Original Article

# Insights gained from applying negate-down during quantification for seismic probabilistic safety assessment

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A B S T R A C T

Approximations such as the delete-term approximation, rare event approximation, and minimal cutset upper bound (MCUB) need to be prudently applied for the quantification of a seismic probabilistic safety assessment (PSA) model. Important characteristics of seismic PSA models indicate that preserving the success branches in a primary seismic event tree is necessary. Based on the authors' experience in modeling and quantifying plant-level seismic PSA models, the effects of applying negate-down to the success branches in primary seismic event trees on the quantification results are summarized along with the following three insights gained: (1) there are two competing effects on the MCUB-based quantification results: one tending to increase and the other tending to decrease; (2) the binary decision diagram does not always provide exact quantification results; and (3) it is identified when the exact results will be obtained, and which combination provides more conservative results compared to the others. Complicated interactions occur in Boolean variable manipulation, approximation, and the quantification of a seismic PSA model. The insights presented herein can assist PSA analysts to better understand the important theoretical principles associated with the quantification of seismic PSA models.

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## 1. Introduction

Probabilistic safety assessment (PSA) of nuclear power plants is performed using integrated PSA software packages such as AIMS-PSA [1], SAREX [2], CAFTA [3], and SAPHIRE [4], which generate minimal cutsets from the event and fault trees. The core damage frequency (CDF) is then calculated using the frequencies of the minimal cutsets. PSA quantification engines such as FTREX (fault tree reliability evaluation expert) [5] that generate minimal cutsets are extensively used for fast quantification. In an internal event PSA, rare event approximation (REA) and minimal cutset upper bound (MCUB) are extensively applied for calculating the CDF under assumptions of low event failure probabilities and relatively loose dependences among the minimal cutsets, respectively.

Seismic PSA includes seismic hazard analysis, seismic fragility analysis, systems analysis, and quantification. Seismic hazard analysis provides seismic hazard curves, which describe the exceedance frequencies of earthquake levels (ground motion levels) along with their uncertainties. Seismic fragility analysis provides the seismic fragility data of structures and equipment. Systems analysis provides a logic model after the occurrence of a seismic initiating event, with event and fault trees in which the seismic failures of the structures and equipment are considered. The logic model is quantified with risk quantification software.

EPRI TR 3002000709 [6] explains two different approaches for the quantification of seismic risk: an approach for developing a discrete hazard interval model and alternative quantification methods based on Monte Carlo simulation and Latin Hypercube sampling. While alternative quantification methods have been extensively used with computer programs such as PRASSE [7], the approach for developing a discrete hazard interval model has been gradually adopted in seismic PSA. One of the most important advantages of the newly adopted approach is the preservation of the logical links between the primary and secondary seismic event trees. By preserving the logical links, the minimal cutsets can include necessary information on the failure combination that leads to core damage.

When developing a discrete hazard interval model for the quantification of seismic risk, it is necessary to appropriately consider high failure probability events that may exist with the increase in the ground motion level [8]. Some of the approximations applicable to internal event PSA may not be applicable to

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seismic PSA. Hence, if a seismic PSA model is quantified similar to an internal event PSA model, significantly conservative results may be obtained. For example, although PSA quantification engines generally delete the success branches of event trees after delete-term approximation [9,10], it may lead to incorrect results if the success branches are not appropriately considered during the quantification of a seismic PSA model [2,3]. In the Surry pilot seismic PSA project [11], a sensitivity analysis was performed to reduce conservatism by quantifying the success factor using minimal cutset post-processor software. For more accurate quantification results, quantification methods based on the binary decision diagram (BDD) and software tools such as the Advanced Cutset Upper Bound Estimator (ACUBE) [12] that implement these methods need to be applied. Han et al. [13] compared the characteristics of several quantification methods by applying them to two simple examples. Lim [14] also applied BDD-based quantification methods to a seismic PSA model with logically linked primary and secondary seismic event trees.

A seismic PSA model includes a primary seismic event tree, secondary seismic event trees, and fault trees associated with the failure branches in the event trees. In the primary seismic event tree, each sequence is modeled to result in either direct core damage or another initiating event such as the loss of offsite power or general transients. If a detailed analysis is required for such an initiating event, the sequence is linked to a secondary seismic event tree. Hence, it is crucial to accurately calculate the sequence frequencies of the primary seismic event tree because they become the frequencies of the initiating events connected to the secondary seismic event trees [8]. Therefore, negate-down of the success branches in the primary seismic event tree has been recently introduced [5]; however, its characteristics remain to be elucidated.

