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ON SOFT ROUGH PRE OPEN SETS IN SOFT ROUGH TOPOLOGICAL SPACES

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Abstract. The notions of soft rough pre open set, soft rough pre closed set, soft rough dense set, soft rough sub maximal, soft rough pre interior and soft rough pre closure are introduced and studied. We also investigate some related properties of these concepts.

1. Introduction

Rough set was introduced by pawlak[11] in 1982 as a tool to deal with vagueness and uncertanity of imprecise data. Considerable amount of works have been done of fundamental results of rough set. Partition of equivalence relation is the core concept behind pawlaks rough set theory. In 1999, molodstov[2] established a completely new approach for modeling uncertainity known as soft set theory and their study on topological spaces were initiated in the year 2011, by Shabir and Naz[7]. Pre open sets were introduced by mashhour et al.[1] and since then different topological properties have been defined in the terms of pre-open sets and investigated by many researchers. In this paper we extend the notion of pre openess to soft rough sets. We define soft rough pre open set and soft rough pre closed set and investigated their properties.

2. Preliminaries

Definition 2.1. [9] Let A be a subset of E. A pair(F, A) is called the Soft Set over X Where $F : A \to P(X)$definied by $F(e) \to P(X)$

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for all $e \in A$. In other words, $F(e)$ may be considered as the set of $e - apporaoximate$ element of the soft set (F, A).

DEFINITION 2.2. [11] Let τ be the collection of soft sets over X, then τ is said to be a soft topology on X if

1. $\emptyset$ and $\tilde{X}$ belong to τ
2. The union of any number of soft sets in τ belong to τ
3. The intersection of any two soft sets in τ belongs to τ

The triplet (X,τ, E)is called a soft topological space over X.

DEFINITION 2.3. [13] Suppose we have an object set $\mathcal{V}$ known as universe, and an indiscernibility relation $\mathfrak{R} \subseteq \mathcal{V} \times \mathcal{V}$ which represents knowledge about elements of $\mathcal{V}$. We take $\mathfrak{R}$ as an equivalence relation and denote it by$\mathfrak{R}(\mathcal{Y})$. The pair $(\mathcal{V}, \mathfrak{R})$ is called the approximation space. Let $\mathcal{Y}$ be any subset of $\mathcal{V}$. We characterize the set $\mathcal{Y}$ with respect to $\mathfrak{R}$.

1. The union of all granules which are entirelly included in the set $\mathcal{Y}$ is called the lower approximation of the set $\mathcal{Y}$ with respect to $\mathfrak{R}$, mathematically defined as
   $\mathfrak{R}_*(\mathcal{Y}) = \bigcup_{\mathcal{Y} \in \mathcal{V}} \{\mathfrak{R}(\mathcal{Y}) : \mathfrak{R}(\mathcal{Y}) \subseteq \mathcal{Y}\}$
2. The union of all the granules having a non-empty intersection with the set $\mathcal{Y}$ is called the upper approximation of the set $\mathcal{Y}$ with respect to $\mathfrak{R}$, mathematically defined as
   $\mathfrak{R}^*(\mathcal{Y}) = \bigcup_{\mathcal{Y} \in \mathcal{V}} \{\mathfrak{R}(\mathcal{Y}) : \mathfrak{R}(\mathcal{Y}) \cap \mathcal{Y} \neq \emptyset\}$
3. The difference between the upper and lower approximations is called the boundary region of the set $\mathcal{Y}$ with respect to $\mathfrak{R}$, mathematically defined as
   $\mathfrak{B}_R(\mathcal{Y}) = \mathfrak{R}^*(\mathcal{Y}) - \mathfrak{R}_*(\mathcal{Y})$
   If $\mathfrak{R}^*(\mathcal{Y}) = \mathfrak{R}_*(\mathcal{Y})$ then $\mathcal{Y}$ is said to be defined. If $\mathfrak{R}^*(\mathcal{Y}) \neq \mathfrak{R}_*(\mathcal{Y})$, i.e $\mathfrak{B}_R(\mathcal{Y}) \neq \emptyset$,the set $\mathcal{Y}$ is said to be (imprecise) rough set with respect to $\mathfrak{R}$

Note. Throughout this paper we denote a rough set $\mathcal{Y}$ by a pair comparising a lower approximation and upper approximation
$\mathcal{Y} = (\mathfrak{R}^*(\mathcal{Y}), \mathfrak{R}_*(\mathcal{Y}))$

DEFINITION 2.4. [6] Let U be the universe of objects, R be an equivalence relation on U and $\tau_R = \{U, \emptyset, R_*(X), R^*(X), B_R(X)\}$ where $X \subseteq U$. τ_R satisfies the following axioms:

1. $\emptyset$ and U belong to τ_R

2. The union of elements of any sub collection of τ_R is in τ_R
3. The intersection of the elements of any finite sub collection of τ_R is in τ_R

   τ_R forms a topology on U called as a rough topology on U with respect to X. We call (U,τ_R, X) as a rough topological space.

DEFINITION 2.5. [3] Consider a soft set $\mathcal{S} = (\mathcal{T}, \mathcal{A})$ over the universe $\mathcal{V}$, where $\mathcal{A} \in \mathcal{E}$ and $\mathcal{T}$ is a mapping defined as $\mathcal{T} : \mathcal{A} \to \mathcal{P}(\mathcal{V})$. Here, soft approximation space is the pair $\mathcal{P} = (\mathcal{V}, \mathcal{S})$. Following the soft approximation space $\mathcal{P}$, we define two operations as follows:
$\mathfrak{R}_{P*}(\mathcal{Y}) = \{\mathsf{v} \in \mathcal{V} : \exists \mathsf{a} \in \mathcal{A}[\mathsf{v} \in \mathcal{T}(\mathsf{a}) \subseteq \mathcal{Y}]\}$
$\mathfrak{R}_{P}{}^{*}(\mathcal{Y}) = \{\mathsf{v} \in \mathcal{V} : \exists \mathsf{a} \in \mathcal{A}[\mathsf{v} \in \mathcal{T}(\mathsf{a}) \cap \mathcal{Y} \neq \emptyset]\}$
regarding every subset $\mathcal{Y} \subseteq \mathcal{V}$, two sets $\mathfrak{R}_{P*}(\mathcal{Y})$ and $\mathfrak{R}_{P}{}^{*}(\mathcal{Y})$, which are called the soft P-lower approximation and soft P-upper approximation of $\mathcal{Y}$, respectively. In general, we refer to $\mathfrak{R}_{P*}(\mathcal{Y})$ and $\mathfrak{R}_{P}{}^{*}(\mathcal{Y})$as $\mathcal{SR}$-approximations of $\mathcal{Y}$ with respect to P. If $\mathfrak{R}_{P*}(\mathcal{Y}) = \mathfrak{R}_{P}{}^{*}(\mathcal{Y})$, then $\mathcal{Y}$is said to be soft P-definable; otherwise,$\mathcal{Y}$ is a soft P-rough set. Then, $Bnd_P = \mathfrak{R}_{P*}(\mathcal{Y}) - \mathfrak{R}_{P}{}^{*}(\mathcal{Y})$is the boundary region. We denote $\mathcal{SR}$-set ($\mathcal{SR}$-set) $\mathcal{Y}$ by a pair comprising $\mathcal{SR}$-lower approximation and $\mathcal{SR}$-upper approximation $\mathcal{Y} = (\mathfrak{R}_{P*}(\mathcal{Y}), \mathfrak{R}_{P}{}^{*}(\mathcal{Y}))$

