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Acoustic Estimation of Phase Velocity of Closed-Cell Kelvin Structure based on Spectral Phase Analysis

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Abstract

In this paper, the effect of porosity on the acoustic phase velocity of the 3D printed Kelvin closed-cell structure was investigated using the spectral phase analysis. Since Kelvin cells bring about the large amount of scattering, acoustic pulses in ultrasonic measurements undergoes a distortion of waveforms due to the dispersion effect. In order to take account on the dispersion, mathematical expressions for calculating the phase velocity of longitudinal waves propagating normal to the plane of the Kelvin structure are suggested by introducing a complex wave number based on Fourier transform. 3D Kelvin structure composed of identical unit-cells, a polyhedron of 14 faces with 6 quadrilateral and 8 hexagonal faces, was developed and fabricated by 3D CAD and 3D printer to represent the micro-structure of porous materials such as aluminum foam and cancellous bone. Total nine samples of 3D Kelvin structure with different porosity were made by changing the thickness of polyhedron. Ultrasonic pulse of 1MHz center frequency was applied to the Kelvin structures for the measurement of the phase velocity of ultrasound using the TOF(time-of-flight) and the phase spectral method. From the experimental results, it was found that the acoustic phase velocity decreased linearly with the porosity.

Keywords: *Porous materials, Time-of-Flight, Ultrasonic test, Kelvin cell, Acoustic phase velocity*

1. INTRODUCTION

Porous materials are widely used in many industries as a lightweight structural material to provide a set of excellent qualities in terms of mechanical, thermal and catalytic characteristics. For example, porous foams attract an interest as biomaterials for implantation purpose as the introduction of pores into the bulk material provides in-growth of living tissues and firm fixation in addition to reducing the alloy density [1-4]. When an elastic wave propagates in porous materials, the acoustic velocity generally depends on the void characteristics and the constituent material properties as well as on the elastic constant. The presence of voids in the matrix decreases the acoustic wave velocity due to a decrease of the effective elastic stiffness of the porous material with increasing void content. Previous studies have attempted to use ultrasonic velocities as a measure of void content or elastic constant in the porous materials. However, if the wavelength is of the same order of magnitude as a characteristic dimension of voids, one would expect the dispersion effect over a large frequency

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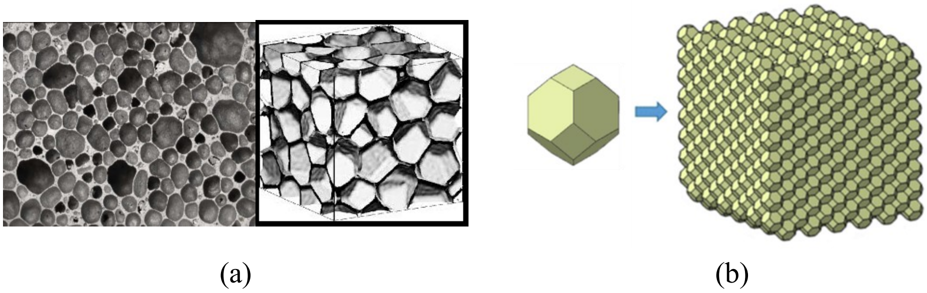
range caused by the dispersion. A pulse, being a superposition of many frequencies, will change its shape as it propagates through a dispersive medium like porous materials[5].

In this paper, a plate-shape closed-cell Kelvin structure composed of identical unit-cells, a polyhedron of 14 faces with 6 quadrilateral and 8 hexagonal faces, was developed and fabricated by 3D printer to represent the complex and irregular micro-structure of porous materials and to measure the phase velocity of ultrasound in the Kelvin structure using the spectral phase analysis. Nine Kelvin-structured samples of different porosity from 10% to 70% were prepared by changing geometric parameter of the Kelvin unit cell, the thickness of polyhedron. The samples were tested by ultrasonic TOF method to determine the phase velocity and to investigate the effect of the porosity on the acoustic phase velocity.

