J. Appl. Math. & Informatics Vol. **40***(2022), No. 3 - 4, pp. 577 - 585*
https://doi.org/10.14317/jami.2022.577

VECTOR EQUILIBRIUM PROBLEMS FOR TRIFUNCTION IN MEASURABLE SPACE AND ITS APPLICATIONS

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ABSTRACT. In this work, we introduced and study vector equilibrium problems for trifunction in measurable space (for short, VEPMS). The existence of solutions of (VEPMS) are obtained by employing Aumann theorem and Fan KKM lemma. As an application, we prove an existence result for vector variational inequality problem for measurable space. Our results in this paper are new which can be considered as significant extension of previously known results in the literature.

AMS Mathematics Subject Classification : 47J40, 90C33.
Key words and phrases : Vector equilibrium problem, Caratheodory functions, KKM-map.

1. Introduction

It is well known that equilibrium problem is firmly analogous to game theory, physics, economics and finance, transportation and operations research, variational inequality, optimization and control problems etc. and is extensively studied and investigated by numerous authors see, for example [2], [5]. Inspired by the notion of vector variational inequality and vector optimization, equilibrium problems have been extended by many researchers, see for example Giannessi [4], Hadjisavvas and Schaible [6], Kim and Salahuddin [8], Kim et al.[9], Konnov [10], Laszlo [11, 12], Ram and Khanna [15], Ram et al.[16] and references therein.

Beuve[1] gave a measurable selection theorem which generalizes Von Neumann Aumann's theorem when the domain of definition is an abstract measurable space and range space is a Suslin space. In 1996, Papageorgiou [14] proved random fixed point theorems for measurable closed and nonclosed multifunctions satisfying general continuity conditions in Banach spaces.

Received June 29, 2021. Revised December 25, 2021. Accepted January 6, 2022.
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From now, unless specified we work under the following settings:

Let $(\Omega, \mathcal{F})$ be a measurable space and let X be a Hausdorff topological vector space with $\mathcal{B}(X)$, a $\sigma-$ algebra of all Borel sets in X and K be a nonempty separable metrizable compact convex subset of X. Let Y be a complete separable metrizable topological vector space with a solid convex cone C and $C \neq Y$ and $\mathcal{B}(Y)$ a $\sigma-$ algebra of all Borel sets of Y. Suppose $f : \Omega \times K \times K \to Y$ is a trifunction.

In this paper, we consider the following *vector equilibrium problems for measurable spaces* (for short, VEPMS) is to find:

$$x^* \in K \text{ such that } f(\omega, x^*, z) \notin -intC, \ \forall \omega \in \Omega, \ z \in K. \tag{1.1}$$

The set of solutions of vector equilibrium problem for measurable space (1.1) is denoted by Sol(VEPMS).

Remark 1.1. If $Y = \mathbb{R}$, and $C = \mathbb{R}^+ = [0, \infty)$, then the above problem (1.1) reduces to equilibrium problem of finding

$$x^* \in K \text{ such that } f(\omega, x^*, z) \geq 0, \forall \omega \in \Omega, z \in K,$$

which is scalar equilibrium problem introduced and study by Laszlo [12].

The aim of the present study is to find the solutions of vector equilibrium problems for trifunction in measurable space by using Fan KKM lemma and Aumann theorem.

The paper is organized as follows:

In the next section, we introduce some preliminary notions and necessary apparatus that we need to obtain the main result. In section 3, we prove an existence result for vector equilibrium problem for measurable spaces. As an application, we apply our result to prove the existence of solutions of vector variational inequality for measurable space.

2. Preliminaries

Definition 2.1. Let Ω be a nonempty set. A collection $\mathcal{F}$ of subsets of Ω is said to be $\sigma-$ *algebra* if

(i) $\phi, \Omega \in \mathcal{F}$
(ii) $A \in F \Longrightarrow \Omega - A \in \mathcal{F}$
(iii) If $\{A_k, k \in \mathbb{N}\} \subseteq \mathcal{F}$ is any countable collection, then $\bigcup_{k \in \mathbb{N}} A_k \in \mathcal{F}$.

Definition 2.2. A measurable space $(\Omega, \mathcal{F})$ is a nonempty set Ω along with a $\sigma-$ algebra $\mathcal{F}$ defined on Ω.

Note: Let X be a complete metric space. Then the smallest $\sigma-$ algebra containing all open sets in X is called the Borel $\sigma-$ algebra on X denoted by $\mathcal{B}(X)$. The measurable space $(X, \mathcal{B}(X))$ is also called the Borel measurable space on X. Moreover, if $A \in \mathcal{B}(X)$, then A is called Borel measurable with respect to X.

