Bull. Korean Math. Soc. **59** (2022), No. 3, pp. 781–787
https://doi.org/10.4134/BKMS.b210478
pISSN: 1015-8634 / eISSN: 2234-3016

# COMPUTATION OF WEDDERBURN DECOMPOSITION OF GROUPS ALGEBRAS FROM THEIR SUBALGEBRA

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Abstract. In this paper, we show that under certain conditions the Wedderburn decomposition of a finite semisimple group algebra $\mathbb{F}_qG$ can be deduced from a subalgebra $\mathbb{F}_q(G/H)$ of factor group $G/H$ of $G$, where $H$ is a normal subgroup of $G$ of prime order $P$. Here, we assume that $q = p^r$ for some prime $p$ and the center of each Wedderburn component of $\mathbb{F}_qG$ is the coefficient field $\mathbb{F}_q$.

## 1. Introduction and main results

Let $G$ be a finite group, $\mathbb{F}_q$ be a finite field with $q = p^k$ elements for some prime $p$ and $k > 0$ and $\mathbb{F}_qG$ denote the semisimple group algebra (cf. [6]), where $p$ is a prime that does not divide $|G|$. The determination of Wedderburn decomposition of a finite group algebra is a well known and extensively studied research problem (cf. [1–3, 5] and the references therein). In the present paper, we continue in this direction and show that under certain conditions, we can directly obtain the Wedderburn decomposition of a group algebra $\mathbb{F}_qG$ from the subalgebra of factor group $G/H$ of $G$, where $H$ is a normal subgroup of $G$ having prime order $P$.

**Main problem:** Suppose that $G$ is a finite group and $H$ is its normal subgroup of order $P$. Let $G$ and $H$ be such that $\mathcal{N}_G = P\mathcal{N}_{G/H}$, where $\mathcal{N}_G$ denotes the number of conjugacy classes of $G$. We want to emphasize that there are infinitely many groups satisfying above-mentioned assumptions. For example, (i) the general linear group of $2 \times 2$ matrices over the field of 4 elements denoted by $GL(2, 4)$ (this group has a normal subgroup of order 3 with the corresponding quotient group isomorphic to $A_5$ with $\mathcal{N}_G = 3\mathcal{N}_{G/H}$), (ii) groups of the form $G = H_1 \times C_P$, where $C_P$ is a cyclic group of order $P$ (this group has a normal subgroup of order $P$ with the corresponding quotient group isomorphic to $H_1$ with $\mathcal{N}_G = P\mathcal{N}_{G/H}$), (iii) $A_5 \rtimes C_4$ (this group has a normal subgroup of order 2 with the corresponding quotient group isomorphic to $S_5$

Received June 27, 2021; Accepted January 17, 2022.
2020 *Mathematics Subject Classification.* Primary 20C05.
*Key words and phrases.* Wedderburn decomposition, unit group, finite field.

with $\mathcal{N}_G = 2\mathcal{N}_{G/H}$) etc. In this paper, we consider the following problem: Suppose that the Wedderburn decomposition of a subalgebra $\mathbb{F}_q(G/H)$ is known, i.e., let $\mathbb{F}_q(G/H) \cong \oplus_{r=1}^{k} M_{t_r}(\mathbb{F}_q)$, and the center of each Wedderburn component of $\mathbb{F}_qG$ be the coefficient field $\mathbb{F}_q$. Then can we compute the Wedderburn decomposition of the group algebra $\mathbb{F}_qG$ from that of $\mathbb{F}_q(G/H)$?

**Main results**: For a group algebra $\mathbb{F}_qG$, let $\mathcal{L}_1$ and $\mathcal{L}_2$ be the lists of degrees of Brauer characters (cf. [7]) having $H$ in the kernel and not having $H$ in the kernel, respectively. The main results of the paper are as follows:

**Theorem 1.1.** *Suppose that $H := \{1, h, \dots, h^{P-1}\}$ is a normal subgroup of $G$ of prime order $P$ with $\mathcal{N}_G = P\mathcal{N}_{G/H}$. Further, let the center of each Wedderburn component in the Wedderburn decomposition of $\mathbb{F}_qG$ be the coefficient field $\mathbb{F}_q$ and the list $\mathcal{L}_2$ exactly contain every element of $\mathcal{L}_1$ $P-1$ times with $|\mathcal{L}_2| = (P-1)|\mathcal{L}_1|$. Then $h^i$ for each $1 \le i \le P-1$ is not the commutators of any two elements of $G$.*