Note. We denote $\mathcal{SR}$ sets by soft rough sets

DEFINITION 2.6. [10] Let $\mathcal{A} = (\mathcal{R}_{P*}(\mathcal{A}), \mathcal{R}_{P}{}^{*}(\mathcal{A}))$ and $\mathcal{B} = (\mathcal{R}_{P*}(\mathcal{B}), \mathcal{R}_{P}{}^{*}(\mathcal{B}))$ be two arbitrary $\mathcal{SR}$-sets and $\mathcal{P} = (\mathcal{V}, \mathcal{S})$ be soft approximation space. Then $\mathcal{A}$ is a $\mathcal{SR}$ subset of $\mathcal{B}$ if $\mathcal{R}_{P*}(\mathcal{A}) \subseteq \mathcal{R}_{P*}(\mathcal{B})$ and $\mathcal{R}_{P}{}^{*}(\mathcal{A}) \subseteq \mathcal{R}_{P}{}^{*}(\mathcal{B})$

DEFINITION 2.7. [10] Let $\mathcal{A} = (\mathcal{R}_{P*}(\mathcal{A}), \mathcal{R}_{P}{}^{*}(\mathcal{A}))$ and $\mathcal{B} = (\mathcal{R}_{P*}(\mathcal{B}), \mathcal{R}_{P}{}^{*}(\mathcal{B}))$ be two arbitrary $\mathcal{SR}$-sets and $\mathcal{P} = (\mathcal{V}, \mathcal{S})$ be soft approximation space. Then the union of $\mathcal{A}$ and $\mathcal{B}$ is defined as $\mathcal{A} \cup \mathcal{B}$=($\mathcal{R}_{P_*}(\mathcal{A}) \cup \mathcal{R}_{P*}(\mathcal{B})$,$\mathcal{R}_{P}{}^{*}(\mathcal{A}) \cup \mathcal{R}_{P}{}^{*}(\mathcal{B})$)

DEFINITION 2.8. [10] Let $\mathcal{A} = (\mathcal{R}_{P*}(\mathcal{A}), \mathcal{R}_{P}{}^{*}(\mathcal{A}))$ and $\mathcal{B} = (\mathcal{R}_{P*}(\mathcal{B}), \mathcal{R}_{P}{}^{*}(\mathcal{B}))$ be two arbitrary $\mathcal{SR}$-sets and $\mathcal{P} = (\mathcal{V}, \mathcal{S})$ be soft approximation space. Then the intersection of $\mathcal{A}$ and $\mathcal{B}$ is defined as $\mathcal{A} \cap \mathcal{B}$=($\mathcal{R}_{P_*}(\mathcal{A}) \cap \mathcal{R}_{P*}(\mathcal{B})$,$\mathcal{R}_{P}{}^{*}(\mathcal{A}) \cap \mathcal{R}_{P}{}^{*}(\mathcal{B})$)

DEFINITION 2.9. [1] Let $\mathcal{V}$ be the Universe of discourse, $\mathcal{P} = (\mathcal{V}, \mathcal{S})$ be a soft approximation space then, $\mathcal{SR}$topology is defined as
$\tau_{\mathcal{SR}}(\mathcal{Y}) = \{\mathcal{V}, \emptyset, \mathfrak{R}_{P*}, \mathfrak{R}_{P*}, Bd(\mathcal{Y})\}$, where $\mathcal{Y} \subseteq \mathcal{V}, \tau_{\mathcal{SR}}(\mathcal{Y})$ satisfies the following axioms:

1. $\emptyset$ and $\mathcal{V}$ belong to $\tau_{\mathcal{SR}}(\mathcal{Y})$

2. The union of elements of any sub collection of $\tau_{\mathcal{SR}}(\mathcal{Y})$ is in $\tau_{\mathcal{SR}}(\mathcal{Y})$
3. The intersection of the elements of any finite sub collection of $\tau_{\mathcal{SR}}(\mathcal{Y})$ is in $\tau_{\mathcal{SR}}(\mathcal{Y})$
   $\tau_{\mathcal{SR}}(\mathcal{Y})$ forms a topology on U called as a soft rough topology on $\mathcal{V}$ with respect to $\mathcal{Y}$. We call $(\mathcal{V}, \tau_{\mathcal{SR}}(\mathcal{Y}), \mathcal{E})$ as a soft rough topological space.

DEFINITION 2.10. [10] Let $(\mathcal{V}, \tau_{\mathcal{SR}}(\mathcal{Y}), E)$ be a $\mathcal{SR}$-topological space. Any subset $\mathcal{A}$ such that$\mathcal{A} \in \tau_{\mathcal{SR}}(\mathcal{A})$ is said to be $\mathcal{SR}$-open, and any subset$\mathcal{A}$is $\mathcal{SR}$-closed if and only if $\mathcal{A}^c \in \tau_{\mathcal{SR}}(\mathcal{Y})$

DEFINITION 2.11. [1] Let $(\mathcal{V}, \tau_{\mathcal{SR}}(\mathcal{X}), E)$ be a soft rough topologial space with respect to $\mathcal{X}$, where $\mathcal{X} \subseteq \mathcal{V}$ and $A \subseteq E$. The soft rough interior of A is defined as the union of all soft rough open subsets of A and it is denoted by $\mathcal{SR} - int(A)$. The soft rough closure of A is defined as the intersecion of all soft rough closed subsets containing A and it is denoted by $\mathcal{SR} - cl(A)$

REMARK 2.12. [15] $\mathsf{s}cl(G, A)$ is the smallest soft closed set containing (G, A) and $\mathsf{s}int(G, A)$ is the largest soft open set contained in (G, A).