Definition 2.3. Suppose $(\Omega, \mathcal{F})$ is a measurable space. A multifunction $F : \Omega \to 2^Y$ is said to be *measurable (or $\mathcal{F}$-measurable) on X if for every open set $O \subseteq Y$, $F^-(O)$ is measurable, that is, $F^-(O) \in \mathcal{F}$, where $F^-(O) = \{\omega \in \Omega : F(\omega) \cap O \neq \phi\}$.*

Remark 2.1. For a measurable function $F : \Omega \to 2^Y$, the sets $F^-(\phi)$ and $Dom(F) = F^-(Y)$ are measurable.

Definition 2.4. Suppose $f : \Omega \times K \times K \to Y$ is a function. Then f is said to be *Caratheodory trifunction if*

(i) *for each fixed $x \in K, z \in K$ $f(., x, z) : \Omega \to Y$ is measurable and*
(ii) *for each fixed $\omega \in \Omega, f(\omega, ., z) : K \to Y$ is continuous for any $z \in K$.*

Definition 2.5. A subset K of a vector space X is called *convex if for all $x, y \in K, \lambda \in [0, 1]$, we have $\lambda x + (1 - \lambda)y \in K$.*

Definition 2.6. [13] Let X be a vector space and K be a convex subset of X. Suppose Y is a topological vector space with a solid convex cone C such that $C \neq Y$, and $f : K \to Y$ is a function. Then f is said to be $C-$ *convex if for any $x, y \in K$ and $\lambda \in [0, 1]$,*

$$f[\lambda x + (1 - \lambda)y] \in \lambda f(x) + (1 - \lambda)f(y) - C.$$

Definition 2.7. A Hausdorff topological space X is called *Suslin if there exist a separable complete metric space P and a continuous function p from P to X.*

Now we give the definition of KKM map and Fan KKM lemma.

Definition 2.8. [7] Let X be a topological vector space and K be a nonempty subset of X. A multifunction $T : K \to 2^Y$ is called a KKM map if for every finite subset $\{x_1, x_2, ..., x_n\}$ of K, $co\{x_1, x_2, ...x_n\} \subseteq \bigcup_{i=1}^{n} T(x_i)$, where co denotes the convex hull.

Lemma 2.9. [7] *Let K be a nonempty subset of a topological vector space X. Let $T : K \to 2^X$ be a KKM map such that for any $y \in K, T(y)$ is closed and $T(y^*)$ is contained in a compact set $B \subseteq X$ for some $y^* \in K$. Then there exist $x^* \in B$ such that $x^* \in T(y)$, for all $y \in K$. That is, $\cap_{y \in K} T(y) \neq \phi$.*

Lemma 2.10. [3] *Let $(\Omega, \mathcal{F})$ be a measurable space and X be a separable metrizable space, Y be a metrizable space and $f : \Omega \times X \to Y$ be a function. If for any fixed $x \in X$, the function $\omega \mapsto f(\omega, x)$ is measurable and for any fixed ω , function $x \mapsto f(\omega, x)$ is continuous, then f is measurable.*

Theorem 2.11. *(Aumann theorem)*[1],[14] *Let* $(\Omega, \mathcal{F})$ *be a complete measurable space,* X *be a Suslin space with the* $\sigma-$ *algebra* $\mathcal{B}(X)$ *of all Borel sets of* X *and* $F : \Omega \to 2^X$ *be a set valued map such that* $Gr(F) = \{(\omega, x) \in \Omega \times X : x \in F(\omega)\} \in \mathcal{F} \times \mathcal{B}(X)$. *Then there exists a measurable function* $g : \Omega \to X$ *such that* $g(\omega) \in F(\omega),$ *for all* $\omega \in \Omega$.

Proposition 2.12. [13] *Let* $(\Omega, \mathcal{F})$ *be a measurable space,* $F : \Omega \to 2^Y$ *be a closed valued map and* Y *be a separable metric space. Then the following statements are equivalent:*

(i) $F^-(C)$ *is measurable for all closed sets* $C \subseteq Y$
(ii) $F^-(O)$ *is measurable for all open sets* $O \subseteq Y$
(iii) $F^-(K)$ *is measurable for all compact sets* $K \subseteq Y$.