In the following theorem, we consider the stronger assumption that $h^i \notin G'$ for all $1 \le i \le M-1$ and show that the list $\mathcal{L}_2$ contains exactly $P-1$ times every element of $\mathcal{L}_1$ with $|\mathcal{L}_2| = (P-1)|\mathcal{L}_1|$. In particular, the stronger assumption $h^i \notin G'$ implicitly implies that $h^i$ for each $1 \le i \le M-1$ are not the commutator of any two elements of $G$.

**Theorem 1.2.** *Suppose that $H = \{1, h, \dots, h^{P-1}\}$ is a normal subgroup of $G$ having prime order $P$ satisfying $\mathcal{N}_G = P\mathcal{N}_{G/H}$ and the center of each Wedderburn component of $\mathbb{F}_qG$ is the coefficient field (i.e., the coefficient field is splitting). Further, suppose that $h^i \notin G'$ for each $1 \le i \le P-1$. Then the list $\mathcal{L}_2$ exactly contains every element of $\mathcal{L}_1$ $P-1$ times with $|\mathcal{L}_2| = (P-1)|\mathcal{L}_1|$.*

We now formulate the final main result of the paper related to Wedderburn decomposition as a corollary to Theorem 1.2.

**Corollary 1.3.** *Suppose that the assumptions of Theorem 1.2 hold and the Wedderburn decomposition of $\mathbb{F}_q(G/H)$ is known, i.e., let*

$$\mathbb{F}_q(G/H) \cong \oplus_{r=1}^{k} M_{t_r}(\mathbb{F}_q).$$

*Then we have that $\mathbb{F}_qG \cong \oplus_{r=1}^{k} M_{t_r}(\mathbb{F}_q)^P$.*

Before giving the proofs of Theorems 1.1 and 1.2, we discuss the motivation behind studying these results. By definition, a group is metabelian if its derived (or commutator) subgroup is abelian. The unit groups of the semisimple group algebras of metabelian groups have been well studied in the literature (cf. [1]). However, there are many groups such as $H_1 \times C_P, GL(2,4), A_5 \rtimes C_4$ etc., that are non-metabelian (i.e., we can not deduce the Wedderburn decomposition of the group algebras of these groups by using [1]) and satisfy the assumptions of Theorem 1.2. Consequently, we can directly determine the Wedderburn decomposition of these group algebras from the Wedderburn decomposition of their subalgebras by utilizing Theorem 1.2.

**Proof of Main results:** Now, we give the proofs of Theorems 1.1 and 1.2.

*Proof of Theorem 1.1.* Since the center of each Wedderburn component is the coefficient field $\mathbb{F}_q$, the coefficient field is a splitting field. Therefore, there is a 1-1 correspondence between the Wedderburn components and the irreducible (Brauer) characters. Let $\mathcal{C}_1, \dots, \mathcal{C}_{n_G}$ represent the conjugacy classes of $G$ and $\mathcal{C}'_1, \dots, \mathcal{C}'_{n_{G/H}}$ represent that of the quotient group $G/H$. Let $\pi : G \to G/H$ be the natural map. Then, as $|H| = P$, $\pi^{-1}(\mathcal{C}'_r)$ can be the union of at most $P$ conjugacy classes of $G$. The assumption $\mathcal{N}_G = P\mathcal{N}_{G/H}$ leads us to conclude that $\pi^{-1}(\mathcal{C}'_r)$ is the union of exactly $P$ conjugacy classes of $G$.

Consequently, the conjugacy class $\mathcal{C}_r$ can be paired in $P$-tuples, let's say $\mathcal{C}_r$ paired with $\mathcal{C}_{r+k}, \dots, \mathcal{C}_{r+(P-1)k}$ so that if the conjugacy class $\mathcal{C}_r$ contains $g_r \in G$, then the conjugacy classes $\mathcal{C}_{r+k}, \dots, \mathcal{C}_{r+(P-1)k}$ contain $hg_r, \dots, h^{P-1}g_r$, respectively, since $H = \langle h \rangle$. This means that $g, hg, \dots, h^{P-1}g$ are not conjugates for every $g \in G$ (since they are in different conjugacy classes).