DEFINITION 2.13. [14] A subset A of X is said to be pre-open (respectively, α-open) if $A \subset Int(Cl(A))$ (respectively, $A \subset Int(Cl(IntA)))$.

DEFINITION 2.14. [8] Let (F, A) be any soft set of a soft topological space (X, τ, E).

1. (F, A)soft pre-open set of X if $(F, A)\tilde{\subset} int(cl((F, A)))$, and
2. (F, A)soft pre-closed set of X if $(F, A)\tilde{\supset} cl(int((F, A)))$.

DEFINITION 2.15. [4] A subset (D, A) of soft rough topological space $(\mathcal{V}, A, \tau)$is said to be a soft dense if $\tilde{\mathsf{s}}cl(D, A) = \mathcal{V}_A$

DEFINITION 2.16. [4] $(\mathcal{V}, A, \tau)$ is said to be soft submaximal if each soft dense subset is soft open.

DEFINITION 2.17. [4] In a soft topological space $(\mathcal{V}, A, \tau)$, a soft set (G, A) is said to be soft regular open if $(G, A) = \tilde{\mathsf{s}}int(\tilde{\mathsf{s}}cl(G, A))$.

DEFINITION 2.18. [8] Let (X, τ, A) be a soft topological space over X and (F, A)be a soft set over X. Then:

1. $\mathsf{sp}int(F, A) = \tilde{\cup}\{(O, E) : (O, E)$is a soft pre open and $(O, E)\tilde{\subset}(F, A)\}$ is called preinterior.
2. $\mathsf{sp}cl(F, A) = \tilde{\cap}\{(F, E) : (F, E)$is a soft pre closed and $(F, A)\tilde{\subset}(F, E)\}$is called preclosure.

Clearly $\mathsf{sp}int(F, A)$ is the largest soft pre-open set over Xwhich is contained in$(F, A)$and $\mathsf{sp}cl(F, A)$ is the smallest soft pre-closed set over$X$which contains (F, A).

3. Some properties of soft rough pre open sets and soft rough closed sets

Let U be an initial universe, $\mathcal{X} \subset U$, $P(U)$ be the set of all subsets of U, E be the set of parameters,$A \subset E, P = (U, S)$ be the approximation space in U, $(U, \tau_{\mathcal{SR}}(\mathcal{X}), E)$ be the soft rough topological space with parameter $E, \mathcal{R}_{P*}(\mathcal{X})$ lower limit and $\mathcal{R}_P{}^*(\mathcal{X})$ upper limit of soft rough topology.

Note.$\tilde{\mathfrak{sr}}\mathsf{p}cl(\mathcal{X})$is soft rough pre closure and $\tilde{\mathfrak{sr}}\mathsf{p}int(\mathcal{X})$ is soft rough pre interior of $\mathcal{X}$

Definition 3.1. Let $\mathcal{A}$ be any soft rough set of a soft rough topological space$(U, \tau_{\mathcal{SR}}(\mathcal{X}), E)$ is called,

1. soft rough pre open set if $\mathcal{A}\tilde{\subseteq} int(cl(\mathcal{A}))$
2. soft rough pre closed set if$\mathcal{A}\tilde{\supseteq} cl(int(\mathcal{A}))$

Example 3.2. *Let $U = \{x_1, x_2, x_3, x_4\}$ be an initial universe, E=$\{e_1, e_2, e_3, e_4\}$ be the set of parameter and A=$\{e_1, e_2, e_3\}\subseteq E$ .Consider the approximation space$P = (U, S)$ where $S = (F, A)$ is a soft set over U given by: $F(e_1) = \{x_1, x_3\}$,$F(e_2) = \{x_2\}$,$F(e_3) = \{x_4\}$. For $\mathcal{Y} = \{x_1, x_2\}$, we have $\mathcal{R}_{P*}(\mathcal{Y}) = \{x_2\}$,$\mathcal{R}_P{}^*(\mathcal{Y}) = \{x_1, x_2, x_3\}$,and $Bnd_P(\mathcal{Y}) = \{x_1, x_3\}$. Thus$\tau_{\mathcal{SR}}(\mathcal{Y}) = \{U, \{\emptyset\}, \{x_2\}, \{x_1, x_2, x_3\}, \{x_1, x_3\}\}$is a soft rough topology. Then the collection of $\mathcal{SR}$ pre open sets are $\{\mathcal{V}, \{\emptyset\}, \{x_1\}, \{x_2\}, \{x_3\}$,$\{x_1, x_2\}, \{x_2, x_3\}, \{x_1, x_3\}, \{x_2, x_3, x_4\}, \{x_1, x_2, x_3\}$, $\{x_1, x_2, x_4\}\}$*

Definition 3.3. A subset $\mathcal{D}$ of a soft rough topological space $(\mathcal{V}, \tau_{\mathcal{SR}}(\mathcal{Y}), E)$ is said to be soft rough dense set if for all soft rough open set X in $\tau_{SR}(Y)$, $\mathcal{D} \cap X \neq \{\emptyset\}$.

