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Do NPV and IRR Measure the Profitability of Investment Opportunities? Conditions as Measures of Profitability

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NPV와 IRR은 투자기회들의 수익성을 측정하는가? 수익성 척도로서 조건들

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Investors must adopt profitable investment opportunities to maximize their wealth. Almost all investment, finance, engineering economics textbooks explain that net present value (NPV) measures the profitability (or value) of investment opportunities in absolute size, and internal rate of return (IRR) measures the profitability of investment opportunities in relative proportions. However, NPV is a measure of the relative size of the return on investment opportunity to do-nothing alternative. Moreover, IRR can occur in multiple investment opportunities and may not exist. To make matters worse, IRR and NPV also have conflicting problems in accept-or-reject decisions. In this study, the reason why NPV and IRR cannot accurately measure the profitability of investment opportunities is identified, and fundamental characteristics that investment opportunity profitability measures should have are presented.

Keywords : Net Present Value, Minimum Attractive Rate of Return, Terminal Value, Internal Rate of Return, External Rate of Return

1. Introduction

Economic factors in investment opportunities, such as investment amounts, investment periods, and profitability, are essential information for investors to assess the economic efficiency of a project. Although the amounts and periods are relatively easy to identify from the cash flows occurring in engineering projects, there are many other ways to measure the profitability. The net present value (NPV) and internal rate of return (IRR) have been known to be the most commonly used measures of investment profitability.

Remer and Nieto [22, 23] compared 25 project evaluation methods and found that companies preferred IRR and NPV as project evaluation methods. In surveys conducted by Gitman and Forrester [11] and Burns and Walker [5], the IRR was favored, whereas in a survey conducted by Moore and Reichert [20], the NPV was slightly more preferred than the IRR. In real-world cases, financial managers and executants have until recently preferred IRR [19]. Currently, the NPV and IRR appear in almost all engineering economics, finance, and management accounting textbooks. However, despite being the two most popular and traditional measures, their correctness when applied to the profitability of an investment opportunity

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has yet to be verified. Furthermore, disputes over multiple IRRs have been ongoing for almost a century.

The purpose of this study is to identify properties that the profitability scales of investment opportunities should have. To this end, the following two sections examine whether the NPV and IRR correctly measure the profitability of the investment. In the last section, the must-have characteristics of profitability measures are established based on the failures of the NPV and IRR.

2. Role and Weakness of NPV

In an investment opportunity, the net cash flow occurring at the end of year t (= 0, 1, 2, ⋯, T) is indicated as A_t. If the discount rate of an investor is r, the NPV sums all cash flows by discounting them. Thus, the present value is indicated by Eq. (1).

$$NPV(r) = \Sigma_{t=0}^{T} A_t \cdot (1+r)^{-t} \tag{1}$$

The NPV can be defined as the effect of an investment opportunity on the wealth of an investor because it is the difference between the present value of all cash inflows and outflows. That is, it measures the net increase or decrease in investor wealth according to the present value. Therefore, if the NPV of the opportunity is greater than zero, the investor adopts it because it may increase investor's wealth. If the NPV is less than zero, the opportunity is rejected. Consequently, the NPV is acknowledged as a rational criterion used to decide the adoption or rejection of an investment opportunity and has long been used without any changes to the original model. However, in addition to the NPV of a project, the size of the investment, return on investment, and project life span are economic factors that cannot be ignored in decision-making. For example, even if the NPV of a project is $100, we do not know how much we should invest to generate that amount of money. Similarly, the NPV does not tell us how long it will take to earn $100. Consider an exercise in which A_t = (-1000, 3900, -5030, or 2145) and the minimum attractive rate of return (MARR) is 6% [21]. Because the NPV of the project is $3.55, the investor adopts the project. However, investors are also interested in how much money they must invest in the project and the revenue that can be accumulated after 3 years. The NPV does not provide such information.

