

A review on a 4 K cryogenic refrigeration system for quantum computing

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Abstract

This paper reviews the literature that has been published since 1980s related to cryogenic refrigeration systems for quantum computing. The reason why such a temperature level of 10-20 mK is necessary for quantum computing is that the superconducting qubit is sensitive to even very small thermal disturbances. The entanglement of the qubits may not be sustained due to thermal fluctuations and mechanical vibrations beyond their thresholds. This phenomenon is referred to as decoherence, and it causes a computation error in operation. For the stable operation of the quantum computer, a low-vibration cryogenic refrigeration system is imperative as an enabling technology. Conventional dilution refrigerators (DR), so called 'wet' DR, are precooled by liquid helium, but a more convenient and economical precooling method can be achieved by using a mechanical refrigerator instead of liquid cryogen. These 'dry' DRs typically equip pulse-tube refrigerators (PTR) for precooling the DRs around 4 K because of its particular advantage of low vibration characteristic. In this review paper, we have focused on the development status of 4 K PTRs and further potential development issues will be also discussed. A quiet 4 K refrigerator not only serves as an indispensable precooler of DR but also immediately enhances the characteristics of low noise amplifiers (LNA) or other cryo-electronics of various type quantum computers.

Keywords: cryogenic refrigeration, dilution refrigerator (DR), pulse-tube refrigerator (PTR), quantum computing

1. INTRODUCTION

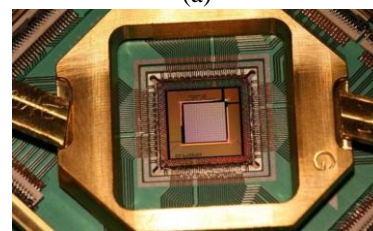
Quantum computers, based on a new idea of using a phenomenon called quantum entanglement, are expected to be used in special fields where proper calculation cannot be performed by conventional computers. In 1981, Richard Feynman [1] predicted the possibility of quantum computing. Since then, as quantum algorithms became further developed, the development of quantum computers and their relevant research has been ongoing having started in 1994 [2].

Fig. 1 shows the world's first quantum computer cooling system and a superconducting quantum qubit circuit board [3]. The state-of-the-art quantum computer, as seen in those manufactured by IBM or Google, is a form of harnessing a superconducting qubit that utilizes quantum states that appear in superconducting phenomena for computations. In order to create a cryogenic temperature, a dilution refrigerator (DR) that can reach a minimum temperature level of 10-20 mK is being adopted for use in quantum computers [4]. The reason why such a low temperature is required for a quantum computer is that the qubit, a basic element that constitutes a quantum computer, is sensitive to even very small disturbances, e.g., the earth's magnetic field. Due to thermal fluctuations and mechanical vibration (or microphonics), the entanglement of the qubits cannot be sustained. This phenomenon is referred to as decoherence,

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(a)



(b)

Fig. 1. (a) Cryogenic refrigeration system for D-Wave and (b) its superconducting quantum qubit circuit [3].

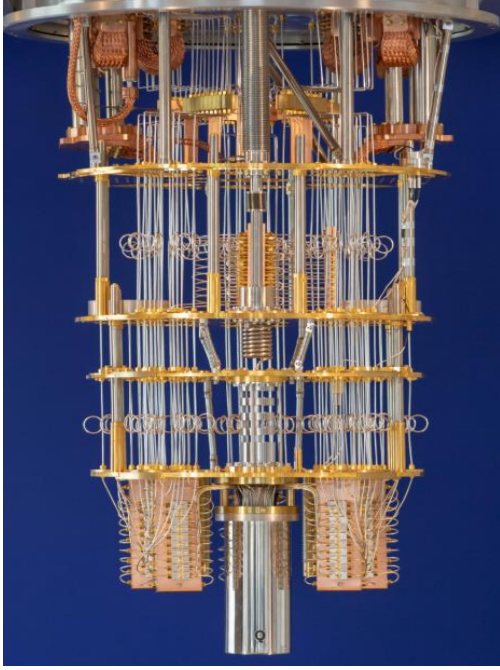


Fig. 2. Quantum computer (Q system one, IBM) [5]

