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RESEARCH ARTICLE

A Study on Effectiveness of Mathematics Teachers' Collaborative Learning: Focused on an Analysis of Discourses

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Abstract

Collaborative learning has been highlighted as an effective method of teachers' professional development in various studies. To disclose teachers' discourse threads in the process of collaborative learning for developing their knowledge, this paper adopted two methods including "content analysis" and "time-sequential analysis" of learning analytics. Such analyses were implemented for mining teachers' updated knowledge and the discourse threads in the discussion during collaborative learning. The materials for analysis involved two aspects: one was from the video-taped lesson observation reports written by teachers before and after discussing, and the other was from their discourses during the discussion process. The results proved that teachers' knowledge for teaching the centroid of a triangle was updated in the collaborative learning period, and also revealed the discourse threads of teachers' collaboration contained "requesting information or opinions", "building on ideas", and "providing evidence or reasoning", with the emphasis on "challenging ideas or re-focusing talk"

Keywords Teacher Collaborative Learning, Teachers' Knowledge, Content Analysis, Time-Sequential Analysis

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I. INTRODUCTION

Collaboration is an activity that promotes professional dialogue through careful consultation, effective communication, joint decision making and learning (Boavida & Ponte, 2002). Mathematics teachers around the world often work and learn in various collaborative ways, including Lesson study and Learning study in Asia, workshop or design-based research in Europe (Ornella, Annalisa, Alison, Barbara, Olive, Cristina, Merrilyn, Masami, & Marie, 2016). Teacher collaboration was encouraged as a policy recommendation through professional development activities or classroom practices (Schleicher, 2015). The qualitative studies from Singapore and the US declared that collaborative learning could help teachers increase their understanding of pedagogical and subject content knowledge (Kamina & Tinto, 2011; Lawrence & Chong, 2010; Lewis, Fischman, Riggs, & Wasserman, 2013). A relatively limited number of studies considered that teacher discussion on classroom performance was linked to their professional understanding (Warwick, Vrikki, Vermunt, Mercer, & Van Halem, 2016). These positive study findings appeared to point to a path forward for teachers. Even when studied using a somewhat organized study model, the rather structured model of contributing relationships in teacher collaboration were not always obvious (Horn & Little, 2010). There is not thoroughly investigated in the process of collaborative learning.

The outcomes of teachers' learning are usually led as intervention features to conceptualize teacher learning, such as knowledge, skills, or attitudes, instead of aiming at the process of teachers' learning. This conceptualization examined the effectiveness of teacher professional development programs (Desimone, 2009) but inadequately located details in the process between intervention and learning outcomes (Vermunt, 2013). Although Bakkenes, Vermunt, and Wubbels (2010) was concerned with the teachers' learning in the light of active and self-regulated learning by content analysis resulting in six categories of teachers' learning activities, the specific contributing moves of the teachers' learning process was not actually revealed. However, the temporal dimension of collaborative learning was explicitly recognized as a significant perspective for analyzing the collaboration (Dyke, Kumar, Ai, & Rosé, 2012; Kapur, 2011; Mercer, 2008; Wise & Chiu, 2011). Without taking into account the temporal component, content analysis is unlikely to provide the effective threads necessary for teachers' knowledge progress. This paper examined the teacher collaborative learning process using content and time-sequential analytic methods, and it investigated how collaborative learning influenced mathematics teachers' knowledge.

II. LITERATURE REVIEW

Knowledge Building Discourse in Collaboration

Knowledge building refers to aiming towards the construction and advancement of shared knowledge artifacts focused on collaborative learning (Scardamalia & Bereiter, 2014). Professional communities provide friendly environments or social structures in cognitive activities to enhance and sustain professional development (Webster-Wright, 2009). The previous studies showed that collaborative learning had many effects on teachers' work or learning, such as teachers' belief, personality, mathematical knowledge in teaching, and teaching practice, which could change in collaborative learning (Huang & Shimizu, 2016). Particularly, the change of teachers' professional knowledge is prominent in collaborative situation. For instance, Heller, Daehler, Wong, Shinohara, and Miratrix (2012) compared the impacts of collaboration-based professional development programs and disclosed the increase of teachers' conceptual knowledge. Lesson Study was considered as a contextual factor of changing teachers' professional knowledge for instructing as identified by Huang & Shimizu (2016), which might also lead to positive learning patterns with teacher learning based on the framework of Vermunt and Endedijk (2011). Effective collaborative learning could be supported by sustained, dynamic, and interactive discourses, as a means of sharing teachers' knowledge with each other and developing collaborative knowledge construction (Eryilmaz, van der Pol, Ryan, Clark, & Mary, 2013).