Moreover, the NPV is not an absolute measure of the return on investment but rather a relative one because it measures the difference (large or small) in comparison to the value of the investment opportunity, which has the MARR as the rate of return. The investment opportunity that generates revenue at the rate of the MARR is commonly referred to as a "do-nothing proposal." For instance, if the NPV of a project is $100, this project can earn $100 more than investing in a "do-nothing proposal." Consequently, when comparing mutually exclusive investment opportunities, the NPV criterion should not compare all investment opportunities at once, but rather in pairs. In simple words, using the NPV, one can distinguish the taller tree when putting two trees side by side but cannot measure their actual height.

Through simple examples, one may analyze the drawbacks of the NPV. An investor can perform project A, which is predicted to make $50 each year for 3 years when investing $100. The MARR is 10%. The investor can conduct this project by raising funds from outside. Project B is the case in which project A is undertaken by borrowing the investment cost ($100) with annual repayments of $38.8 for a 3-year period. Project C is the case in which project A is carried out using borrowed funds ($100) under the condition in which funds of $30, $40, and $50 are repaid at the end of the first, second, and third years, respectively. Project D is the case in which project A was conducted without paying the initial investment. Instead, the investor spends $108 a year later. Project E is the case in which project A is conducted with internal ($10) and borrowed ($90) funds. The loan should be repaid at $120 after 3 years. The investor also have other project options. In addition to the initial $100, Project F consists of an additional $10,000 investment after two years, and its net cash flow is {-100, 50, -10000, 11105}. The investment costs of project G are $100, but the income is double that of project A. <Table 1> shows the cash flows and NPVs of the projects.

<Table 1> Cash Flows and NPVs of Investment Opportunities

Time	Project A	Project B	Project C	Project D
0	-100	0	0	0
1	50	11.2	20	-58
2	50	11.2	10	50
3	50	11.2	0	50
NPV(.1)	24.34	27.85	26.45	26.16

Time	Project E	Project F	Project G
0	-10	-100	-100
1	50	50	100
2	50	-10,000	100
3	-70	11,105	100
NPV(.1)	24.18	24.34	148.69

The NPV values presented in this table are relative measurements compared to the NPV (= 0) of the "do-nothing proposal." Therefore, the NPV can be an appropriate method for deciding whether to accept or reject a project, depending on its impact on investor wealth. However, if the investor expect new investment opportunities in three years, the total revenue generated from the currently adopted investment opportunities needs to be estimated. The NPV does not provide such information. Because projects A and F have the same NPV (= $24.34), the effect on the increase in investor wealth is the same, whereas the investment costs are largely different. However, the NPV method did not distinguish between the two projects. Regarding project G, the initial investment is the same at $100, but the profit is twice that of project A. However, NPVA (10%) is not measured twice as large as NPVG (10%). Accordingly, the NPV corresponds to an interval scale defined by Stevens [27] and has a limitation in which all four fundamental rules of arithmetic operations are not applicable; only addition and subtraction can be applied for the NPV. Moreover, the NPV method may cause the investor to misunderstand projects B or C as a 1- or 2-year project rather than a 3-year project.

In summary, the NPV is an indicator measuring the relative value of investment opportunities, not their profitability. If the profitability can be measured, investors can not only simultaneously compare the investment opportunities but also obtain other information, such as the size of the funds available for subsequent new businesses. In addition, investors are interested in the profitability of investment opportunities and the size and duration of their investments. Therefore, for investors to properly evaluate the economics of the investment opportunities, the size and duration of the investment opportunities must be specified, along with their profitability.