2. KELVIN MODEL FOR POROUS MATERIALS

Porous materials or foams are aggregates of bubbles, normally of different sizes in a matrix. Since the process of foam formation is driven by energy minimization and since the energy decreases with decreasing surface area, the structure that would best represent the geometry of foams is thought to be that of lowest surface area. This translates into the geometrical question of how to divide space into equal volume cells with minimum partitional area, which is known as the Kelvin problem. Various approaches to solve this question have led to a number of new structures[6]. A family of surfaces - derived by the minimal surfaces - has been adapted for use in the modelling of highly porous materials for structural applications. In this work, a Kelvin cell structure as shown in Fig. 1(b) was proposed to model the complex geometry of porous materials in Fig. 1(a) and to describe the porosity and mechanical properties in mathematical way.



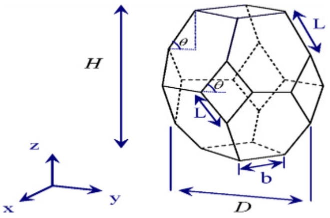
(a) (b)

Figure 1. Microstructures, (a) porous materials, (b) Closed-cell Kelvin structure

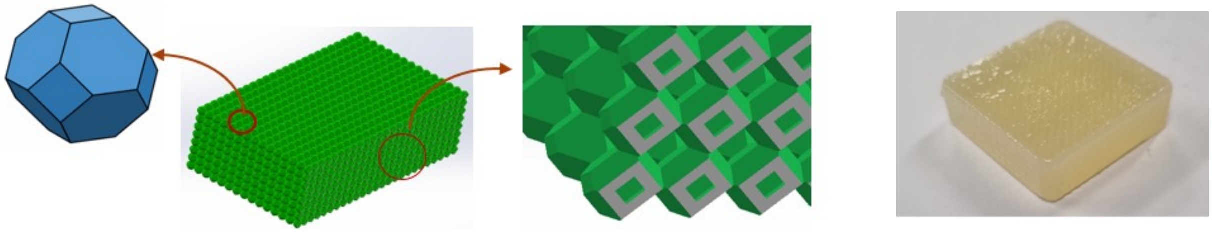
3. FABRICATION OF CLOSED-CELL KELVIN STRUCTURE

The Kelvin structure developed in this paper is made of a Kelvin unit closed-cell shown in Fig. 2(a), a 14-sided polyhedron with eight hexagonal and six quadrilateral sides. These cells put together and assembled periodically in the space so they can completely construct a three-dimensional structure like Fig. 2(b) and Fig. 2(c). Fig. 2(d) shows a 3D Kelvin structure of plate-shape made of 8x25x25 unit-cell elements fabricated by 3D printer for the measurement of porosity using sound velocity in the Kelvin structure. Geometric parameters of the Kelvin unit cell are described in Table 1, where the height and width of the unit cell are designed as about 1.68mm in order to be producible by 3D printer and not to be larger than the wavelength of the acoustic wave. The thicknesses of each faces are identical for the convenience of 3D fabrication and varies from 0.1mm to 0.25mm to change the porosity (volume % of solid) of the Kelvin structure. When the thickness of the closed Kelvin cell is the lowest, i.e., 0.1mm, the porosity of the corresponding 3D Kelvin structure is about 35% in volume ratio. The porosity of the thickest Kelvin structure was about 70%. Fig. 2 shows a 3D CAD model of plate–shape Kelvin structure of 70% porosity. In Fig. 2(d), a thick plate composed of 5,000 Kelvin-unit cells was produced using 3D printer from the CAD data of Fig. 2(b), where the empty spaces in Kelvin closed-cell was filled with Paraffin wax during the 3D printing because the acoustic wave cannot be transmitted through empty spaces.

Table 1. Dimensions of Kelvin unit cell



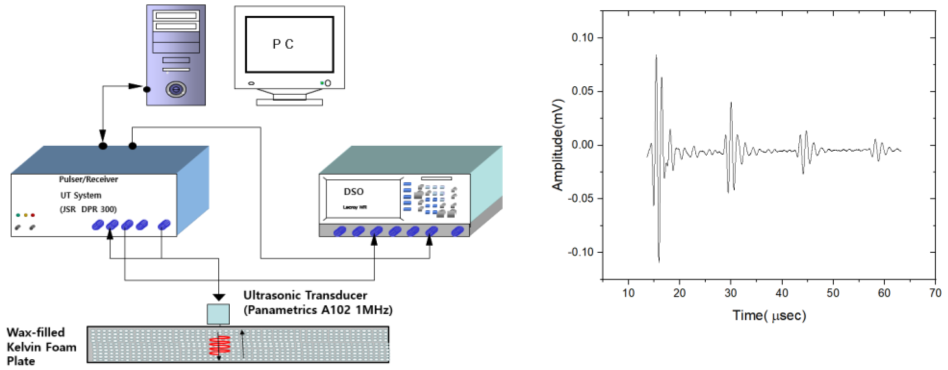
Geometric parameters	Size(μm)
Height(H)	1680
Width(D)	1680
Length of Hexagon(L)	800
Length of Rectangle(b)	800
Angle(θ)	60