Let $\mathcal{A} = \binom{P}{2}$. The assertion, i.e., $g, hg, \dots, h^{P-1}g$ are not conjugates for every $g \in G$ implicitly means that there are no $g_1, g_2, \dots, g_{\mathcal{A}} \in G$ such that

$$g_i^{-1}(h^i g)g_i = g \text{ for } 1 \le i \le P-1,$$

$$g_{P-1+i}^{-1}(h^{i+1}g)g_{P-1+i} = hg \text{ for } 1 \le i \le P-2,$$

$$g_{2P-3+i}^{-1}(h^{i+2}g)g_{2P-3+i} = h^2 g \text{ for } 1 \le i \le P-3,$$

$$\cdots\cdots\cdots\cdots$$

$$g_{\mathcal{A}}^{-1}(h^{P-1}g)g_{\mathcal{A}} = h^{P-2}g.$$

From the first $P-1$ relations, we see that $h, h^2, \dots, h^{P-1}$ are not the commutators of any two elements of $G$. This is because $g_i^{-1}(h^i g)g_i = g$ for $1 \le i \le P-1$ implies that $h^i = g_i g g_i^{-1} g^{-1}$ for $1 \le i \le P-1$ for any $g_1, \dots, g_{P-1}$ in $G$. The second set of $P-2$ relations is $g_{P-1+i}^{-1}(h^{i+1}g)g_{P-1+i} = hg$. We post multiply both sides by $(h^{i+1}g)^{-1}$ to see that

$$g_{P-1+i}^{-1}(h^{i+1}g)g_{P-1+i}(h^{i+1}g)^{-1} = hg(h^{i+1}g)^{-1} = h^{P-i} \text{ for } 1 \le i \le P-2.$$

That is $h^{P-i}$ for $1 \le i \le P-2$ must not be a commutator of any two elements of $G$ which we already know from the first $P-1$ relations. The third set of $P-3$ relations is $g_{2P-3+i}^{-1}(h^{i+2}g)g_{2P-3+i} = h^2 g$. From these relations, we see that

$$g_{2P-3+i}^{-1}(h^{i+2}g)g_{2P-3+i}(h^{i+2}g)^{-1} = h^2 g(h^{i+2}g)^{-1} = h^{P-i} \text{ for } 1 \le i \le P-3.$$

That is $h^{P-i}$ for $1 \le i \le P-3$ must not be a commutator of any two elements of $G$. On continuing similarly, we reach to the relation $g_{\mathcal{A}}^{-1}(h^{P-1}g)g_{\mathcal{A}} = h^{P-2}g$ and deduce that $g_{\mathcal{A}}^{-1}(h^{P-1}g)g_{\mathcal{A}}(h^{P-1}g)^{-1} = h^{P-2}g(h^{P-1}g)^{-1} = h^{P-1}$, i.e., $h^{P-1}$ must not be commutator of any two elements of $G$. This completes the proof. □

*Remark* 1.4. We want to emphasize that Theorem 1.1 holds for any normal subgroup $H$ of $G$ of order $n$ with $\mathcal{N}_G = n\mathcal{N}_{G/H}$. The proof for the same can be obtained on the similar lines of proof of Theorem 1.1 by replacing $P$ with $n$.

To this end, now we discuss the proof of Theorem 1.1.