For accurate quantification and appropriate comprehension of the quantification results of seismic PSA, it is important to understand the characteristics of the quantification methods and their effect on the quantification results under a seismic PSA environment. This study intends to present the insights gained on the application of negate-down to the success branches in the primary seismic event tree based on the authors' experience in modeling and quantifying plant-level seismic PSA models. Section 2 offers a brief review on PSA quantification methods and the relationship among them. Section 3 describes the quantification characteristics in seismic PSA and introduces the negate-down of success branches in primary seismic event trees. Section 4 presents the insights gained with the application of negate-down in primary seismic event trees during seismic PSA quantification. Section 5 concludes the study.

## 2. PSA quantification methods

To examine the applicability to seismic PSA models, we compare the quantification methods frequently used for internal event PSA models. There are several quantification methods such as the REA, MCUB, and BDD. REA is a commonly used quantification method that provides more accurate results when the probabilities of all the basic events are significantly lesser than unity. MCUB provides accurate results, if all the minimal cutsets are independent of each other. BDD provides accurate quantification results.

According to Jung et al. [15], a fault tree is coherent if its Boolean function is nondecreasing and each basic event is relevant. A coherent fault tree can be represented with AND OR logic, without using NOT logic. When $P_{REA}$, $P_{MCUB}$, and $P_{BDD}$ are the top event probabilities for any coherent fault tree using REA, MCUB, and BDD, respectively, the relationship among the top event probabilities is [16].

$$P_{BDD} \leq P_{MCUB} \leq P_{REA}. \tag{1}$$

Hence, while BDD provides exact quantification results, MCUB provides conservative (or equal) quantification results compared to BDD, and REA provides conservative (or equal) quantification results compared to both BDD and MCUB.

Fig. 1 depicts an example event tree for examining the PSA quantification methods. $\%I$ is an initiating event and $C_i$ is a logical expression where $\%I \cap C_i$ becomes a minimal cutset. Two minimal cutsets $\{\%I \cap C_1, \%I \cap C_2\}$ correspond to sequence-2, whereas another two minimal cutsets $\{\%I \cap C_3, \%I \cap C_4\}$ correspond to sequence-3.

Table 1 lists the mathematical expressions of the different quantification methods applied to calculate the CDF for the example shown in Fig. 1. $\Pr(\%I)$ is the frequency of the initiating event and $\Pr(C_i)$ is the probability of. $C_i$.

REA approximates the CDF using the sum of the frequencies of the minimal cutsets. Although it provides a simple method to approximate the CDF, it yields conservative results compared to the other quantification methods, as indicted by Eq. (1).

In Table 1, three MCUB-based methods are applied. If MCUB is used in its original form without suitably considering the initiating event (MCUB without separating the initiating events), the frequency of the initiating event may not be separated from the other probabilities. When the initiating event is suitably separated, MCUB can be applied in two different ways depending on whether the minimal cutsets are separated according to their associated sequences. MCUB with mutually exclusive sequences applies MCUB to the minimal cutsets in the same sequence, assuming that the minimal cutsets in different sequences are mutually exclusive. MCUB with ignoring sequences applies MCUB to all the minimal cutsets, regardless of the sequences. In PSA, both types of MCUB-based quantifications with initiating event separation are used.

BDD is the graphical representation of a logical expression based on the Shannon decomposition. BDD-based quantification may provide exact quantification results for relatively small PSA models.

## 3. Seismic PSA quantification characteristics

There are several important characteristics in the quantification of a seismic PSA model compared to that of an internal event PSA model. The CDF is significantly affected by the ground motion level due to a seismic event. As the ground motion level increases, the failure probabilities of the systems, structures, and components also increase to values that cannot be considered rare events.

As an example, Fig. 2 shows 5%, 50%, 95%, and the mean of the seismic failure probabilities of the offsite power, when the median capacity ($A_m$) is 0.3 g and uncertainty parameters ($\beta_r$, $\beta_u$) are 0.3 and 0.45, respectively. The fragility data ($A_m$, $\beta_r$, $\beta_u$) are from a previous study [17]. When an earthquake with a ground motion level of 0.6 g occurs, the mean failure probability of the offsite power approaches 0.9. The quantification results may be inaccurate when REA is used because it is suitable only when the basic events have low probabilities.