(a) (b) (c) (d)

Figure 2. 3D Kelvin structure, (a) unit cell, (b) 3D CAD model, (c) Cross-sectional view (d) Fabricated 3D Kelvin structure (8x25x25 Kelvin closed cells)

4. MEASUREMENT OF ACOUSTIC WAVE VELOCITY IN KELVIN STRUCTURE



(a) (b)

Figure 3. Ultrasonic test of Kelvin structure, (a) Configuration, (b) Reflected acoustic waveforms

Measurement of acoustic velocity of Kelvin structure shown in Fig. 2(d) was conducted by using ultrasonic pulse-echo method, which uses the time-of-flight (TOF) measurement to calculate the wave velocity. The configuration of ultrasonic measurement is set up as illustrated in Fig. 3(a) to measure the TOF of ultrasonic pulse inside the Kelvin structure. In the TOF method, acoustic transducer(Panametrics V103-RB) generates an acoustic pulse signal on the surface of the Kelvin structure and records the reflected waves from the Kelvin structure as shown in Fig. 3(a) in digital oscilloscope (Lecroy wave runner 530). Fig. 3(b) is an example of reflected acoustic waves, where the first large echo signal is reflected from the interface between the acoustic transducer and the Kelvin structure, and followed by the multiple echo signals from the back surface of the Kelvin structure. The duration of the reflected waves (TOF) depends on the thickness of the structure as well as on the acoustic wave velocity of the traveling medium (Kelvin structure) [7-9]. The acoustic velocity c is expressed by the TOF and the thickness of the Kelvin structure, d.

$$c = 2d / \mathrm{TOF} \tag{1}$$

5. SPECTRAL ANALYSIS FOR THE CALCULATION OF PHASE VELOSITY

The TOF method is very easy and simple to realize on actual transducer unless it suffers from low signal-to-noise ratio (SNR) or signal shape distortion. However, unlike solid metals, the Kelvin structure has a great number of small cells and boundaries that makes the acoustic wave scatter during the propagation through the Kelvin structure. This characteristic of porous structure leads to a dispersion of acoustic wave, which appears as a distortion of acoustic waveform. A pulse, being a superposition of many frequencies, will change its shape as it propagates through a dispersive medium. Dispersion effects appears as the frequency dependency of ultrasonic velocities.

In order to overcome this problem, the most dominant single frequency component of the acoustic signal was selected from Fourier transform(FFT), and used to calculate its phase velocity using the spectral phase analysis. In this work, this frequency of the acoustic pulse signal was about 0.75MHz. To describe the dispersion effect, a frequency-dependent wave-number k(ω) is defined to express a broadband longitudinal pulse u(x, t) propagating in the positive x direction as a linear combination of all plane harmonic waves, that is

$$u(x,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{i(k(\omega)x-\omega t)}d\omega \quad \text{or} \quad u(0,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F_0(\omega)e^{i(-\omega t)}d\omega \tag{2}$$

Where, $F(\omega)$ and $F_0(\omega)$ are the Fourier transforms of $u(x,t)$ and $u(0,t)$. If this wave transmits into the Kelvin structure without diffraction and energy loss, the wave after traveling the thickness d of the Kelvin structure is given by the similar expression[10].

$$\begin{aligned} u(d,t) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} F_0(\omega)e^{i(k(\omega)d-\omega t)}d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} F_0(\omega)e^{ik(\omega)d}e^{-i\omega t}d\omega \\ &= \frac{1}{2\pi}\int_{-\infty}^{\infty} F_d(\omega)e^{-i\omega t}d\omega \end{aligned} \tag{3}$$

In Eq. (3), $F_d(\omega)$ is the Fourier transforms of the acoustic signal $u(d,t)$ at the distance d. From Eq. (2) and (3), the phase velocity c of the frequency ω can be determined by calculating $F_0(\omega)$ and $F_d(\omega)$ as follows

$$\frac{F_d(\omega)}{F_0(\omega)} = e^{ik(\omega)d} \quad \text{or} \quad tan^{-1}\left(\frac{F_d(\omega)}{F_0(\omega)}\right) = k(\omega)d = \frac{\omega d}{c(\omega)} \tag{4}$$

Eq. (4) gives the phase velocity $c(\omega)$ of acoustic wave at the frequency ω in terms of the distance d and the ratio of two Fourier transforms, $F_d(\omega)/F_0(\omega)$.