*Proof of Theorem 1.2.* It is given that $h^i \notin G'$ for each $1 \leq i \leq P-1$ which means $h^iG' \neq e$ for each $1 \leq i \leq P-1$. Consequently, $G$ has a set of $P-1$ linear representations $\zeta_i$ for $1 \leq i \leq P-1$ with $\zeta_i(h^i) \neq 1$. This means that $\zeta_1(h) = \alpha$, $\zeta_2(h^2) = \alpha, \ldots, \zeta_{P-1}(h^{P-1}) = \alpha$, where $\alpha$ is primitive $P^{\text{th}}$ root of unity. Therefore, $\zeta_1(hg) = \alpha\zeta_1(g)$, $\zeta_2(h^2g) = \alpha\zeta_2(g), \ldots, \zeta_{P-1}(h^{P-1}g) = \alpha\zeta_{P-1}(g)$ for every $g \in G$. Due to this, for the representation $\zeta_1$, we obtain that

$$\zeta_1(h^2) = \alpha\zeta_1(h) = \alpha^2, \ldots, \zeta_1(h^{P-1}) = \alpha^{P-2}\zeta_1(h) = \alpha^{P-1}.$$

Further, for the representation $\zeta_2$, we have that

$$\begin{aligned}\zeta_2(h) = \zeta_2(h^{P+1}) = \alpha\zeta_2(h^{P-1}) = \alpha^2\zeta_2(h^{P-3}) = \cdots = \alpha^{\frac{P-4+1}{2}}\zeta_2(h^{P-(P-4)})\\ = \alpha^{\frac{P-2+1}{2}}\zeta_2(h^{P-(P-2)}) = \alpha^{\frac{P-1}{2}+1} = \alpha^{\frac{P+1}{2}}.\end{aligned}$$

Similarly, we see that $\zeta_2(h^3) = \alpha\zeta_2(h) = \alpha^{\frac{P+3}{2}}$, $\zeta_2(h^4) = \alpha\zeta_2(h^2) = \alpha^2$. We continue similarly to deduce that

$$\begin{aligned}\zeta_2(h^{P-2}) = \alpha\zeta_2(h^{P-4}) = \alpha^2\zeta_2(h^{P-6}) = \cdots = \alpha^{\frac{(P-1)-2}{2}}\zeta_2(h^{P-(P-1)})\\ = \alpha^{\frac{P-3}{2}}\alpha^{\frac{P+1}{2}} = \alpha^{P-1},\end{aligned}$$

and $\zeta_2(h^{P-1}) = \alpha^{\frac{p-1}{2}}$. Next, we look for the values taken by the representation $\zeta_3$ at $h$ and $h^{P-1}$. We have $\zeta_3(h^3g) = \alpha\zeta_3(g)$. If $p \equiv -1 \pmod 3$, then one can verify $\zeta_3(h) = \alpha^{\frac{p+1}{3}}$, and $\zeta_3(h^{P-1}) = \alpha^{\frac{2p-1}{3}}$. And if $p \equiv 1 \pmod 3$, then one can verify $\zeta_3(h) = \alpha^{\frac{2p+1}{3}}$, and $\zeta_3(h^{P-1}) = \alpha^{\frac{p-1}{3}}$. For both the cases, we obtain that $\zeta_3(h)\zeta_3(h^{P-1}) = 1$. We continue in the similar manner and obtain the remaining values of representations $\zeta_i$ for $4 \leq P-1$ at $h$ and $h^{P-1}$. In fact, one can verify that $\zeta_i(h)\zeta_i(h^{P-1}) = 1$ for all $4 \leq i \leq P-1$.

To this end, we see that $\zeta_t\Psi$, for $1 \leq t \leq P-1$, are the representations of $G$ for every irreducible character $\Psi$ of $G$. Next, let $\Psi$ be a representation of $G$ having $H$ in the kernel, i.e., $\Psi(H) = 1$. This means that $\Psi(hg) = \Psi(g)$ for every $g \in G$. Let the conjugacy classes of $G$ be $\mathcal{C}_1, \mathcal{C}_2, \ldots, \mathcal{C}_{n_G}$. Now, as discussed in the proof of Theorem 1.1, we have that the conjuacy classes of $G$ can be paired, since $h, h^2, \ldots, h^{P-1} \notin G'$. We write them as $\mathcal{C}_1, \ldots, \mathcal{C}_{n_{G/H}}$, $h\mathcal{C}_1, \ldots, h\mathcal{C}_{n_{G/H}}, \ldots, h^{P-1}\mathcal{C}_1, \ldots, h^{P-1}\mathcal{C}_{n_{G/H}}$. Further, we see that