Example 3.4. *$U = \{x_1, x_2, x_3, x_4\}$,$E$=$\{e_1, e_2, e_3, e_4\}$, A=$\{e_1, e_2, e_3\}\subseteq E$ .Consider the approximation space$P = (\mathcal{V}, \mathcal{S})$ where $\mathcal{S} = (F, A)$ over $\mathcal{V}$ given by: $F(e_1) = \{x_2, x_3\}$,$F(e_2) = \{x_1\}$,$F(e_3) = \{x_4\}$. For $\mathcal{Y} = \{x_1, x_2\}$, we have $\underline{R_P}(\mathcal{Y}) = \{x_1\}$,$\overline{R_P}(\mathcal{Y}) = \{x_1, x_2, x_3\}$,and $Bnd_P(\mathcal{Y}) = \{x_2, x_3\}$. Thus$\tau_{SR}(\mathcal{Y}) = \{\mathcal{V}, \emptyset, \{x_1\}, \{x_1, x_2, x_3\}, \{x_2, x_3\}\}$is a soft rough topology. Let $\mathcal{B} = \{x_1, x_2, x_3\} \subseteq \mathcal{V}, \mathcal{B} \cap \mathcal{V} = \{x_1, x_2, x_3\}, \mathcal{B} \cap \{x_1\} =$*

$\{x_1\}, \mathcal{B} \cap \{x_1, x_2, x_3\} = \{x_1, x_2, x_3\}, \mathcal{B} \cap \{x_2, x_3\} = \{x_2, x_3\}$. *Then* $\mathcal{B}$ *is soft rough dense set in a soft rough topological space* $(\mathcal{V}, \tau_{\mathcal{SR}}(\mathcal{Y}), E)$.

DEFINITION 3.5. Let $(U, \tau_{\mathcal{SR}}(\mathcal{Y}), E)$ is said to be a soft rough sub maximal if each soft rough dense subset is soft rough open.

DEFINITION 3.6. In a soft rough topological space $(U, \tau_{\mathcal{SR}}(\mathcal{C}), E)$ a soft rough set $\mathcal{C}$is said to be a soft rough regular open if $\mathcal{C} = int(cl(\mathcal{C}))$.

REMARK 3.7. $\{\emptyset\}$and U are always soft rough pre open set and soft rough pre closed set.

REMARK 3.8. Every soft rough open (soft rough closed) set is soft rough pre open (Soft rough pre closed) set.
Converse of this remark need not be true.

EXAMPLE 3.9. *Let* $U = \{x_1, x_2, x_3, x_4\}$ *be an initial universe,* E=$\{e_1, e_2, e_3, e_4\}$ *be the set of parameters and* A=$\{e_1, e_2, e_3\}\subseteq E$ *.Consider the approximation space*$P = (U, S)$ *where* $S = (F, A)$ *is a soft set over* U *given by:* $F(e_1) = \{x_2, x_3\}$,$F(e_2) = \{x_1\}$,$F(e_3) = \{x_4\}$ *. For* $\mathcal{Y} = \{x_1, x_2\}$, *we have* $\mathcal{R}_{P*}(\mathcal{Y}) = \{x_1\}$,$\mathcal{R}_{P}{}^{*}(\mathcal{Y}) = \{x_1, x_2, x_3\}$,*and* $Bnd_P(\mathcal{Y}) = \{x_2, x_3\}$. *Thus*$\tau_{\mathcal{SR}}(\mathcal{Y}) = \{U, \{\emptyset\}, \{x_1\}, \{x_1, x_2, x_3\}, \{x_2, x_3\}\}$*is a soft rough topology. Then* $\mathcal{SR}$ *pre open set*=$\{U, \{\emptyset\}, \{x_1\}, \{x_2\}, \{x_3\}, \{x_1, x_2\}, \{x_2, x_3\}, \{x_1, x_3\}, \{x_2, x_3, x_4\}, \{x_1, x_2, x_3\}, \{x_1, x_2, x_4\}\}$
From the above example $\{x_1, x_2\}, \{x_1, x_3\}, \{x_2, x_3, x_4\}, \{x_1, x_2, x_4\}$ *are soft rough pre open but not soft rough open.*

REMARK 3.10. If $\{(\mathcal{X})_\alpha : \alpha \in I\}$ be a collection of all soft rough pre open sets, Then

1. $\tilde{\cup} int(\mathcal{X}_\alpha) \tilde{\subseteq} int(\tilde{\cup}\mathcal{X}_\alpha)$
2. $\tilde{\cup} cl(\mathcal{X}_\alpha) \tilde{\subseteq} cl(\tilde{\cup}\mathcal{X}_\alpha)$

REMARK 3.11. If $\mathcal{X}$ and $\mathcal{Y}$ are any two $\mathcal{SR}$-sets in $(U, \tau_{\mathcal{SR}}(\mathcal{C}), E)$ then $U - (\mathcal{X} \tilde{\cap} \mathcal{Y}) = (U - \mathcal{X}) \tilde{\cup} (U - \mathcal{Y})$

THEOREM 3.12. *Arbitrary union of soft rough pre open sets is a soft rough pre open.*

Proof. If $\{(\mathcal{X})_\alpha : \alpha \in I\}$ be a collection of soft rough pre open sets of a soft rough topological space $(U, \tau_{\mathcal{SR}}(\mathcal{X}), E)$
Then for each $\alpha, \mathcal{X}_\alpha \tilde{\subseteq} int(cl(\mathcal{X}_\alpha))$

$$\begin{aligned}\Rightarrow \tilde{\cup}\mathcal{X}_\alpha \tilde{\subseteq} \tilde{\cup}\ int(cl(\mathcal{X}_\alpha))\\ \tilde{\subseteq}\ int(\tilde{\cup} cl(\mathcal{X}_\alpha)) \text{ (by Remark 3.10(i))}\\ \tilde{\subseteq}\ int(cl(\tilde{\cup}\mathcal{X}_\alpha)) \text{ (by Remark 3.10(ii))}\end{aligned}$$

Therefore, $\tilde{\cup}\mathcal{X}_\alpha \tilde{\subseteq}\ int(cl(\tilde{\cup}\mathcal{X}_\alpha))$ □

Remark 3.13. Finite intersection of soft rough pre open sets need not be a soft rough pre open set.
In Example 3.2, $\mathcal{SR}$ pre open set=$\{U, \{\emptyset\}, \{x_1\}, \{x_2\}, \{x_3\}, \{x_1, x_2\}, \{x_2, x_3\}, \{x_1, x_3\}, \{x_2, x_3, x_4\}, \{x_1, x_2, x_3\}, \{x_1, x_2, x_4\}\}$
Here $\{x_1, x_2, x_4\}$ and $\{x_2, x_3, x_4\}$ are soft rough pre open sets but$\{x_1, x_2, x_4\} \cap \{x_2, x_3, x_4\} = \{x_2, x_4\}$ is not a soft rough pre open set.