6. RESULTS AND DISCUSSIONS

Total nine samples of Kelvin structure were tested by ultrasonic pulse-echo method to evaluate the phase speed using the spectral analysis. The samples have a different porosity depending on the wall thickness of Kelvin closed-cell ranging from 30% to 70% in volume percent. Table 2 summarizes the measurement results that the phase velocity decreases with porosity. This relationship is represented again in graph in the Fig. 4, where a linear relation of porosity to the phase velocity is clearly seen.

As shown in Eq. (4), the phase velocity of the Kelvin structure was obtained by calculating the phase of the most dominant harmonic component of the acoustic signal from FFT. Since the ATAN function keeps the phase in the interval $(-\pi,+\pi)$, phase unwrapping routines often have to be used to obtain the continuous phase spectrum. The routine simply inserts a 2π radians (or 360°) correction whenever there is a jump of more than π radians. Thus, the phase uncertainty of $\mp 2\pi m$ may arise and the continuous phase spectra may always contain spurious 2π errors that lead to wrong phase velocity results. This error was eliminated in this study by comparing the phase spectral velocity and the approximate velocity from the time-of-flight approach.

Table 2. Experimental results for the phase velocity

Porosity of Specimen	Geometric and acoustic properties	
	Cell thickness(μm)	Phase speed(m/sec)
30%	83	1624
35%	97	1631
40%	110	1635
45%	125	1649
50%	140	1664
55%	163	1662
60%	188	1676
65%	210	1673
70%	237	1691

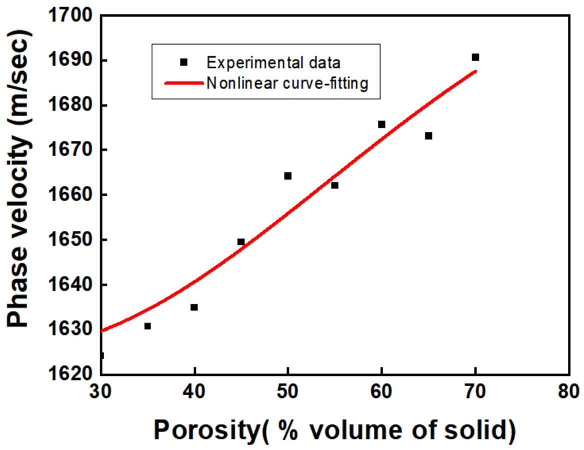


Figure 4. Relationship of porosity to phase velocity

7. CONCLUSIONS

The phase velocities of 3D-printed Kelvin structures with different porosities from 30% to 70% are evaluated by the ultrasonic pulse-echo method based on TOF measurement and spectral analysis. A plate-like 3D Kelvin structures were fabricated by assembling and duplicating the Kelvin closed-unit cell in 3D space with different wall thickness to change the porosity of the Kelvin structure. The Kelvin structures were excited on the surface by 1MHz ultrasonic transducer to penetrate ultrasound into the structure and to record the reflected acoustic waves from the back surface. Fourier transform of the reflection waves was performed to select the most dominant frequency component of the acoustic signal, from which the accurate phase velocity of the harmonic wave at the frequency was calculated by the spectral phase analysis to eliminate the dispersion effect of the highly scattering Kelvin structures. From the experimental results, the phase velocity of the Kelvin samples is estimated successfully that it decreases linearly with the increase of porosity from 1690 m/sec to 1630 m/sec. It can be concluded that the ultrasonic method with spectral phase analysis is effective to evaluate the phase velocity of the Kelvin structures in a fast and simple way. It is also expected that the phase velocity is a great potential tool for the measurement of volume density in medical and industrial applications like bones or metal foams.

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