$$\begin{aligned}[\zeta_1\Psi, \zeta_1\Psi] = \frac{1}{|G|}\sum_{i=1}^{n_{G/H}} \big(|\mathcal{C}_i|\zeta_1(g)\Psi(g)\zeta_1(g^{-1})\Psi(g^{-1})\\ + |h\mathcal{C}_i|\ \zeta_1(hg)\Psi(hg)\zeta_1(g^{-1}h^{-1})\Psi(g^{-1}h^{-1}) + \cdots\end{aligned}$$

$$+ |h^{P-1}\mathcal{C}_i|\ \zeta_1(h^{P-1}g)\Psi(h^{P-1}g)\zeta_1(g^{-1}h^{-(P-1)})\Psi(g^{-1}h^{-(P-1)})\big)$$

$$= \frac{1}{|G|}\sum_{i=1}^{n_{G/H}}\sum_{j=0}^{P-1}\big(|h^j\mathcal{C}_i|\zeta_1(h^jg)\Psi(h^jg)\zeta_1(g^{-1}h^{-j})\Psi(g^{-1}h^{-j})\big). \tag{1.1}$$

Since we know that $|\mathcal{C}_i| = |h\mathcal{C}_i| = \cdots = |h^{P-1}\mathcal{C}_i|$ for each $i$, we can write (1.1) as

$$\begin{aligned}
[\zeta_1\Psi, \zeta_1\Psi] &= \frac{1}{|G|}\sum_{i=1}^{n_{G/H}}\big(|\mathcal{C}_i|\big)\sum_{j=0}^{P-1}\big(\zeta_1(h^jg)\Psi(h^jg)\zeta_1(g^{-1}h^{-j})\Psi(g^{-1}h^{-j})\big)\\
&= \frac{1}{|G|}\sum_{i=1}^{n_{G/H}}\big(|\mathcal{C}_i|\big)\sum_{j=0}^{P-1}\big(\alpha^j\zeta_1(g)\Psi(g)\zeta_1(g^{-1}h^{P-j})\Psi(g^{-1})\big)\\
&= \frac{1}{|G|}\sum_{i=1}^{n_{G/H}}\big(|\mathcal{C}_i|\big)\sum_{j=0}^{P-1}\big(\alpha^j\zeta_1(g)\Psi(g)\alpha^{P-j}\zeta_1(g^{-1})\Psi(g^{-1})\big)\\
&= \frac{1}{|G|}\sum_{i=1}^{n_{G/H}}\big(|\mathcal{C}_i|\big)\sum_{j=0}^{P-1}\big(\zeta_1(g)\Psi(g)\zeta_1(g^{-1})\Psi(g^{-1})\big)\\
&= \frac{1}{|G|}\sum_{i=1}^{n_{G/H}}\big(|\mathcal{C}_i|\big)\sum_{j=0}^{P-1}\big(\Psi(g)\Psi(g^{-1})\big) = [\Psi, \Psi] = 1.
\end{aligned}$$

Now, similar to the case for $[\zeta_1\Psi, \zeta_1\Psi]$, we see that for any $2 \le t \le P-1$, one may obtain

$$[\zeta_t\Psi, \zeta_t\Psi] = \frac{1}{|G|}\sum_{i=1}^{n_{G/H}}\big(|\mathcal{C}_i|\big)\sum_{j=0}^{P-1}\big(\zeta_t(h^jg)\Psi(h^jg)\zeta_t(g^{-1}h^{-j})\Psi(g^{-1}h^{-j})\big)$$

$$= \frac{1}{|G|}\sum_{i=1}^{n_{G/H}}\big(|\mathcal{C}_i|\big)\sum_{j=0}^{P-1}\big(\alpha^{u_t}\zeta_t(g)\Psi(g)\alpha^{v_t}\zeta_t(g^{-1})\Psi(g^{-1})\big), \tag{1.2}$$

where $u_t, v_t \in \mathbb{Z}^+$. We have already shown that for any $2 \le t \le P-1$, $u_t$ and $v_t$ are such that $\alpha^{u_t+v_t} = 1$. Therefore, (1.2) yields

$$\begin{aligned}
[\zeta_t\Psi, \zeta_t\Psi] &= \frac{1}{|G|}\sum_{i=1}^{n_{G/H}}\big(|\mathcal{C}_i|\big)\sum_{j=0}^{P-1}\big(\zeta_t(g)\Psi(g)\zeta_t(g^{-1})\Psi(g^{-1})\big)\\
&= \frac{1}{|G|}\sum_{i=1}^{n_{G/H}}\big(|\mathcal{C}_i|\big)\sum_{j=0}^{P-1}\big(\Psi(g)\Psi(g^{-1})\big) = [\Psi, \Psi] = 1.
\end{aligned}$$