Lemma 2.13. [13] *Suppose* X, Y *be two locally convex Hausdorff topological vector spaces and* K *be a bounded subset of* X. *Suppose* $L(X, Y)$ *is the set of all continuous linear operators from* X *to* Y, *equipped with the topology of bounded convergence. Define a vector valued function* $f : L(X, Y) \times K \to Y$ *by* $f(g, x) = g(x),\ g \in L(X, Y)$ *and* $x \in K$. *Then* f *is continuous.*

3. Main Result

In this section, we prove an existence result for the solution of vector equilibrium problem for measurable spaces VEPMS (1.1) by using Fan- $KKM-$ lemma and Aumann theorem.

Theorem 3.1. *Let* $(\Omega, \mathcal{F})$ *be a complete measurable space and Let* X *be a Hausdorff topological vector space with* $\mathcal{B}(X)$, *a* $\sigma-$*algebra of all Borel sets in* X *and* K *be a nonempty separable metrizable compact convex subset of* X. *Let* Y *be a complete separable metrizable topological vector space with a solid convex cone* C *and* $C \neq Y$ *and* $\mathcal{B}(Y)$, *a* $\sigma-$ *algebra of all Borel sets of* Y. *Suppose* $f : \Omega \times K \times K \to Y$ *is a Caratheodory trifunction. Assume that following conditions holds:*

(i) f *is C-convex and continuous in the 3rd argument, that is, for* $\omega \in \Omega, x \in K,\ f(\omega.x, .) : K \to Y$ *is C-convex and continuous,*
(ii) $f(\omega, x, x) \in C$ *for any* $x \in K$ *and* $\omega \in \Omega$.

Then there exists $x^* \in K$ *such that* $f(\omega, x^*, z) \notin -intC,$ *for all* $\omega \in \Omega,\ z \in K$.

Proof. For each $\omega \in \Omega$ and $z \in K$, let $F(\omega, z) = \{x \in K : f(\omega, x, z) \notin -intC\}$.

So $F : \Omega \times K \to 2^K$ is a multifunction. We shall show that

$$\bigcap_{z \in K} F(\omega, z) \neq \phi,\ \forall \omega \in \Omega.$$

For this, we first show that the multifunction $F(\omega, .) : K \to 2^K$, for fixed $\omega \in \Omega$ is a KKM mapping.

If possible, suppose on contrary, there exist a finite set $\{z_1, z_2, \cdots, z_n\} \in K$ and

$$x = \sum_{i=1}^{n} \lambda_i z_i \left(\sum_{i=1}^{n} \lambda_i = 1,\ \lambda_i > 0 \right) \in co\{z_1, z_2, \cdots, z_n\}$$

such that

$$x \notin \bigcup_{i=1}^{n} F(\omega, z_i).$$

This implies

$$f(\omega, x, z_i) \in -intC, i = 1, 2, \cdots, n.$$

Now

$$\begin{aligned} f(\omega, x, x) &= f(\omega, x, \sum_{i=1}^{n} \lambda_i z_i) \\ &\in \sum_{i=1}^{n} \lambda_i f(\omega, x, z_i) - C, \text{ by the C-convexity of a function f} \\ &\subseteq -intC - C = -intC. \end{aligned}$$

Also by (ii), $f(\omega, x, x) \in C$, we know that $0 \in intC$, which is not true, because $C \neq Y$. Therefore our supposition is wrong.

This proves that for fixed $\omega \in \Omega$, $F(\omega, .) : K \to 2^K$ is a KKM map.

Next, define a multifunction $S : \Omega \to 2^K$ by

$$S(\omega) = \bigcap_{z \in K} F(\omega, z), \forall \omega \in \Omega.$$

Since f is a Caratheodory trifunction, for each fixed $\omega \in \Omega, f(\omega, ., z) : K \to Y$ for any $z \in K$ is continuous. This implies that for fixed $\omega \in \Omega, F(\omega, z)$ is closed for any $z \in K$. Thus by Fan-KKM Lemma 2.9, we have

$$\cap_{z \in K} F(\omega, z) \neq \phi \text{ for fixed } \omega \in \Omega.$$

$$\implies S(\omega) \neq \phi, \forall \omega \in \Omega.$$

Also, since K is separable, so there exists a countable dense subset say D of K such that $\bar{D} = X$