Hence, we need BDD-based quantification, which provides exact results; however, it requires considerable computing resources. For large PSA models such as those for nuclear power plants, the method developed by Jung [18] is applied to quantify the risk. In this method, BDD-based quantification is applied to a specified number of high-ranking minimal cutsets, whereas MCUB-based quantification is applied to the remaining low-ranking minimal cutsets. This is because the high-ranking minimal cutsets generally account for a major portion of the risk, whereas the remaining low-ranking minimal cutsets account for a minor portion. This method renders it possible to quantify the seismic PSA model of a nuclear
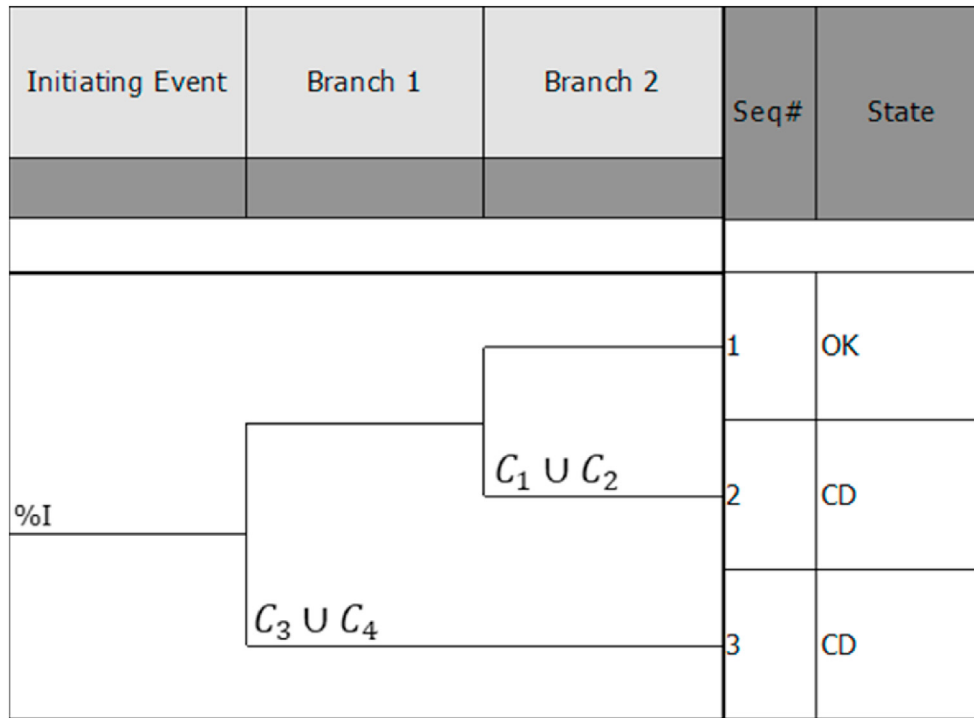
**Fig. 1.** Example event tree for examining the PSA quantification methods.

**Table 1**
Mathematical expressions to calculate the CDF using the quantification methods.

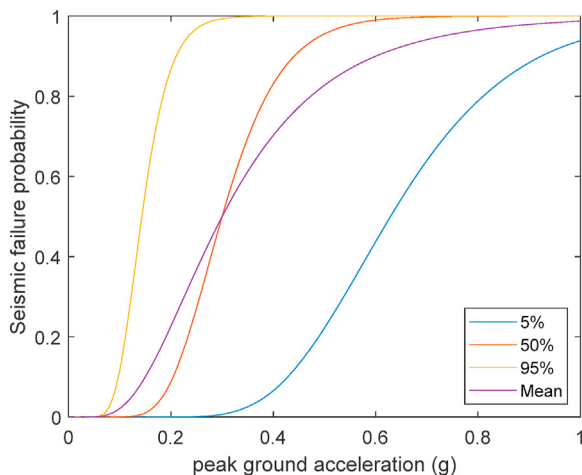| Quantification method | Mathematical expression to calculate the CDF |
|---|---|
| REA | $\Pr(\%I) \cdot [\Pr(C_1) + \Pr(C_2) + \Pr(C_3) + \Pr(C_4)]$ |
| MCUB without separating the initiating events | $1 - [(1 - \Pr(\%I)\Pr(C_1))(1 - \Pr(\%I)\Pr(C_2))(1 - \Pr(\%I)\Pr(C_3))(1 - \Pr(\%I)\Pr(C_4))]$ |
| MCUB with mutually exclusive sequences | $\Pr(\%I) \cdot [(1 - (1 - \Pr(C_1))(1 - \Pr(C_2))) + (1 - (1 - \Pr(C_3))(1 - \Pr(C_4)))]$ |
| MCUB with ignoring sequences | $\Pr(\%I) \cdot [1 - (1 - \Pr(C_1))(1 - \Pr(C_2))(1 - \Pr(C_3))(1 - \Pr(C_4))]$ |
| BDD (exact result) | $\Pr(\%I) \cdot [\Pr(C_1) + \Pr(\overline{C_1} \cap C_2) + \Pr(\overline{C_1} \cap \overline{C_2} \cap C_3) + \Pr(\overline{C_1} \cap \overline{C_2} \cap \overline{C_3} \cap C_4)]$ |



**Fig. 2.** Seismic failure probability of the loss of offsite power for generic data.

power plant with a moderate level of computing resources. While BDD-based quantification provides exact results for the major portion of the risk, MCUB-based quantification may overestimate the minor portion of the risk.