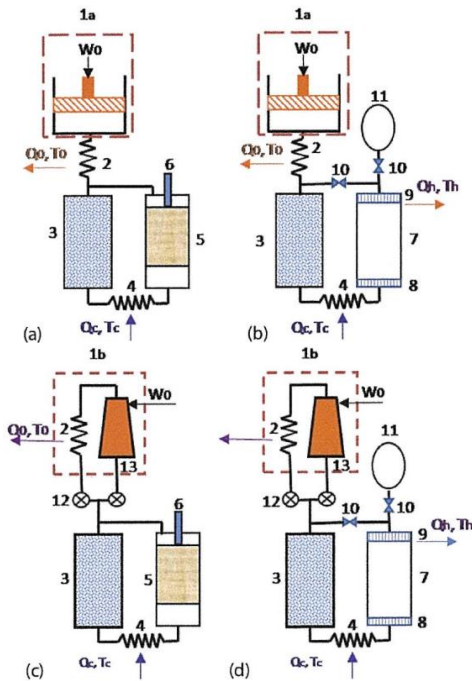


Fig. 3. Schematics of regenerative cryocoolers [6]; (a) Stirling cryocooler, (b) Stirling-type PTR, (c) GM cryocooler and (d) GM-type PTR.

and it causes an error in operation. For stable computer operation, the external environment of the qubit must be manipulated to slow down decoherence as much as possible. In other words, a low-vibration cryogenic refrigeration system that can quickly dissipate the heat that is generated during computation and minimize thermal/mechanical noise is an enabling technology for quantum computing. Fig. 2 shows IBM's quantum computer (Q system one), one of the most advanced

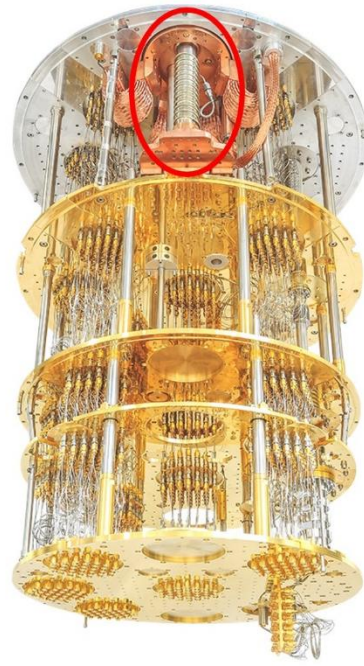


Fig. 4. Dilution refrigerator (Bluefors, GM-type PTR installed at the top side) [11].

quantum computers to date. From the top, there is a pulse-tube refrigerator (PTR) for cooling from room temperature to 4.2 K, a dilution refrigerator (DR) for cooling down to 20 mK and finally, at the center of the base, a superconducting quantum qubit [5].

The upper part of the cooling system for such a quantum computer can be divided into a classical method using liquid helium (normal boiling point of 4.2 K) and a dry system introducing a mechanical refrigerator. Among these, in the case of a dry system, a DR is pre-cooled by means of a PTR [7–9]. In this review paper, 4.2 K GM-type PTR, which is used in most dry DR, will be investigated, and the characteristics of current dry DRs and further development potential will be also discussed.

2. CRYOGENIC REFRIGERATION SYSTEM

As shown in Fig. 3(a) and 3(c), the solid displacer in Stirling and GM (Gifford-McMahon) cryocoolers has several drawbacks. It makes the system to have a short lifetime, create a source of mechanical vibration and contribute not only to axial conduction loss but a shuttle loss. In the PTR, shown in Fig. 3(b), the solid displacer is eliminated and it is replaced to a fictitious gas displacer. The phase controller can provide a proper phase difference between mass flow and pressure within the pulse tube. The PTR is driven by a valveless compressor and it is commonly called as a Stirling-type PTR. Since the pressure oscillates everywhere within the Stirling-type PTR, excessive dead-volume must be minimized to maximize pressure magnitude for a given swept volume of the compressor. For this reason, oil-removal system thus cannot be equipped, which means that the moving piston