Theorem 3.14. *If $\mathcal{X}$ is soft rough pre open set such that $\mathcal{Y} \tilde{\subseteq} \mathcal{X} \tilde{\subseteq} cl(\mathcal{Y})$ then $\mathcal{Y}$ is also a soft rough pre open set.*

Proof. Let $\mathcal{Y} \tilde{\subseteq} \mathcal{X} \tilde{\subseteq} cl(\mathcal{X}) \tilde{\subseteq} cl(\mathcal{Y})$ and $\mathcal{X}$ is soft rough pre open, $\mathcal{X} \tilde{\subseteq} int(cl(\mathcal{X}))$
Now $cl(\mathcal{X}) \tilde{\subseteq} cl(\mathcal{Y}) \Rightarrow int(cl(\mathcal{X})) \tilde{\subseteq} int(cl(\mathcal{Y}))$ and
$\Rightarrow \mathcal{Y} \tilde{\subseteq} \mathcal{X} \tilde{\subseteq} int(cl(\mathcal{X})) \tilde{\subseteq} int(cl(\mathcal{Y}))$
$\Rightarrow \mathcal{Y} \tilde{\subseteq} int(cl(\mathcal{Y})) \Rightarrow \mathcal{Y}$ is soft rough pre open set. □

Note: We shall denote the family of all soft rough pre open sets [soft rough pre closed sets] of a soft rough topological space$(U, \tau_{\mathcal{SR}}(\mathcal{X}), E)$ by $SRPO(U)[SRPC(U)]$.

Definition 3.15. Let$(U, \tau_{\mathcal{SR}}(\mathcal{X}), E)$ be a soft rough topological space and $\mathcal{X}$ be a soft rough set over$U$.

1. The soft rough pre closure of $\mathcal{X}$ is a soft rough set, $\tilde{\mathfrak{sr}}\mathfrak{p}cl(\mathcal{X}) = \tilde{\cap}\{\mathcal{S} : \mathcal{X} \tilde{\subseteq} \mathcal{S} \text{ and } \mathcal{S} \in SRPC(U)\}$
2. The soft rough pre interior of $\mathcal{X}$ is a soft rough set, $\tilde{\mathfrak{sr}}\mathfrak{p}int(\mathcal{X}) = \tilde{\cup}\{\mathcal{S} : \mathcal{S} \tilde{\subseteq} \mathcal{X} \text{ and } \mathcal{S} \in SRPO(U)\}$

Remark 3.16. $\tilde{\mathfrak{sr}}\mathfrak{p}cl(\mathcal{X})$is the smallest soft rough pre closed set containing $\mathcal{X}$ and $\tilde{\mathfrak{sr}}\mathfrak{p}int(\mathcal{X})$ is the largest soft rough pre open set contained in $\mathcal{X}$.

Theorem 3.17. *Let $(U, \tau_{\mathcal{SR}}(\mathcal{X}), E)$ be a soft rough topological space and $\mathcal{X}$ be a soft rough set over U. Then*

1. $\tilde{\mathfrak{sr}}\mathfrak{p}cl(\{\emptyset\}) = \{\emptyset\}$ *and* $\tilde{\mathfrak{sr}}\mathfrak{p}cl(U) = U$
2. $\tilde{\mathfrak{sr}}\mathfrak{p}int(\{\emptyset\}) = \{\emptyset\}$ *and* $\tilde{\mathfrak{sr}}\mathfrak{p}int(U) = U$
3. $(\tilde{\mathfrak{sr}}\mathfrak{p}cl(\mathcal{X}))^c = \tilde{\mathfrak{sr}}\mathfrak{p}cl(\mathcal{X})^c$
4. $(\tilde{\mathfrak{sr}}\mathfrak{p}int(\mathcal{X}))^c = \tilde{\mathfrak{sr}}\mathfrak{p}int(\mathcal{X})^c$

Proof. Let $\mathcal{X}$ be a soft rough set over U.

1. Since $\{\emptyset\}$ and U are soft rough pre closed sets, $\tilde{\mathfrak{sr}}\mathfrak{p}cl(\{\emptyset\}) = \{\emptyset\}$ and $\tilde{\mathfrak{sr}}\mathfrak{p}cl(U) = U$.
2. Since $\{\emptyset\}$ and U are soft rough pre open sets, $\tilde{\mathfrak{sr}}\mathfrak{p}int(\{\emptyset\}) = \{\emptyset\}$ and $\tilde{\mathfrak{sr}}\mathfrak{p}int(U) = U$.

3. $(\tilde{\mathfrak{srp}}cl(\mathcal{X}))^c = (\tilde{\cap}\{\mathcal{S} : \mathcal{X}\tilde{\subseteq}\ \mathcal{S}\ and\ \mathcal{S} \in SRPC(U)\})^c$
$= \tilde{\cup}\{(\mathcal{S})^c : (\mathcal{S})^c\tilde{\subseteq}(\mathcal{X})^c\ and\ (\mathcal{S})^c \in SRPO(U)\}$
$= \tilde{\cup}\{\mathcal{S}^c : \mathcal{S}^c\tilde{\subseteq}\mathcal{X}^c\ and\ \mathcal{S}^c \in SRPO(U)\}$
$= \tilde{\mathfrak{srp}}int(\mathcal{X})^c$
4. $(\tilde{\mathfrak{srp}}int(\mathcal{X}))^c = (\tilde{\cup}\{\mathcal{S} : \mathcal{S}\tilde{\subseteq}\ \mathcal{X}\ and\ \mathcal{S} \in SRPO(U)\})^c$
$= \tilde{\cap}\{(\mathcal{S})^c : (\mathcal{X})^c\tilde{\subseteq}(\mathcal{S})^c\ and\ (\mathcal{S})^c \in SRPC(U)\}$
$= \tilde{\cap}\{\mathcal{S}^c : \mathcal{X}^c\tilde{\subseteq}\mathcal{S}^c\ and\ \mathcal{S}^c \in SRPC(U)\}$
$= \tilde{\mathfrak{srp}}cl(\mathcal{X})^c$

□

THEOREM 3.18. *Let $\mathcal{X}$ be any soft rough set in a soft rough topological sapce$(U, \tau_{\mathcal{SR}}(\mathcal{X}), E)$. Then*
1. $\tilde{\mathfrak{srp}}cl(\mathcal{X}^c) = U - \tilde{\mathfrak{srp}}int(\mathcal{X})$ *and*
2. $\tilde{\mathfrak{srp}}int(\mathcal{X}^c) = U - \tilde{\mathfrak{srp}}cl(\mathcal{X})$.