Some of the sequence frequencies of the primary seismic event tree become the initiating event frequencies of the secondary seismic event trees; hence, accurate quantification of the sequence frequencies becomes critical. While generating minimal cut sets from a PSA model, the delete-term approximation is normally used. As discussed by Epstein and Rauzy [19], delete-term approximation may cause errors when high failure probability events are involved in the success branches of the primary seismic event tree. Therefore, a method for preserving the success branches in the primary seismic event tree is necessary by exempting delete-term approximation for success branches with high failure probability events. FTREX recently implemented a new feature involving the negate-down of selected success branches (NOT gates in fault trees) in the primary seismic event tree by expanding the success branches (NOT gates) to success events (basic events with NOT logic) using de Morgan's theorem shown below.

$$\overline{A \cap B} = \overline{A} \cup \overline{B} \tag{2}$$

$$\overline{A \cup B} = \overline{A} \cap \overline{B} \tag{3}$$

Fig. 3 displays an example in which negate-down is applied to a success branch ($\overline{G2}$). The frequency of the initiating event (%I) is 0.1 (per year). The probabilities of basic events A, B, and C are 0.1, 0.2, and 0.3, respectively. For the fault tree on the left, delete-term approximation is applied to A and $\overline{G2}$; hence, the minimal cutset is
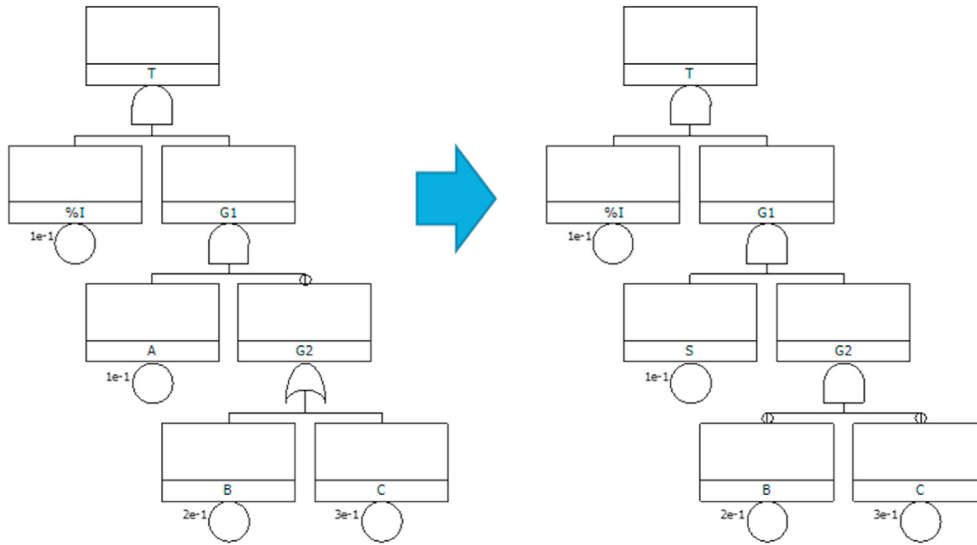
**Fig. 3.** Example of negate-down application to a success branch ($\overline{G2}$).

$$T = \%I \cap (A \cap \overline{G2}) = \%I \cap (A \cap \overline{B \cup C}) \approx \%I \cap A. \tag{4}$$

The probability of $T$ is approximated as the product of the initiating event frequency and the probability of $A$, which is 0.01 ($= 0.1 \times 0.1$).

For the fault tree on the right, negate-down is applied to success branch ($\overline{G2}$) by expanding it to success events (basic events with NOT logic, i. e., $\overline{B}, \overline{C}$). The minimal cutset is

$$T = \%I \cap (A \cap \overline{B \cup C}) = \%I \cap (A \cap (\overline{B} \cap \overline{C})) = \%I \cap A \cap \overline{B} \cap \overline{C}. \tag{5}$$

From Eq. (3), the exact probability of $T$ is calculated to be 0.0056 ($= 0.1 \times 0.1 \times 0.8 \times 0.7$).

By applying negate-down to the success branches in the primary seismic event tree, the minimal cutsets for different sequences become mutually exclusive and the sequence frequencies become exact. The sequence frequencies are important because some of them become the initiating event frequencies of the secondary seismic event trees.