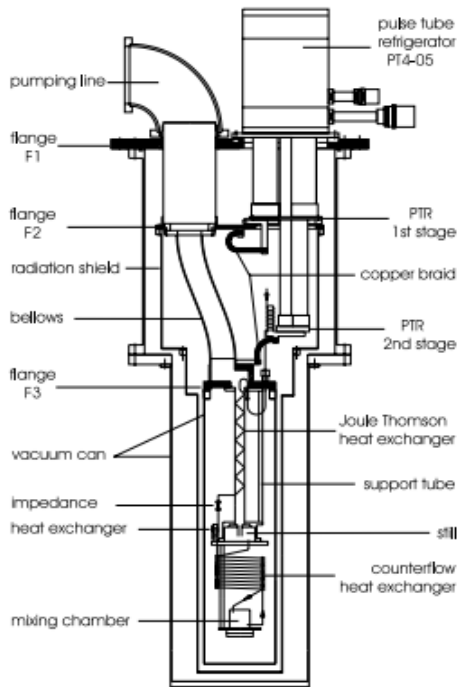


Fig. 5. Schematic of dry DR [24].

must be oil-free and the Stirling-type PTR is compact. PTR also can be driven by a valved compressor like that for the GM cryocooler, as shown in Fig. 3(d). This is commonly referred to as a GM-type PTR. Although the valve loss significantly reduces the efficiency of the cycle, the advantage of this approach is that it allows for an oil-lubricated scroll compressor with oil-removal system including a coalescer and adsorber. The scroll compressors are readily available from the air-conditioning industry [10]. The PTR can be summarized into two different types with respect to the operating mechanism;

- Stirling-type: PTR driven by an oil-free, valve-less linear compressor (frequency: several tens of Hz, pressure ratio: 1.2-1.4)
- GM-type: PTR driven by oil-sealed, valved screw compressor (frequency: 1-2 Hz, pressure ratio: 3-4)

The absence of the solid displacer is advantageous in that it has higher reliability, lower vibration and less electromagnetic interference (EMI) compared to that Stirling or GM cryocoolers have. Since a system using liquid helium requires additional effort and cost to handle the cryogen, dry dilution refrigerator (DR) with a PTR could be more attractive. Despite the advantages mentioned above, the PTR used for dry DRs cannot provide a perfect vibration-free environment because of the vibrations that are transmitted from the compressor and those caused by the pulsating pressure from the PTR itself. When configuring a cryogenic environment for cooling sensitive parts, (i) *mechanical vibrations (or microphonics)*, (ii) *temperature fluctuations* and (iii) *EMI (electromagnetic interference)* need to be minimized. Fig. 4 shows a DR manufactured by Bluefors which is equipped with a Cryomech's GM-type PTR [11].

2.1. Development of a 4.2 K pulse-tube refrigerator (PTR)

When looking at cryocooler development, there were two historically significant events. *First*, using a single compressor, a refrigerator that can reach the temperature of liquid helium down to 4.2 K was developed. Up until the 1980s, there had been active development of regenerative materials used for the second-stage regenerator carried out as a result of continuous efforts to increase the performance of the second-stage regenerator of GM or Stirling cryocoolers. In particular, it was possible to reach down to 4.2 K by applying intermetallic rare earth materials, e.g., Er_3Ni , HoCu_2 [12–15] and $\text{Gd}_2\text{O}_2\text{S}$ (GOS) [16–18], which has a higher volumetric specific heat than lead. As a state of the arts, GOS spheres are packed at the lowest section, HoCu_2 spheres are juxtaposed at the intermediate section and Pb (or Sn or Bi for RoHS compliance) spheres are at the highest section of the second-stage regenerator. *Second*, a GM-type PTR that does not use a mechanical expander was invented. Since the concept of the GM-type PTR was first invented in 1963 [19], it has now achieved an efficiency comparable to that of the conventionally commercialized GM and Stirling cryocoolers. This improvement was made possible through the development of a phase shifter that dramatically adjusts the phase of the pressure and mass flow rate inside the PTR. Interestingly, 4.2 K GM-type PTR can be seen as the result of the two historical efforts described above.

In 1994, Matsubara and Gao firstly reached a temperature below 4 K with a three-stage GM-type PTR. An axillary regenerator tube was introduced at the warm-side of the third pulse tube. This PTR was recorded a base temperature of 3.6 K and cooling power of 30 mW at 4.2 K [20,21].