Proof.

1. Let $\mathcal{Y}$ be the soft rough pre open in $\mathcal{X}$ such that $\mathcal{Y}\tilde{\subseteq}\mathcal{X}$
$\Rightarrow \mathcal{Y}^c\tilde{\supseteq}\mathcal{X}^c$ By definition 3.15 $\tilde{\mathfrak{srp}}int(\mathcal{X}) = \tilde{\cup}\{\mathcal{Y} : \mathcal{Y}\tilde{\subseteq}\ \mathcal{X}\ and\ \mathcal{Y} \in SRPO(U)\}$
$\Rightarrow \tilde{\mathfrak{srp}}int(\mathcal{X}) = \tilde{\cup}\{\mathcal{Y}^c : \mathcal{Y}^c\tilde{\supseteq}\ \mathcal{X}^c\ and\ \mathcal{Y}^c \in SRPC(U)\}$
$= U - \tilde{\cap}\{\mathcal{Y}^c : \mathcal{Y}^c\tilde{\supseteq}\ \mathcal{X}^c\ and\ \mathcal{Y}^c \in SRPC(U)\}$
$= U - \tilde{\mathfrak{srp}}cl(\mathcal{X}^c)$
Hence, $\tilde{\mathfrak{srp}}cl(\mathcal{X}^c) = U - \tilde{\mathfrak{srp}}int(\mathcal{X})$.
2. Let $\mathcal{P}$ be the soft rough pre closed in $\mathcal{X}$ such that $\mathcal{P}\tilde{\supseteq}\mathcal{X}$
$\Rightarrow \mathcal{P}^c\tilde{\subseteq}\mathcal{X}^c$ By definition 3.15 $\tilde{\mathfrak{srp}}cl(\mathcal{X}) = \tilde{\cap}\{\mathcal{P} : \mathcal{P}\tilde{\supseteq}\ \mathcal{X}\ and\ \mathcal{P} \in SRPC(U)\}$
$\Rightarrow \tilde{\mathfrak{srp}}cl(\mathcal{X}) = \tilde{\cap}\{\mathcal{P}^c : \mathcal{P}^c\tilde{\subseteq}\ \mathcal{X}^c\ and\ \mathcal{P}^c \in SRPO(U)\}$
$= U - \tilde{\cup}\{\mathcal{P}^c : \mathcal{P}^c\tilde{\subseteq}\ \mathcal{X}^c\ and\ \mathcal{P}^c \in SRPO(U)\}$
$= U - \tilde{\mathfrak{srp}}int(\mathcal{X}^c)$
Hence, $\tilde{\mathfrak{srp}}int(\mathcal{X}^c) = U - \tilde{\mathfrak{srp}}cl(\mathcal{X})$.

□

THEOREM 3.19. *In a soft rough topological space $(U, \tau_{\mathcal{SR}}(\mathcal{X}), E)$, a soft rough set $\mathcal{X}$ is soft rough pre closed [soft rough pre open] set iff $\mathcal{X} = \tilde{\mathfrak{srp}}cl(\mathcal{X})$ [$\mathcal{X} = \tilde{\mathfrak{srp}}int(\mathcal{X})$].*

Proof. Let $\mathcal{X}$ be a soft rough pre closed set in $(U, \tau_{\mathcal{SR}}(\mathcal{X}), E)$. Since $\mathcal{X}$ is a soft rough pre closed set we have by the definition 3.14 $\mathcal{X} \in \{\mathcal{Y} : \mathcal{X}\tilde{\subseteq}\ \mathcal{Y}\ and\ \mathcal{Y} \in SRPC(U)\}$ and $\mathcal{X}\tilde{\subseteq}\mathcal{Y}$ implies that $\mathcal{X} = \tilde{\cap}\{\mathcal{Y} : \mathcal{X}\tilde{\subseteq}\ \mathcal{Y}\ and\ \mathcal{Y} \in SRPC(U)\}$, i.e. $\mathcal{X} = \tilde{\mathfrak{srp}}cl(\mathcal{X})$. Conversely suppose that $\mathcal{X} = \tilde{\mathfrak{srp}}cl(\mathcal{X})$,i.e.$\mathcal{X} = \tilde{\cap}\{\mathcal{Y} : \mathcal{X}\tilde{\subseteq}\ \mathcal{Y}\ and\ \mathcal{Y} \in SRPC(U)\}$. This implies that $\mathcal{X} \in \{\mathcal{Y} : \mathcal{X} \tilde{\subseteq} \mathcal{Y} \ and \ \mathcal{Y} \in SRPC(U)\}$. Hence $\mathcal{X}$ is soft rough pre closed.

Similarly we can prove soft rough pre open sets. □

THEOREM 3.20. *Let $(U, \tau_{\mathcal{SR}}(\mathcal{X}), E)$ be a soft rough topological space and $\mathcal{X}$ and $\mathcal{Y}$be two soft rough set over U, then*

1. $\mathcal{X} \tilde{\subseteq} \mathcal{Y} \Rightarrow \widetilde{\mathfrak{srp}}int(\mathcal{X}) \tilde{\subseteq} \widetilde{\mathfrak{srp}}int(\mathcal{Y})$
2. $\mathcal{X} \tilde{\subseteq} \mathcal{Y} \Rightarrow \widetilde{\mathfrak{srp}}cl(\mathcal{X}) \tilde{\subseteq} \widetilde{\mathfrak{srp}}cl(\mathcal{Y})$
3. $\widetilde{\mathfrak{srp}}cl(\mathcal{X} \tilde{\cup} \mathcal{Y}) = \widetilde{\mathfrak{srp}}cl(\mathcal{X}) \tilde{\cup} \widetilde{\mathfrak{srp}}cl(\mathcal{Y})$
4. $\widetilde{\mathfrak{srp}}int(\mathcal{X} \tilde{\cap} \mathcal{Y}) = \widetilde{\mathfrak{srp}}int(\mathcal{X}) \tilde{\cap} \widetilde{\mathfrak{srp}}int(\mathcal{Y})$
5. $\widetilde{\mathfrak{srp}}cl(\mathcal{X} \tilde{\cap} \mathcal{Y}) \tilde{\subset} \widetilde{\mathfrak{srp}}cl(\mathcal{X}) \tilde{\cap} \widetilde{\mathfrak{srp}}cl(\mathcal{Y})$
6. $\widetilde{\mathfrak{srp}}int(\mathcal{X} \tilde{\cup} \mathcal{Y}) \tilde{\supset} \widetilde{\mathfrak{srp}}int(\mathcal{X}) \tilde{\cup} \widetilde{\mathfrak{srp}}int(\mathcal{Y})$