3. IRR and Variants

The IRR of an investment opportunity is defined as the discount rate that makes the results of Eq. (1) zero or satisfies Eq. (2).

$$\begin{aligned} NFV(r) &= NPV_j(r) \cdot (1+r)^T \\ &= \Sigma_{t=0}^{T} A_t \cdot (1+r)^{T-t} = 0 \end{aligned} \tag{2}$$

If the IRR of an investment opportunity is measured, the investor compares it to the MARR (also called the hurdle rate, cutoff rate, or opportunity cost) to accept or reject the project. MARR is defined as the ratio of revenue earned over a period of time (nominally 1 year) per $1 of investment. Therefore, because the definitions of the MARR and IRR differ, we must first demonstrate that their definitions have the same meaning for their comparisons to have valid practical meanings. In addition, because MARR is always defined as a unique and real value, the IRR of a project must also be a unique and real value. Furthermore, because any investment has only one of the three results of profit, loss, and breakeven, an IRR must represent one of them to be a measure of profitability.

However, because an IRR is defined as a Tth degree polynomial, as shown in Eq. (2), it has up to T roots, including imaginary numbers that do not actually exist. Therefore, most scholars agree that having multiple IRRs is a serious flaw, including Solomon [25], Teichroew et al. [29, 30], Cannaday et al. [8], Riggs and West [24], Hajdasinski [12], Lohmann [16], Blank and Tarquin [3], Thuesen and Fabrycky [31], Bussey and Eschenbach [6], Steiner [26], Young [33], Fleischer [10], Eschenbach [9], Canada et al. [7], Park [21], White et al. [32], Sullivan et al. [28], Hazen [14], Hartman and Schafrick [13], and Magni [17, 18].

Numerous studies have been conducted to solve the problem of multiple IRRs. For example, Cannaday et al. [8] classified solutions to the problem of multiple IRRs into four categories: (1) two-period analysis solution, (2) truncation solution, (3) relevant rate solution, and (4) reinvestment/financing rate solution.

The two-period and truncation solutions use only part of the investment opportunity cash flows to create a unique rate. Thus, they obtain a unique root; however, this unique root is calculated using investment opportunities that are entirely different from the original. Therefore, these approaches cannot solve the problem of multiple IRRs.

The relevant rate solution is the illogical assertion that the appropriate and inappropriate return rates can be discerned without the ground that polynomial roots can have different meanings. The relevant IRR discerned in the relevant rate solution is just one of several discounts by which the investment opportunity does not increase or decrease investor wealth (not gaining or losing anything), but it is not the rate of return. Although Cannaday et al. [8] argued for a fifth solution by revising the relevant rate solution, they ironically recommended the use of the NPV rather than the relevant IRR.

The reinvestment/financing rate solution defines an IRR using other rates (e.g., reinvestment rate, market interest rate, cost of capital, or marginal growth rate). However, this generates more variants and disputes than the other solutions. Solomon [25] proposed the idea that the interim cash flows of a project should not be reinvested at an internal rate of return and they should instead be used explicitly by assuming an annual expected rate of return. Baldwin [2] presented an example of obtaining annual returns under the assumption that all investment-related cash, including the input and recovery of working capital, is discounted to the time value of money, and returns are reinvested at the average rate of return of the company by the end of the project lifetime. Lin [15] proposed a modified rate of return as a measure of profitability. In addition, Bernhard [4] showed that external rates of return (ERRs), such as Solomon's rate [25], Baldwin's rate [2], Lin's first and second rate [15], and Athanasopoulos's rate [1] can be expressed as a single equation. However, it is not proven which rate is a true profitability measure because different rates are calculated for the same project. In addition, ERRs are not defined, despite being common investment opportunities, such as projects B and C in Table 1. Also, there is criticism that ERRs do not solve the multiple IRRs problem; rather, it defines a new rate of return.