Next, we shall show that $\bigcap_{z \in K} F(\omega, z) = \bigcap_{z \in D} F(\omega, z)$.
For this, it is sufficient to show that

$$\cap_{z \in D} F(\omega, z) \subseteq \cap_{z \in K} F(\omega, z).$$

Suppose on the contrary, $\cap_{z\in D}F(\omega,z) \nsubseteq \cap_{z\in K}F(\omega,z)$. Then there exists $x_0 \in \cap_{z\in D}F(\omega,z)$ but $x_0 \notin \cap_{z\in K}F(\omega,z)$. This implies $x_0 \in \cap_{z\in D}F(\omega,z)$ and there exists $z_0 \in K$ such that $x_0 \notin F(\omega,z_0)$ and so

$$f(\omega,x_0,z_0) \in -intC. \tag{3.1}$$

Again, since $\bar{D} = K$, there exists a sequence say $\{z_n\}_{n\in\mathbb{N}}$ in D such that $z_n \longrightarrow z_0$.
Now

$$x_0 \in \cap_{z\in D}F(\omega,z) \implies x_0 \in \cap_{n\in\mathbb{N}}F(\omega,z_n) \text{ and so } F(\omega,x_0,z_n) \notin -intC.$$

Thus by (i), $f(\omega,x_0,z_0) = \lim_{n\to\infty} f(\omega,x_0,z_n) \notin -intC$, which contradicts (3.1).

Therefore

$$\cap_{z\in K}F(\omega,z) = \cap_{z\in D}F(\omega,z).$$

Now the multifunction $S:\Omega \to 2^K$ becomes $S(\omega) = \cap_{z\in D}F(\omega,z),$ for all $\omega \in \Omega$, that is, a countable intersection.

Finally

$$\begin{aligned} Gr(S) &= \{(\omega,x) \in \Omega\times K : x \in S(\omega) \\ &= \cap_{z\in D}F(\omega,z)\} \\ &= \{(\omega,x) \in \Omega\times K : x \in F(\omega,z), \text{ for all } z \in D\} \\ &= \cap_{z\in D}\{(\omega,x) \in \Omega\times K : x \in F(\omega,z)\} \\ &= \cap_{z\in D}\{(\omega,x) \in \Omega\times K : f(\omega,x,z) \notin -intC\}. \end{aligned}$$

Since for each fixed $x \in K, z \in K$ $f(.,x,z)$ is measurable and for each fixed $\omega \in \Omega, f(\omega,.,z)$ is continuous for any $z \in K$. As f is a Caratheodory trifunction, so by Lemma 2.10 for any fixed $z \in K, f(.,.,z)$ is measurable and so

$$\{(\omega,x) \in \Omega\times K : f(\omega,x,z) \notin -intC\} \in \mathcal{F}\times\mathcal{B}(K).$$

This implies

$$Gr(S) = \cap_{z\in D}\{(\omega,x) \in \Omega\times K : f(\omega,x,z) \notin -intC\} \in \mathcal{F}\times\mathcal{B}(K).$$

Therefore, by Theorem 2.11, there exists a measurable function $g:\Omega\to K$ such that $g(\omega) \in F(\omega,z),$ for all $\omega \in \Omega$. This means that there exists $g(\omega) = x^*$ (say) for some $\omega \in \Omega$ such that $x^* \in F(\omega,z),$ for all $\omega \in \Omega$.
Hence there exists $x^* \in K$ such that $f(\omega,x^*,z) \notin -intC$ for all $\omega \in \Omega$, for all $z \in K$. □

Theorem 3.2. *Let $(\Omega,\mathcal{F})$ be a complete measurable space and let X be a Hausdorff topological vector space with $\mathcal{B}(X)$ a $\sigma-$algebra of all Borel sets in X and K be a nonempty separable metrizable compact convex subset of X. Let Y be a complete separable metrizable topological vector space with a solid convex cone C*

and $C \neq Y$ *and* $\mathcal{B}(Y)$ *a* $\sigma-$ *algebra of all Borel sets of* Y*. Let* $f : \Omega \times K \times K \to Y$ *be a trifunction. Assume that for fixed* $z \in K$*,* $f(.,.,z) : \Omega \times K \longrightarrow Y$ *is measurable and for fixed* $\omega \in \Omega, z \in K, f(\omega,.,z) : K \longrightarrow Y$ *is continuous. Then the multifunction* $F : \Omega \times K \to 2^K$ *defined by*

$$F(\omega, z) = \{x \in K : f(\omega, x, y) \notin -intC\}, \text{ for all } (\omega, z) \in \Omega \times K$$

is measurable.

Proof. By the ordering of a cone C in Y, it is easy to see that $F(.,.)$ is closed valued. Since K is separable, let D be a countable dense subset of Y and $H \subseteq Y$ be a closed set. Then $H \cap D$ is a countable dense subset of H, say $H \cap D = \{x_1, x_2, \cdots, \},$ for some $x \in H$.
Now

$$\begin{aligned} F^-(H) &= \{(\omega, z) \in \Omega \times K : F(\omega, z) \cap H \neq \phi\} \\ &= \bigcup_{x \in H} F^-(x) \\ &= \{(\omega, z) \in \Omega \times K : f(\omega, x, z) \notin -intC\}. \end{aligned}$$

Since $f(\omega,.,z)$ is continuous, for $x^* \in K, f(\omega, x^*, z) \notin -intC$ implies that there exists a neighborhood $U(x^*)$ such that

$$f(\omega, x, z) \notin -intC, \text{for all} x \in U(x^*). \tag{3.2}$$

Again, since $H \cap D$ is dense in H, each neighborhood in H meets with $D \cap H$ say, $x_n \in U(x^*) \cap (D \cap H)$, for some $n \in \mathbb{N}$.