Proof. 1. By definition 3.15, $\widetilde{\mathfrak{srp}}int(\mathcal{X}) = \tilde{\cup}\{\mathcal{S} : \mathcal{S} \tilde{\subseteq} \mathcal{X} \ and \ \mathcal{S} \in SRPO(U)\}$ and $\widetilde{\mathfrak{srp}}int(\mathcal{Y}) = \tilde{\cup}\{\mathcal{T} : \mathcal{T} \tilde{\subseteq} \mathcal{Y} \ and \ \mathcal{T} \in SRPO(U)\}$
   Now $\widetilde{\mathfrak{srp}}int(\mathcal{X}) \tilde{\subseteq} \mathcal{X} \tilde{\subseteq} \mathcal{Y} \Rightarrow \widetilde{\mathfrak{srp}}int(\mathcal{X}) \tilde{\subseteq} \mathcal{Y}$.
   Since $\widetilde{\mathfrak{srp}}int(\mathcal{Y})$ is the largest soft rough pre open set contained in $\mathcal{Y}$, we have $\widetilde{\mathfrak{srp}}int(\mathcal{Y}) \tilde{\subseteq} \mathcal{Y}$ and hence $\widetilde{\mathfrak{srp}}int(\mathcal{X}) \tilde{\subseteq} \widetilde{\mathfrak{srp}}int(\mathcal{Y})$.
2. By definition 3.15 $\widetilde{\mathfrak{srp}}cl(\mathcal{X}) = \tilde{\cap}\{\mathcal{S} : \mathcal{X} \tilde{\subseteq} \mathcal{S} \ and \ \mathcal{S} \in SRPC(U)\}$ and $\widetilde{\mathfrak{srp}}cl(\mathcal{Y}) = \tilde{\cap}\{\mathcal{T} : \mathcal{Y} \tilde{\subseteq} \mathcal{T} \ and \ \mathcal{T} \in SRPC(U)\}$
   Since $\mathcal{X} \tilde{\subseteq} \widetilde{\mathfrak{srp}}cl(\mathcal{X})$ and $\mathcal{Y} \tilde{\subseteq} \widetilde{\mathfrak{srp}}cl(\mathcal{Y})$
   $\Rightarrow \mathcal{X} \tilde{\subseteq} \mathcal{Y} \tilde{\subseteq} \widetilde{\mathfrak{srp}}cl(\mathcal{Y}) \Rightarrow \mathcal{X} \tilde{\subseteq} \widetilde{\mathfrak{srp}}cl(\mathcal{Y})$
   But $\widetilde{\mathfrak{srp}}cl(\mathcal{X})$ is the smallaest soft rough pre closed set containing $\mathcal{X}$
   Therefore, $\widetilde{\mathfrak{srp}}cl(\mathcal{X}) \tilde{\subseteq} \widetilde{\mathfrak{srp}}cl(\mathcal{Y})$.
3. We have, $\mathcal{X} \tilde{\subset} \mathcal{X} \tilde{\cup} \mathcal{Y}$ and $\mathcal{Y} \tilde{\subset} \mathcal{X} \tilde{\cup} \mathcal{Y}$
   By (ii), $\mathcal{X} \tilde{\subseteq} \mathcal{Y} \Rightarrow \widetilde{\mathfrak{srp}}cl(\mathcal{X}) \tilde{\subseteq} \widetilde{\mathfrak{srp}}cl(\mathcal{Y})$
   $\widetilde{\mathfrak{srp}}cl(\mathcal{X}) \tilde{\subset} \widetilde{\mathfrak{srp}}cl(\mathcal{X} \tilde{\cup} \mathcal{Y})$
   $\widetilde{\mathfrak{srp}}cl(\mathcal{Y}) \tilde{\subset} \widetilde{\mathfrak{srp}}cl(\mathcal{X} \tilde{\cup} \mathcal{Y})$
   $\Rightarrow \widetilde{\mathfrak{srp}}cl(\mathcal{X}) \tilde{\cup} \widetilde{\mathfrak{srp}}cl(\mathcal{Y}) \tilde{\subset} \widetilde{\mathfrak{srp}}cl(\mathcal{X} \tilde{\cup} \mathcal{Y}) \ldots\ldots (1)$
   Now $\widetilde{\mathfrak{srp}}cl(\mathcal{X}), \widetilde{\mathfrak{srp}}cl(\mathcal{Y}) \in SRPC(U)$
   $\Rightarrow \widetilde{\mathfrak{srp}}cl(\mathcal{X}) \tilde{\cup} \widetilde{\mathfrak{srp}}cl(\mathcal{Y}) \in SRPC(U)$
   Then $\mathcal{X} \tilde{\subset} \widetilde{\mathfrak{srp}}cl(\mathcal{X})$ and $\mathcal{Y} \tilde{\subset} \widetilde{\mathfrak{srp}}cl(\mathcal{Y})$
   $\Rightarrow \mathcal{X} \tilde{\cup} \mathcal{Y} \tilde{\subset} \widetilde{\mathfrak{srp}}cl(\mathcal{X}) \tilde{\cup} \widetilde{\mathfrak{srp}}cl(\mathcal{Y})$
   i.e., $\widetilde{\mathfrak{srp}}cl(\mathcal{X}) \tilde{\cup} \widetilde{\mathfrak{srp}}cl(\mathcal{Y})$is a soft rough pre closed set containing $\mathcal{X} \tilde{\cup} \mathcal{Y}$. But, $\widetilde{\mathfrak{srp}}cl(\mathcal{X} \tilde{\cup} \mathcal{Y})$ is the largest soft rough pre closed set containing $\mathcal{X} \tilde{\cup} \mathcal{Y}$.
   Hence $\widetilde{\mathfrak{srp}}cl(\mathcal{X} \tilde{\cup} \mathcal{Y}) \tilde{\subset} \widetilde{\mathfrak{srp}}cl(\mathcal{X}) \tilde{\cup} \widetilde{\mathfrak{srp}}cl(\mathcal{Y}) \ldots\ldots (2)$
   By (1) and (2) , $\widetilde{\mathfrak{srp}}cl(\mathcal{X} \tilde{\cup} \mathcal{Y}) = \widetilde{\mathfrak{srp}}cl(\mathcal{X}) \tilde{\cup} \widetilde{\mathfrak{srp}}cl(\mathcal{Y})$.
4. We have, $\mathcal{X} \tilde{\cap} \mathcal{Y} \tilde{\subset} \mathcal{X}$ and $\mathcal{X} \tilde{\cap} \mathcal{Y} \tilde{\subset} \mathcal{Y}$
   By(i), $\mathcal{X} \tilde{\subseteq} \mathcal{Y} \Rightarrow \widetilde{\mathfrak{srp}}int(\mathcal{X}) \tilde{\subseteq} \widetilde{\mathfrak{srp}}int(\mathcal{Y})$