In 1997, Dr. Chao Wang, who had worked towards a two-stage GM-type PTR for 4.2 K, went on to work for Cryomech Inc. and was able to commercialize his work. A multi-layer regenerator including $\text{ErNi}_{0.9}\text{Co}_{0.1}$, ErNi and lead were used, and a double-inlet type orifice was adopted as a phase shifter. The GM-type PTR was able to reach a minimum temperature of 2.23 K at the second-stage. It is also interesting to note that the pulse tubes of two different stages are connected to an identical gas reservoir, which is placed at room temperature. The cooling power was recorded to be 370 mW at 4.2 K under an input power of 6.3 kW [22]. After commercializing model PT405, which can provide a cooling power of 500 mW at 4.2 K with an input power of 4.7 kW, in the late 1990s, Cryomech, Inc. has continued to develop GM-type PTRs of various capacities [23]. As a state of the art, a series of 4.2 K GM-type PTR from 250 mW to 2.7 W at 4.2 K (models PT403, PT405, PT407, PT410, PT415, PT420 and PT425), have been developed and commercialized.

2.2. GM-type PTR as a pre-cooler for a dilution refrigerator (DR)

As mentioned above, since the classical wet DR system using liquid helium requires significant effort to handle the cryogen, the demand of dry DRs is ever increasing. Kurt Uhlig, a pioneer in this field, proposed a concept of a dry DR using GM-type PTR in 2002 [24]. Since then, commercial systems have been developed by Leiden

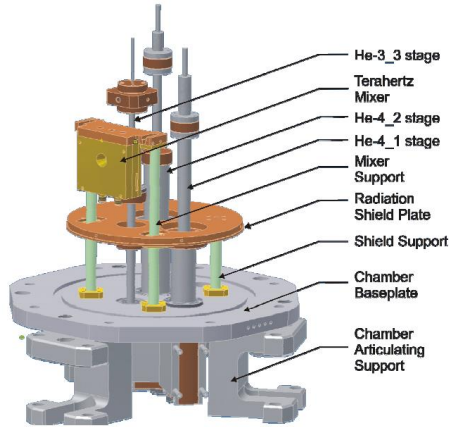


Fig. 6. Three-stage Stirling-type PTR for mobile terahertz detector cooling [30].

Cryogenics [25], Air Liquide [26], Oxford Instruments [27], LTLab [28], Cryoconcept, Janis, BlueFors. Fig. 5 shows the dry DR system proposed by Kurt Uhlig [24]. This system is mainly composed of a DR using $^3\text{He}/^4\text{He}$ and a two-stage GM-type PTR (PT405, Cryomech) that precools the DR. The GM-type PTR that was used provides a cooling capacity of 500 mW at the temperature of 4.2 K. The radiation shield is cooled conductively by the first-stage and the remaining parts including the DR are cooled by the first-stage of the GM-type PTR.

2.3. Stirling-type PTR as a precooler for DR

Although the GM-type PTR has been well commercialized up until now, its efficiency is still lower than that of the Stirling-type PTR due to the loss that originates from the existence of a rotary valve to generate a pressure waveform. In addition, since the operational frequency is in the range of 1-2 Hz, temperature fluctuations due to periodic compression/expansion of the working gas may become a problem at the low temperature region. This problem can be solved unquestionably by a Stirling-type PTR for 4.2 K, which equips a valve-less linear compressor that generates a sinusoidal pressure around several tens of hertz. As the operational frequency increases, the working gas does not have enough time for heat transfer with the regenerative material. For this reason, the controversy over the effectiveness of the 4.2 K Stirling-type PTR is still ongoing.

Next, we will examine multi-stage Stirling-type PTRs for 4.2 K. Sunpower is a leading manufacturer of free-piston Stirling cryocoolers and has attempted to develop 4.2 K Stirling-type PTR [29]. A three-stage structure was adopted and Er_3Ni was juxtaposed at the lowest temperature stage. The base temperature, however, did not reach below 10 K. The National Institute of Standards and Technology (NIST) of the US also developed a Stirling-type PTR operating at the 4 K region for the purpose of mobile terahertz detector cooling [30]. As shown in Fig. 6, the system was constructed through three different PTR, which are ‘thermally-coupled’ with each other. In other words, the regenerators of the PTR, which is responsible for the lowest temperature, is