From (3.2), we have

$$f(\omega, x_n, z) \notin -intC, \quad \text{for some} \quad n \in \mathbb{N}.$$

Therefore, we have for some $n \in \mathbb{N}$

$$\begin{aligned} F^-(H) &= \{(\omega, z) \in \Omega \times K : f(\omega, x_n, z) \notin -intC\} \\ &= \bigcup_{n \in \mathbb{N}} \{(\omega, z) \in \Omega \times K : f(\omega, x_n, z) \notin -intC\}. \end{aligned}$$

Since for each $n \in N$, $\{(\omega, z) \in \Omega \times K : f(\omega, x_n, z) \notin -intC\}$ is measurable, since $f(.,.,z)$ is measurable. So, $F^-(H) \in \mathcal{F} \times \mathcal{B}(K)$. This proves that F is a measurable function by Preposition 2.12.

□

Remark 3.1. It is very interesting to be noted that if we define a multifunction $F : \Omega \times K \to 2^K$ by $F(\omega, z) = Sol\,(VEPMS)$, then by above Theorem 3.2, F is a measurable function, which means that the multifunction from $\Omega \times K$ to 2^K constant to Sol(VEPMS) is measurable.

4. Applications

In this section, we give an existence result for vector variational inequality problem for measurable spaces.

Throughout this section, we denote $L(X,Y)$, the space of all continuous linear operators from X to Y with the topology of bounded convergence.

Let $(\Omega, \mathcal{F})$ be a measurable space and let X be a Hausdorff topological vector space with $\mathcal{B}(X)$, a $\sigma-$ algebra of all Borel sets in X and K be a nonempty separable metrizable compact convex subset of X. Let Y be a complete separable metrizable topological vector space with a solid convex cone C and $C \neq Y$ and $\mathcal{B}(Y)$ a $\sigma-$ algebra of all Borel sets of Y. Let $\langle f, x\rangle$ be the value of an operator $f \in L(X,Y)$ at $x \in X$ and $T : \Omega \times K \to L(X,Y)$ be a given mapping. Then the *vector variational inequality problem for measurable spaces* (for short, VVIPMS) is to find:

$$x^* \in K \text{ such that } \langle T(\omega, x^*), z - x^*\rangle \notin -intC,\ \forall \omega \in \Omega,\ z \in K. \tag{4.1}$$

The following corollary gives the existence result for (VVIPMS).

Corollary 4.1. *Let $(\Omega, \mathcal{F})$ be a measurable space and let X be a Hausdorff topological vector space with $\mathcal{B}(X)$, a $\sigma-$ algebra of all Borel sets in X and K be a nonempty separable metrizable compact convex subset of X. Let Y be a complete separable metrizable topological vector space with a solid convex cone C and $C \neq Y$ and $\mathcal{B}(Y)$ a $\sigma-$ algebra of all Borel sets of Y. Let $T : \Omega \times K \to L(X,Y)$ be a Caratheodory function. Then there exist $x^* \in K$ such that*

$$\langle T(\omega, x^*), z - x^*\rangle \notin -intC,\ \forall \omega \in \Omega,\ z \in K.$$

Proof. Define a trifunction $f : \Omega \times K \times K \to Y$ by

$$f(\omega, x, z) = \langle T(\omega, x), z - x\rangle.$$

In view of Lemma 2.13, it is easy to see that all the assumptions of Theorem 3.1 are satisfied and hence there exists at least one $x^* \in K$ such that

$$\langle T(\omega, x^*), z - x^*\rangle \ \notin -intC, \text{ for all } \omega \in \Omega,\ z \in K.$$

□

5. Conclusion

In this article the existence of solutions of vector equilibrium problems for trifunction in measurable spaces are obtained by using KKM Fan theorem and Aumann theorem. As an application, existence result for the solutions of vector variational inequalities problem are also established for measurable spaces. These results can be further generalized in many directions using novel and innovative techniques which will motivate the researchers working in the area of variational inequality.

Conflicts of Interest : The authors declare no conflict of interest.

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