Hazen [14] demonstrated that each of the multiple IRRs has a valid meaning, unlike the authors who pursued a unique real-valued IRR. Hazen interpreted the IRR as the rate of return if it made unrecovered balances of an investment opportunity into an investment stream. The author also interpreted the IRR as the rate of borrowing if it made unrecovered balances into a borrowing stream. Thus, the author showed that accept-or-reject decisions are consistent with the NPV criterion, regardless of which IRR is used among multiple IRRs. However, the author encouraged decisions to be made based on the simple NPV criterion rather than the complicated and convoluted IRR criterion. Furthermore, the author did not prove that IRRs are the rate of return of a project. For example, although project E shown in Table 1 procures investment costs through borrowing, it is a project for which \$100 should be invested. However, according to Hazen [14], this project may be considered an investment project with a rate of return of 466%, or a borrowing project with a borrowing rate of -17%. The investor will accept the project because the return on investment is greater than the MARR (10%), or the borrowing rate of -17% is lower than the MARR. This decision is consistent with that obtained using the NPV criterion. However, because 466% is an extremely unrealistic rate of return for a project, it cannot be recognized as the rate of return.

Magni [17, 18] claims to have solved the complex IRRs that Hazen [14] was unable to overcome. Magni created an investment or borrowing stream from the investment opportunity cash flows at his discretion, obtaining an average internal rate of return (AIRR) from one stream, and showed that the AIRR was consistent with the NPV criterion. However, because many AIRRs are generated at the discretion of the investor, the rate of return as a unique real value is still undecided. Magni's AIRR can only accept or reject investment opportunities through a complicated and convoluted process like Hazen [14]. Moreover, there is no evidence that AIRR is the rate of return.

Although numerous efforts have been made thus far, it has only been confirmed that there is no unique root that can replace multiple roots of a polynomial. Instead, as we saw in the cases of ERRs, a new definition of the rate of return is needed.

The IRRs of many investment opportunities are often calculated as unique real numbers. Investors compare IRRs to MARRs. However, even if the IRR obtained is a unique real number, it must be verified that the two measures are at the same scale for comparison. For example, if the IRR of project A (23.38%) means a rate of return, it is expected that \$187.79 (=100 x (1.2338)3) will be earned after 3 years as a result of investing \$100 now. However, except for the \$100 initial cost, the total future cash inflow is estimated to be \$165.50. Therefore, there is a mismatch between the revenue at the end calculated on the basis of the IRR and the revenue at the end cumulated on the basis of the net cash flows: \$187.79 ≠ \$165.50. This mismatch was also evident in the comparison of other projects. Project G had an IRR of 89.3%. Suppose the investor planned a new 3-year project (project H) with a 50% rate of return for the \$100 investment. The investor would prefer project G because its rate of return is approximately 1.8-times higher than that of project H. However, the terminal revenue of project H (\$337.5) is larger than that of project G (\$331.0). The contradiction that a project with a lower rate of return makes more money proves that the IRR is not a rate of return, despite existing as a unique real number. Although cash flows of long-term investments into shares with dividends or bonds with regular interest occur annually, only one change in sign occurs in each cash flow stream. Because the IRR of these financial instruments is calculated as a unique real number, it is used as a profitability measure [3, 26]. However, this is a very serious fallacy, as it means that the rates of return on some financial instruments in the financial market have long been miscalculated. This is because previous scholars focused only on the multi-IRR problem and omitted to verify that a unique real-valued IRR can be considered as the rate of return on investment opportunities.

In Eq. (1), the value of each investment opportunity relies on project life T, cash flow A_t, and discount rate r. Therefore, although T and A_t are fixed values, r can vary depending on the investors. Consequently, the NPVs of investment opportunities present different values depending on the discount rates of the investors. If the total value added to the wealth of an investor is different when the same amount is invested over the same period, the value earned by $1 from among the total investment should also be different. However, the IRR is not the rate of return of an investment opportunity because it is equally defined for all investors. This is another reason why the IRR is unreliable as a measure of profitability of an investment opportunity.