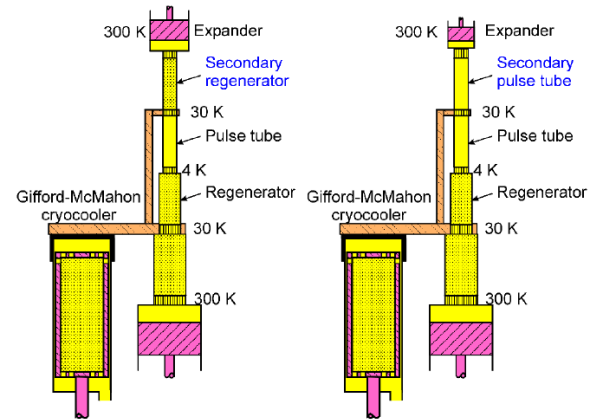


Fig. 7. Novel phase shifter by means of secondary regenerator and secondary pulse tube [31].

precooled by the other remaining PTRs. ^3He was used as a working fluid. A novel phase shifter was also implemented by connecting the pulse tube to a linear motor located at room temperature through an additional regenerator or pulse tube, as shown in Fig. 7. A base temperature of 5.2 K was marked with ^3He working gas [31]. Through the Advanced Cryocooler Technology Development Project funded by the National Aeronautics and Space Administration (NASA), Lockheed Martin developed a 4.2 K Stirling-type PTR. This Stirling-type PTR has four-stage structure and the base temperature of 3.8 K was achieved [32]. As the most recent research, Dang, H., et al. developed a four-stage Stirling-type PTR as shown in Fig. 8 [33]. Similar to the system from NIST above, upper stages were used to precool the fourth-stage PTR and its gas reservoir. Using ^3He and ^4He , the base temperatures were recorded to be 3.3 K and 4.2 K, respectively. However, as seen from the figures, the geometrical configuration is very complicated compared to the typical 4.2 K GM-type PTRs. The Stirling-type PTR still seems advantageous in that it can be driven by a compact linear compressor without requiring any valves, nor does it require any maintenance service that is involved in the use of a lubricating oil.

3. DISCUSSIONS

3.1. Further improvements of GM-type PTR

Summarizing the results of previous research, GM-type PTR has been adopted for precooling of dry DRs, and the required cooling temperature and capacity are to be 4.2 K and 1 W, respectively. Although it is true that mechanical vibration is greatly reduced by implementing a GM-type PTR, it is still necessary to further suppress the vibration transmitted to the milli-Kelvin region for quantum computing. Karla, R., et al. found that vibrations by GM-type PTR affect electrical signals, and that these vibrations also affect qubits [34]. The vibration problems will intensify as the number of qubits in quantum computers increases. The vibration generated by the PTR originates mainly from the gas flow inside the pulse tube

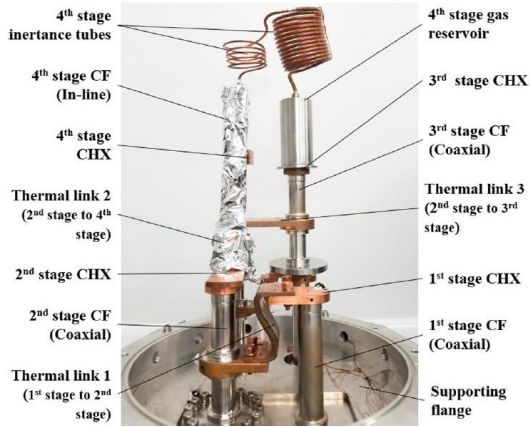


Fig. 8. Four-stage Stirling-type PTR [33].

rather than by the compressor. In order to reduce flow-induced vibration, it is recommended that a co-axial configuration be introduced to make the structure more rigid to attenuate the vibration [35].

As seen in the previous chapter, the Stirling-type PTR seems to not be adequate for 4.2 K applications. The linear compressor, however, is highly efficient. A well-made linear compressor typically rates electrical to mechanical pressure-volume (PV) conversion of 70-80% or higher. As the operational temperature of the regenerator decreases, the heat penetration depth, $\delta_{th}=(2\alpha/\omega)^{1/2}$, of helium is also drastically reduced. Where, α is the thermal diffusivity ($m^2 s^{-1}$) and ω is the angular frequency ($rad s^{-1}$). For the Stirling-type PTRs, the frequency is much higher by at least ten times that of the GM-type ones. These facts signify that the regenerative material of the Stirling-type PTR should be tinier than that of the GM-type PTRs. The tiny regenerator creates exceedingly high thermal and pressure loss. Therefore, it seems more realistic to utilize GM-type PTRs for the precooling stage of the DR. However, since the operating frequency is as low as 1 Hz, temperature fluctuations at the cold part are relatively large. K. Uhlig reported a temperature fluctuation of 0.15 K [24]. In the case of Stirling-PTR, on the other hand, the temperature fluctuation can be mitigated because the operating frequency is relatively high. In addition, since the pressure ratio is small, the magnitude of the flow-induced vibration itself is also small. Therefore, it seems informative to minimize temperature fluctuations by using a GM type pulsating tube refrigerator and to figure out how the fluctuation affect the performance of DRs. As an innovative idea, if the GM-type PTR proposed by J. Jung, et al., which is driven by two pulsating pressures of opposite phase with each other, is used, temperature fluctuations at the 4.2 K-stage and vibrations can be effectively canceled [36]. This tandem system can operate two PTRs by a single compressor. At this point, the capacity of the compressor required for the same cooling capacity can also be reduced.