Thus, $\zeta_t\Psi$ for each $1 \le t \le P-1$ is an irreducible character of $G$, corresponding to $\Psi$. Consequently, the map $\Psi \mapsto (\zeta_1\Psi, \ldots, \zeta_{P-1}\Psi)$ is a bijection, since $\mathcal{N}_G = P\mathcal{N}_{G/H}$ and 1-1 correspondence between the Wedderburn components and the irreducible (Brauer) characters (since coefficient field is splitting). To be more precise, corresponding to a Brauer character having $H$ in the kernel, there are $P-1$ Brauer characters not having $H$ in the kernel. All in all, the list $\mathcal{L}_2$ contains exactly $(P-1)$ times every element of $\mathcal{L}_1$. This completes the proof. □

Now, we discuss the proof of Corollary 1.3

*Proof of Corollary 1.3.* It is given that $\mathbb{F}_q(G/H) \cong \oplus_{r=1}^{k} M_{t_r}(\mathbb{F}_q)$. From [6], we know that if $H$ is a normal subgroup of a group $G$, then $\mathbb{F}_qG \cong \mathbb{F}_q\big(G/H\big) \oplus \Delta(G,H)$, where $\Delta(G,H)$ denotes the left ideal of $\mathbb{F}_qG$ generated by the set $\{h-1 : h \in H\}$. Finally, we can utilize Theorem 1.2 to see that $\mathbb{F}_qG \cong \oplus_{r=1}^{k} M_{t_r}(\mathbb{F}_q)^P$. □

To this end, we discuss the practicality of Theorem 1.2 by deducing the Wedderburn decomposition of groups algebra $GL(2,4)$ from that of the Wedderburn decomposition of its subalgebras formed by one of its quotient group. By choice of $p$, we know that the group algebra $\mathbb{F}_qGL(2,4)$ is semisimple. We also assume that the coefficient field is a splitting field. We observe that $GL(2,4)$ has a normal subgroup $H$ of order 3 such that $GL(2,4)/H \cong A_5$. Moreover, the number of conjugacy classes of $GL(2,4)$ are 15 and that of $A_5$ are 5. In addition, one can verify that no element of $H$ is in $GL(2,4)'$. Therefore, all the conditions of Theorem 1.2 are satisfied for $P=3$. Consequently, as $\mathbb{F}_qA_5 \cong \mathbb{F}_q \oplus M_3(\mathbb{F}_q)^2 \oplus M_4(\mathbb{F}_q) \oplus M_5(\mathbb{F}_q)$ (cf. [4]), we have that $\mathbb{F}_qGL(2,4) \cong \mathbb{F}_q^3 \oplus M_3(\mathbb{F}_q)^6 \oplus M_4(\mathbb{F}_q)^3 \oplus M_5(\mathbb{F}_q)^3$. Thus, we have obtained the Wedderburn decomposition of $\mathbb{F}_qGL(2,4)$. Further, one can also utilize Theorem 1.2 for many other groups of the form $H \times C_P$, $A_5 \rtimes C_4$ etc to deduce the Wedderburn decomposition.

## 2. Discussion

We have shown that under certain conditions, one can obtain the Wedderburn decomposition of a group algebra from the subalgebra of its factor group. We have proved Theorem 1.2 for any normal subgroup of prime order $P$. By utilizing this theorem, one can obtain the Wedderburn decompositions of group algebras of groups of arbitrary (but finite) order (provided, the assumptions of theorem hold). Moreover, it is likely that Theorem 1.2 can be extended for normal subgroups of order $n$, where $n$ is any arbitrary positive integer. Therefore, it is an important future task.

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