-near rings*, International Journal of mathematical sciences and Applications **6** (2016), 901-908.
13. U. Mustafa, M.A. Ozturk, and Y.B. Jun, *On prime amma near rings with derivations*, Communication Korean Mathematical society **19** (2004), 427-433.
14. I.S. Rakhimov, K.K. Dey, and A.C. Paul, *Prime Gamma Near Rings with Derivations*, International Journal of pure and Applied Mathematics **83** (2013), 221-231.
15. B. Satyanarayana, *Contributions to near ring theory*, Doctoral Dissertation, Acharya Nagarjuna University, 1984.
16. Y.B. Jun, H.K. Kim and Y.U. Cho, *On gamma derivations in gamma near rings*, Soochow Journal of Mathematics **29** (2003), 275-282.
17. O. Golbasi, *On prime near rings with generalized* (α, τ) *derivations*, Kyungpook Mathematical Journal **45** (2005), 249-249.
18. Y. Shang, *A note on the commutativity of prime near-rings*, Algebra Colloquium **22** (2015), 361-366.

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