After minimal cut sets are generated, the combination of BDD and MCUB approximation is used in seismic PSA, instead of REA used in internal event PSA, owing to the existence of high failure probability events. Because different approaches are used for their purposes, the combination of different approaches does not result in significant problems if the approaches are properly applied.

While the negate-down approach helps increase the accuracy of seismic PSA quantification, it also increases the complexity of the seismic PSA model and hence requires more computing resources. Owing to its significant computing resource requirement, the application of the negate-down approach to an entire plant-size PSA model is not possible. For applying negate-down, the success branches should be carefully selected by analysts, considering their effect on the quantification results. The best way to use the negate-down approach is to apply it to success branches with simple logic and high failure probability events, such as seismic failure events, that may significantly affect the quantification results. Such events are generally found in primary seismic event trees; however, the application of negate-down to success branches is not limited to primary seismic event trees. While the negate-down approach can be applied to secondary seismic event trees to which large fault trees are linked, inadequate selection of the success branches to

which negate-down is applied may significantly complicate quantification without producing a meaningful difference in the quantification results or even make the quantification impossible.

Depending on the modeling approach, secondary seismic event trees may include high probability events, such as seismic failures. If success branches with high probability events, irrespective of whether they are in the primary or secondary seismic event tree, are not selected for negate-down application, a similar quantification problem may occur. BDD-based quantification for the entire seismic PSA model can be a solution for such a problem. As this solution is not widely available for seismic PSA, the current practice involves the application of negate-down to carefully selected success branches in event trees to provide the best approximation for the seismic risk of a nuclear power plant.

In other words, the cut set approach with delete-term approximation can be applied to PSA models without high failure probability events, and hence is good for most internal event PSA models. When high failure probability events exist in a PSA model such as those for seismic PSA, the cut set approach with delete-term approximation combined with the conversion of cut sets to BDD provides better quantification results. Only finite number of high-ranking minimal cut sets can be converted to a BDD owing to computational resource limitation. However, errors may still exist when the delete-term approximation is applied to success branches with high failure probability events. When a PSA model can be fully converted to a BDD, such errors can be eliminated but such a full-sized BDD can be constructed only when the PSA model is very small. The use of negate-down approach to those success branches with high failure probability events, may significantly reduce such errors in exchange of slightly increased quantification complexity.

In summary, high failure probability events may exist in a seismic PSA model, especially when the ground motion level increases. Owing to the existence of high failure probability events, the quantification methods valid in internal event PSA may result in excessively conservative results in seismic PSA. The use of BDD is introduced for more exact quantification, but BDD is only partially used due to limitations in computational resources. Currently, the combination of BDD and MCUB is used for the quantification of a seismic PSA model.

Delete-term approximation, which is an effective approximation method in internal event PSA, may result in conservative

quantification results in seismic PSA when high failure probability events exist. A new feature, negate-down of selected success branches, is introduced for more accurate quantification, but the effect of the newly introduced feature on the quantification results has not been widely investigated. In the following section, the effects of the negate-down of success branches on the seismic PSA quantification results are summarized along with the three insights gained.

## 4. Insights from seismic PSA quantification

As negate-down of the success branches in the primary seismic event tree has been newly introduced in seismic PSA, its effect on the quantification results must be carefully examined. Because high failure probability events are involved in seismic PSA models, MCUB and BDD-based quantification methods are mainly focused upon. The effects of the negate-down of success branches on the quantification results are summarized along with the three insights gained.

### 4.1. Competing effects on the sequence frequencies and CDF

When MCUB-based quantification is used, two competing effects due to the negate-down of success branches in the primary seismic event tree are identified. One effect tends to reduce the frequency of each minimal cutset by multiplying the probabilities less than unity. For example, NOR gate $\overline{A \cup B}$ can be expanded to the product of the basic events with NOT logic $\overline{A} \cap \overline{B}$, according to de Morgan's theorem. Because the probabilities of $\overline{A}$ and $\overline{B}$ are always lesser than unity, the frequency of each minimal cutset additionally containing $\overline{A} \cap \overline{B}$ will be reduced. For the example depicted in Fig. 3, the probability of $T$ is reduced from 0.01 (delete-term approximation result) to 0.0056 (exact result by applying negate-down to success branch $\overline{G2}$) because the minimal cutset additionally contains $\overline{B} \cap \overline{C}$, whose probability is 0.56 ($= 0.8 \times 0.7$).

The other effect tends to increase the number of minimal cutsets. For example, NAND gate $\overline{A \cap B}$ can be expanded to $\overline{A} \cup \overline{B}$, which doubles the number of minimal cutsets. By considering the equation for MCUB-based quantification, doubling the number of minimal cutsets is expected to almost double the frequencies associated with the minimal cutsets.