$\tilde{\mathfrak{srp}}int(\mathcal{X}\tilde{\cap}\mathcal{Y}) \tilde{\subset} \tilde{\mathfrak{srp}}int(\mathcal{X})$ and $\tilde{\mathfrak{srp}}int(\mathcal{X}\tilde{\cap}\mathcal{Y}) \tilde{\subset} \tilde{\mathfrak{srp}}int(\mathcal{Y})$
$\tilde{\mathfrak{srp}}int(\mathcal{X}\tilde{\cap}\mathcal{Y}) \tilde{\subset} \tilde{\mathfrak{srp}}int(\mathcal{X}) \tilde{\cap} \tilde{\mathfrak{srp}}int(\mathcal{Y})$ . . . . . (3)
Now $\tilde{\mathfrak{srp}}int(\mathcal{X})$, $\tilde{\mathfrak{srp}}int(\mathcal{Y}) \in SRPO(U)$
$\Rightarrow \tilde{\mathfrak{srp}}int(\mathcal{X}) \tilde{\cap} \tilde{\mathfrak{srp}}int(\mathcal{Y}) \in SRPO(U)$
Then $\tilde{\mathfrak{srp}}int(\mathcal{X}) \tilde{\subset} \mathcal{X}$ and $\tilde{\mathfrak{srp}}int(\mathcal{Y}) \tilde{\subset} \mathcal{Y}$
$\Rightarrow \tilde{\mathfrak{srp}}int(\mathcal{X}) \tilde{\cap} \tilde{\mathfrak{srp}}int(\mathcal{Y}) \tilde{\subset} \mathcal{X}\tilde{\cap}\mathcal{Y}$
i.e., $\tilde{\mathfrak{srp}}int(\mathcal{X}) \tilde{\cap} \tilde{\mathfrak{srp}}int(\mathcal{Y})$ is a soft preopen set contained in $\mathcal{X}\tilde{\cap}\mathcal{Y}$. But, $\tilde{\mathfrak{srp}}int(\mathcal{X}\tilde{\cap}\mathcal{Y})$ is the largest soft preopen set contained in $\mathcal{X}\tilde{\cap}\mathcal{Y}$.
Hence $\tilde{\mathfrak{srp}}int(\mathcal{X}) \tilde{\cap} \tilde{\mathfrak{srp}}int(\mathcal{Y}) \tilde{\subset} \tilde{\mathfrak{srp}}int(\mathcal{X}\tilde{\cap}\mathcal{Y})$ . . . . . (4)
By (3) and (4), $\tilde{\mathfrak{srp}}int(\mathcal{X}) \tilde{\cap} \tilde{\mathfrak{srp}}int(\mathcal{Y}) = \tilde{\mathfrak{srp}}int(\mathcal{X}\tilde{\cap}\mathcal{Y})$

5. We have, $\mathcal{X} \tilde{\cap} \mathcal{Y} \tilde{\subset} \mathcal{X}$ and $\mathcal{X} \tilde{\cap} \mathcal{Y} \tilde{\subset} \mathcal{Y}$
$\Rightarrow \tilde{\mathfrak{srp}}cl(\mathcal{X}\tilde{\cap}\mathcal{Y}) \tilde{\subset} \tilde{\mathfrak{srp}}cl(\mathcal{X})$ and $\tilde{\mathfrak{srp}}cl(\mathcal{X}\tilde{\cap}\mathcal{Y}) \tilde{\subset} \tilde{\mathfrak{srp}}cl(\mathcal{Y})$
$\Rightarrow \tilde{\mathfrak{srp}}cl(\mathcal{X}\tilde{\cap}\mathcal{Y}) \tilde{\subset} \tilde{\mathfrak{srp}}cl(\mathcal{X}) \tilde{\cap} \tilde{\mathfrak{srp}}cl(\mathcal{Y})$
6. We have, $\mathcal{X} \tilde{\subset} \mathcal{X} \tilde{\cup} \mathcal{Y}$ and $\mathcal{Y} \tilde{\subset} \mathcal{X} \tilde{\cup} \mathcal{Y}$
$\Rightarrow \tilde{\mathfrak{srp}}int(\mathcal{X}) \tilde{\subset} \tilde{\mathfrak{srp}}int(\mathcal{X}\tilde{\cup}\mathcal{Y})$ and $\tilde{\mathfrak{srp}}int(\mathcal{Y}) \tilde{\subset} \tilde{\mathfrak{srp}}int(\mathcal{X}\tilde{\cup}\mathcal{Y})$
$\tilde{\mathfrak{srp}}int(\mathcal{X}) \tilde{\cup} \tilde{\mathfrak{srp}}int(\mathcal{Y}) \tilde{\subset} \tilde{\mathfrak{srp}}int(\mathcal{X}\tilde{\cup}\mathcal{Y})$

□

THEOREM 3.21. *Every soft rough dense is a soft rough pre open set.*

Proof. The proof follows from the definition 3.1 and 3.3. □

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