Because any investment results in one of three outcomes, the profitability scale should be measurable for all investment opportunities. However, there are investment opportunities for which the IRR is not measured. If the net cash flows of the investment opportunities occur only positively or negatively, such as projects B and C in Table 1, there is no IRR for them. Technically, their IRRs are imaginary numbers that are always present in even numbers. Thus, the IRR not only has the problem of multiplicity, it also creates the problem of not being able to set rates of return on investment opportunities. Even worse, an IRR may not be calculated even if an investment opportunity has one or more different sign cash flows. Polynomial (2) is not defined at r = -1; therefore, an IRR cannot be calculated. Hazen [14] and Magni [17] also developed a logic that assumes $r \neq -1$. However, this does not mean that the investment opportunity does not exist because Eq. (2) is not defined for r = -1. If all of the investment money is lost, the rate of return becomes -100%. Therefore, the IRR cannot be trusted as a profitability measure because it cannot be defined for all investment opportunities.

In summary, the IRR may not be defined as a unique real number, does not contain different assessments of investors, cannot measure the profitability of investment opportunities, and cannot be defined across all investment opportunities.

4. Relationships between Measures of Profitability

Many investment opportunities in financial markets have no limits on the amount and duration of the investments. Therefore, their profitability is not expressed on an absolute scale, but in rates of return, as in the MARR. The rate of return is the percentage of revenue earned by a $1 investment over 1-year period. Therefore, if the investment amount and duration of the investment opportunity are specified along with the rate of return, the amount of revenue that can be earned at the end of the investment opportunity can be calculated. Consequently, if the investment amount and period of investment opportunities are the same, the greater the rate of return, the greater the revenue at the end of the period, and vice versa. The correlation between these two factors can be mathematically tested using the following equations:

Investing a limited amount ($P) in an investment opportunity with a rate of return, r, allows us to earn $\$F$ after a specified period of time (T years), as shown in Eq. (3).

$$F = P(1+r)^T \tag{3}$$

Equation (3) is then rearranged into Eq. (4).

$$r = \left(\frac{F}{P}\right)^{1/T} - 1 \tag{4}$$

Through Equations (3) and (4), we can identify the compatibility between profitability in terms of absolute size and the rate of return:

(1) If $\$P$ is invested today as an investment opportunity with a rate of return r, then $\$F$ is returned after T years.
(2) If $\$P$ is invested today and $\$F$ is returned T years later, the rate of return for this investment opportunity is r.

If the aforementioned relationship between F and r is not established in the investment opportunity, then F and r are not homogeneous scales. If this relationship is established, the same decision can be made whether the profitability of an investment opportunity is measured in absolute or proportional measurements. This relationship is a critical feature in verifying the validity of the profitability measurements of the investment opportunities, as was also used in the previous section to verify that the NPV and IRR are not homogeneous profitability measures.

5. Conclusions

Investors are interested in the amount they need to invest in projects and their potential profits. Although the NPV has the advantage of measuring the extent to which an investment opportunity increases or decreases the wealth of an investor, it also has the disadvantage of not measuring the magnitude of the return. Therefore, investors prefer the IRR over the simpler NPV criterion because of the expectation that the IRR measures the profitability of investment opportunities. Although the IRR may be an appropriate index for measuring the profitability of some financial instruments, it cannot be used as a measure of profitability for financial instruments or engineering projects in which frequent cash inflows occur over the investment period. Consequently, a new measure of profitability of an investment opportunity with complex cash flows, such as engineering projects, should be defined and have the following characteristics:

(1) The profitability should be measurable for all investment opportunities.
(2) The profitability should be a unique real number.
(3) When the revenue is high, the profitability must be measured high and vice versa.
(4) The profitability scales with characteristic (3) must be homogeneous.
(5) If investors have different discount rates, the measured profitability can be different, even for the same investment opportunity.
(6) The profitability should be indicated alongside the investment period and amount.

If the profitability of different investment opportunities is appropriately measured, one can compare their profitability simultaneously and decide whether to adopt or reject them. Furthermore, this measure can be the basis for extended analyses, such as stochastic models and real options..

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