When these two competing effects combine, MCUB-based quantification results are conservative at low ground motion levels, and gradually decrease as the ground motion level increases. This is because the frequency of each minimal cutset decreases with the increase in the ground motion level, whereas the number of minimal cutsets does not depend on the ground motion level. However, when the success branches to which negate-down is applied do not include NAND gates, the MCUB-based quantification results may not be conservative.

Fig. 4 displays an example event tree to demonstrate the competing effects on the sequence frequencies and CDF when negate-down is applied to the success branches. Fig. 4 can be considered as an example primary event tree. If a sequence, for example Sequence 1, needs to be further analyzed, a secondary event tree linked to Sequence 1 can be developed. The frequency of the initiating event, %I, is given as 0.1 (per year). The probabilities of basic events X, Y, K, L, and O are given as 0.1, 0.3, 0.1, 0.1, and 0.1, respectively. Table 2 compares the minimal cutsets, their frequencies, and the corresponding sequences, depending on whether negate-down is applied (newly introduced or not (conventional). The frequency of each minimal cutset corresponding to sequence-2 is reduced from 1.00E-02 (when negate-down is not applied) to
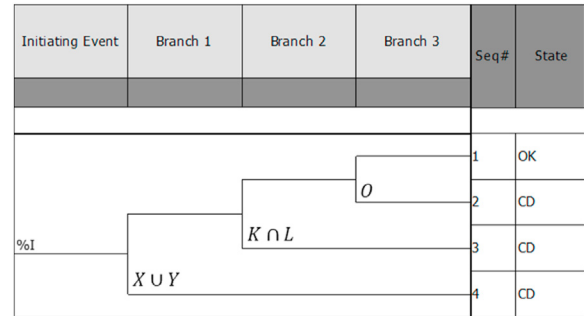


**Fig. 4.** Example event tree for demonstrating the competing effects on the sequence frequencies and CDF.

5.67E-03 when negate-down is applied. Moreover, the frequency of sequence-3 is reduced from 1.00E-03 to 6.30E-04 when negate-down is applied to the success branches.

However, the expansion of $\overline{K \cap L}$ to $\overline{K} \cup \overline{L}$, according to de Morgan's theorem, doubles the number of minimal cutsets for sequence-2 when negate-down is applied. This almost doubles the sequence frequency of sequence-2 and increases the CDF as well.

Table 3 compares the quantification results of this specific example for two different values 0.1 and 0.2 of Pr(K) and Pr(L) using REA, MCUB with mutually exclusive sequences, MCUB with ignoring sequences, and BDD, depending on whether negate-down is applied (newly introduced) or not (conventional). The results of the REA method are depicted in Table 3 for comparison only and are not considered significant. When Pr(K) and Pr(L) are 0.1, the REA and MCUB quantification results are more conservative when negate-down is applied to the success branches, compared to those when negate-down is not applied. On the other hand, when Pr(K) and Pr(L) are increased to 0.2, the REA and MCUB quantification results are more optimistic when negate-down is applied to the success branches, compared to those when negate-down is not applied. This is an example of the two competing effects. BDD provides exact results in all the cases.

Fig. 5 illustrates the two competing effects by presenting the changes in the quantification results with the increase in Pr(K) and Pr(L) when negate-down is applied to the success branches. $P_{NoDTA}$ and $P_{DTA}$ denote the quantification results when negate-down is applied to the success branches (delete-term approximation is not applied) and when it is not (delete-term approximation is applied), respectively. The $P_{NoDTA}/P_{DTA}$ ratio decreases with the increase in the ground motion level.

In summary, the negate-down of success branches has two competing effects. When these two competing effects combine, the MCUB-based quantification results tend to be conservative when negate-down is applied to the success branches ($P_{NoDTA}$) compared to those when it is not applied ($P_{DTA}$), at low ground motion levels. As the ground motion level increases, the MCUB-based quantification results with negate-down of the success branches ($P_{NoDTA}$) tend to become smaller than those without negate-down ($P_{DTA}$).

### 4.2. BDD does not always provide exact quantification results

It is a common belief in the PSA field that BDD provides exact quantification results. However, when delete-term approximation is applied, this is true only under specific conditions. The quantification results in Table 3 are an example that BDD provides exact results when delete-term approximation is applied. In general, there are certain cases where BDD may not provide exact quantification results.