3.2. High efficiency and low disturbance

Fig. 9(a) shows an alternative way to eliminate the valves that the GM-type PTR has with a low-speed linear

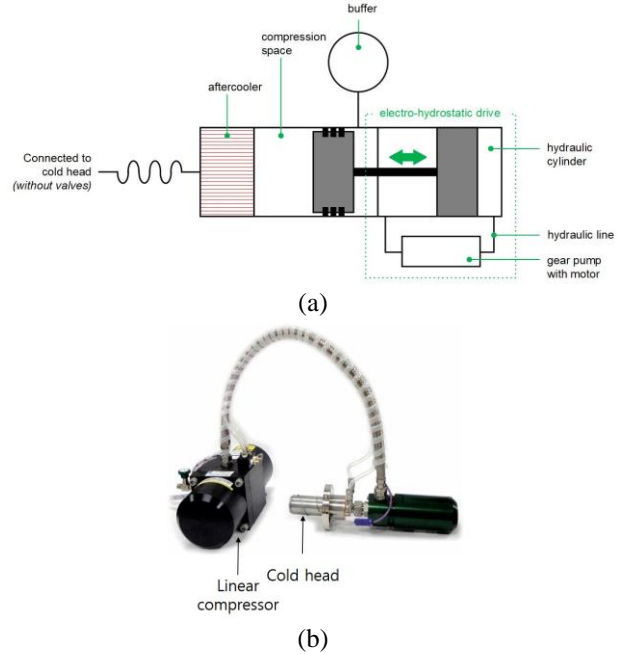


Fig. 9. (a) GM-type PTR driven by a low-speed linear compressor and (b) split-type PTR [38].

compressor. Radebaugh reported second law efficiency of 25% when the regenerator operating between 30 K and 4.2 K under 30 Hz operational frequency with 3He [31, 37]. This system could be more efficient, but it has several drawbacks. There are additional mechanical parts and it is not compact. It is mainly because the compressor is not a resonant system like a typical Stirling-type PTR. In order to efficiently transmit the mass flow generated by the linear compressor, the linear compressor and Stirling-type PTR's cooling parts are located close to each other. Therefore, even if the double-acting structure is introduced to cancel the vibration caused by the movement of the pistons at the linear compressor, residual vibrations that are caused by asymmetry cannot be ignored. As shown in Fig. 9(b), a split-type also can be considered [38].

Since the pressure wave of the GM-type PTR is not a sinusoidal form, there are high-order harmonic terms. It is hard to mitigate this vibration with a passive damper. An active vibration control (AVC) system that creates force in an equal and opposite fashion to the force by external vibration can be a good candidate by utilizing a cryogenically compatible actuator [39]. Another way is to apply the concept of an acoustic black hole that exploits the vibration damping properties of spiral beam structures [40]. Both methods seem to be very interesting but have not yet been applied at cryogenic temperatures.

4. CONCLUDING REMARKS

Since quantum computers are very sensitive to vibration and noise generated by mechanical refrigerators, this review paper focused on pulse tube refrigerators (PTR) installed on top of the dilution refrigerator (DR), which reaches to tens of milli-Kelvin. Most dry DR uses 4.2 K

GM-type PTR. Several researchers have worked towards developing a Stirling-type PTR to get the temperature below 4.2 K because it has the advantages of thermal stability and high efficiency. In order to alleviate flow-induced vibrations and temperature fluctuations GM-type PTRs have, several ideas have been also proposed, i.e., tandem operated GM-type PTR and Stirling-like PTR driven by a linear actuator.

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