Fig. 6 shows such an example. In the event tree, sequences 3 and

**Table 2**
Comparison of the minimal cutsets, their frequencies, and the corresponding sequences, depending on whether negate-down is applied (newly introduced) or not (conventional).

| Negate-down | Sequence number | Minimal cutset frequency | Minimal cutset | | | | |
|---|---|---|---|---|---|---|---|
| No (conventional) | Sequence 2 | 1.00E-02 | %I | O | | | |
| | Sequence 3 | 1.00E-03 | %I | K | L | | |
| | Sequence 4 | 3.00E-02 | %I | Y | | | |
| | Sequence 4 | 1.00E-02 | %I | X | | | |
| Yes (newly introduced) | Sequence 2 | 5.67E-03 | %I | /K | O | /X | /Y |
| | Sequence 2 | 5.67E-03 | %I | /L | O | /X | /Y |
| | Sequence 3 | 6.30E-04 | %I | K | L | /X | /Y |
| | Sequence 4 | 3.00E-02 | %I | Y | | | |
| | Sequence 4 | 1.00E-02 | %I | X | | | |

**Table 3**
Quantification results for the example event tree when $\Pr(K) = \Pr(L) = 0.1$ and $\Pr(K) = \Pr(L) = 0.2$

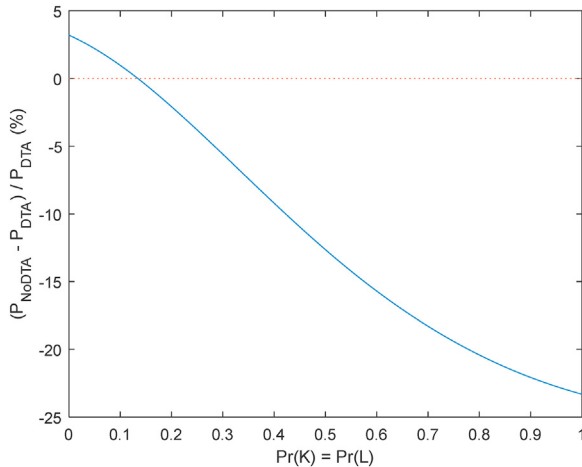| $\Pr(K) = \Pr(L)$ | Negate-down | REA | MCUB with mutually exclusive sequences | MCUB with ignoring sequences | BDD |
|---|---|---|---|---|---|
| 0.1 | No (conventional) | 5.10E-02 | 4.80E-02 | 4.39E-02 | 4.39E-02 |
| | Yes (newly introduced) | 5.20E-02 | 4.86E-02 | 4.43E-02 | 4.39E-02 |
| 0.2 | No (conventional) | 5.40E-02 | 5.10E-02 | 4.56E-02 | 4.56E-02 |
| | Yes (newly introduced) | 5.26E-02 | 4.93E-02 | 4.43E-02 | 4.56E-02 |



**Fig. 5.** Relative changes in the quantification results with the increase in $\Pr(K)$ and $\Pr(L)$ when negate-down is applied to the success branches.

5 lead to core damage. Based on the logical relationship shown in the event tree, the exact CDF, $F_{CD}$, can be calculated as

$$F_{CD} = \Pr(\%I)[\Pr(A \cap C) + \Pr(\overline{A} \cap B \cap D)]. \tag{6}$$



**Fig. 6.** Example event tree to demonstrate that BDD does not always provide exact quantification results.

This exact CDF can be calculated when negate-down is applied; hence, delete-term approximation is not applied to success branch ($\overline{A}$) in the second term in Eq. (6).

When delete-term approximation is applied, the minimal cutsets are {A, C} and {B, D}. The application of BDD to these minimal cutsets results in the following CDF quantification result denoted as $F_{CD,DTA}$:

$$\begin{aligned}
F_{CD,\,DTA} &= \Pr(\%I)[\Pr(A \cap C) + \Pr(\overline{A \cap C} \cap B \cap D)] \\
&= \Pr(\%I)[\Pr(A \cap C) + \Pr((\overline{A} \cup \overline{C}) \cap (B \cap D))] \\
&= \Pr(\%I)[\Pr(A \cap C) + \Pr((\overline{A} \cup (\overline{\overline{A}} \cap \overline{C})) \cap (B \cap D))] \\
&= \Pr(\%I)[\Pr(A \cap C) + \Pr((\overline{A} \cup (A \cap \overline{C})) \cap (B \cap D))] \\
&= \Pr(\%I)[\Pr(A \cap C) + \Pr(\overline{A} \cap B \cap D) + \Pr(A \cap \overline{C} \cap B \cap D)] \neq F_{CD.}
\end{aligned} \tag{7}$$

Eq. (7) includes an additional term $\Pr(\%I) \cdot \Pr(A \cap \overline{C} \cap B \cap D)$ along with the first two terms for the exact probability, which renders the BDD-based quantification result slightly conservative. Therefore, when delete-term approximation is applied, BDD-based quantification may produce slightly conservative results compared to the exact ones. More specifically, when all the sequences after the failure of A in Fig. 6 lead to core damage, we can expect exact results. However, if there are sequences that do not lead to core damage after the failure of A in Fig. 6, such as sequence 4, we can expect conservative quantification results. Therefore, this example establishes that BDD-based quantification does not always provide exact quantification results when delete-term approximation is applied.

### 4.3. Effect on BDD and MCUB-based quantification results

The expected effects on the quantification result using BDD and MCUB when negate-down is applied to the success branches are summarized in Table 4. Without negate-down, BDD-based quantification is expected to produce slightly conservative results. Exact results can be obtained under specific conditions where all the failure paths lead to core damage. In this condition, there is no additional term from BDD-based quantification, even though negate-down is not applied. When negate-down is applied to the success branches, BDD-based quantification is expected to produce exact results by rendering the Boolean expressions for the sequences mutually exclusive.
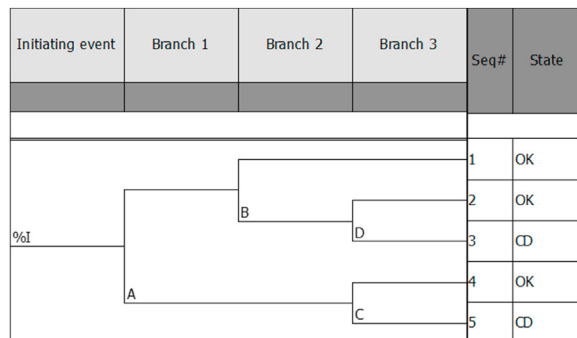
**Table 4**
Effect of avoiding delete-term approximation on BDD-based and MCUB-based quantification results.

| Negate-down | BDD-based quantification | MCUB-based quantification |
|---|---|---|
| No (conventional) | Conservative (Exact under specific conditions) | Conservative (Exact when all the minimal cutsets are independent) |
| Yes (newly introduced) | Exact | Competing effects |

Without negate-down, MCUB-based quantification (with ignoring sequences) is expected to produce conservative results. Exact quantification results can be obtained only when all the minimal cutsets are independent. When negate-down is applied to the success branches, it is difficult to determine whether the MCUB-based quantification results are smaller or greater than those when negate-down is not applied owing to the two competing effects explained in Section 4.1.

Table 4 indicates that the MCUB part may underestimate the risk when negate-down is applied to the selected success branches, whereas the BDD part provides exact results. One possible solution to this problem is to apply BDD-based quantification to the entire seismic PSA model. Even though this solution is not easy to achieve, it may significantly improve the quantification accuracy of seismic PSA models.

## 5. Conclusions

Seismic PSA includes hazard and fragility analyses, the results of which need to be reflected in a seismic PSA model. Seismic failure probabilities are often sufficiently large at high ground motion levels, rendering the quantification methods valid for internal event PSA invalid for seismic PSA. Moreover, the sequence frequencies in the primary seismic event tree need to be calculated as exactly as possible because some of them become the initiating event frequencies of the secondary seismic event trees. Hence, BDD-based quantification needs to be adopted for the quantification of seismic PSA models.

Delete-term approximation is an important approximation technique used in internal event PSA. For more accurate quantification of the sequence frequencies, negate-down of selected success branches in the primary seismic event tree has been newly introduced by exempting the application of delete-term approximation to the success branches. The negate-down approach can be best applied to the success branches with simple logic and high failure probability events, such as seismic failure events, that may significantly affect the quantification results. Based on the authors' experience in modeling and quantifying plant-level seismic PSA models, the effect of negate-down on the quantification results are summarized as the insights gained:

- When MCUB-based quantification is used, there are two competing effects on the sequence frequencies and CDF. Owing to these effects, the MCUB-based quantification results may increase or decrease. In general, there is a decreasing trend with the increase in the ground motion level.
- Contrary to common belief, BDD-based quantification does not always provide exact results. BDD-based quantification with delete-term approximation may provide slightly conservative results.
- For different combinations of quantification methods and the application of negate-down, it can be determined whether exact results can be obtained, and the combination that provides comparatively more conservative results can be identified.

These insights gained are not exhaustive, and the quantification characteristics of the newly introduced negate-down of the success branches in seismic PSA warrant further elucidation.

As BDD-based quantification cannot be solely applied to a large plant-size PSA model, a combination of BDD and MCUB is used in software tools. Complicated interactions occur in Boolean variable manipulation, approximation, and quantification. It is recognized that insufficient understanding of the theoretical backgrounds and quantification processes may lead to significantly incorrect results. The insights gained presented in this study can assist PSA analysts in understanding important theoretical principles beyond the numerical results that different combinations of approximation and quantification methods provide during the quantification of seismic PSA models.

## Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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