The features of collaboration indicated the central role of the interaction, the sharing of ideas with group members, resulting in advantages for teachers. Vygotsky (1978) elaborated that people's intramental functioning was shaped through social interaction and communication with others to the importance of collaborative learning, as a common feature, particularly in functioning professional groups with joint goals including teachers. From this conception, teachers' learning in a community could be regarded as a process of accumulating and building professional knowledge, and that analyzing the content or quality of teachers' communication could give a means to explain the details for productive outcomes or impacts (Vrikki, Warwick, Vermunt, Mercer, & Van Halem, 2017). Discourse may be considered as one of the crucial roles in the learning process of teacher collaboration for their professional development. A wide range of research brought out the discourse as a tool to enable people to understand others' knowledge, and it was encouraged to develop new ideas (Mercer, 2000). Starting with an initial proposal, a discussion may be triggered by clarifying or comparing different ideas. When group members are doubtful, to actively promote the proposal, their talks and discourses seem to be the important traits and supportive conditions (Nemeth, Rogers, & Brown, 2001). However, the details and characteristics related to teachers' improvement require more clear evidence in collaborative situations. The previous studies were mostly quantitative including survey methods (Doppenberg, den Brok, & Bakx, 2012; Ronfeldt, Farmer, McQueen, & Grissom, 2015) or experiments of collaborative teacher professional development programs (Saunders, Goldenberg, & Gallimore, 2009). Although there are a variety of approaches to investigate the process of collaborative learning, including using sources of information such as discourses, few studies have revealed the essential indicators for the correlation and effect of learning from the multifarious information.

Content Analysis and Time-Sequential Analysis in Learning Analytics

Understanding the discourses and movements in face-to-face or online learning environments, or what happens in the generation process, are crucial issues in the study of collaborative learning (Lossman & So, 2010). The familiar sources like text or speech often appear in studies, as records are not hard to collect from the face-to-face activities and online systems with rich information, including varied types of explanations and topics (Scardamalia & Bereiter, 2014). When it comes to the previous content analysis schemes, online discourse is regarded to be popular and promotive (De Wever, Schellens, Valcke, & Van Keer, 2006), which is rather beneficial for the assessment of group knowledge in a given point of time. Identifying and labeling the content systematically, which can be analyzed quantitatively using statistical methods or from qualitatively digging into the meanings of content within texts, is a major feature. The frequencies for considering themes or words are the simplest and most objective form of content analysis of unambiguous characteristics. However, the quantitative measures and analysis hardly considered shared knowledge produced by a group, nor mentioned the dynamic and interactive process of inquiry that contributes to the advancement of such knowledge (Chen, Resendes, Chai, & Hong, 2017). Mercer (2005) pursued the joint intellectual activity like collaborative learning and advocated the complementary use of qualitative and quantitative methods, proposing Sociocultural Discourse Analysis (SCDA) for studying classroom talk. Influenced by SCDA, Vrikki and Warwick (Vrikki et al., 2017; Warwick et al., 2016) constructed a reliable coding scheme for teachers' learning to identify the dialogic moves. However, clarifying the effective discourse threads remains a challenge. It is unlikely to reach a contributing conclusion based just on content analysis without taking into account the temporary.

Temporality is gaining attention gradually for the process of collaborative learning, such as regarding the temporal patterns of knowledge construction in online discussions (Wise & Chiu, 2011). Learning analytics refers to the analysis and interpretation of data related to learners' behaviors and interactions during the learning process, as well as learners' profiles and the learning contexts they are situated in (Hwang, Chu, & Yin, 2017). To understand deeply the collaborative learning, learning analytics was recently applied and has contributed greatly to a multitude of studies, which got explicit recognition as a vital perspective for analyzing collaboration. One of the popular methods of learning analytics is time-sequential analysis, which can provide the learning behavioral or interactive models by finding the sequential relationships between learning behaviors or interactive content to analyze learning patterns (Hwang et al., 2017). For analyzing the discourses of learning in collaboration, Chen, Resendes, Chai, & Hong (2017) applied this method to uncover sequential patterns that distinguished productive threads of knowledge-building discourse, advocating for more analytics tapping into the temporality of learning. Thus, characteristics such as the discourse thread may be retrieved as an indication to detect knowledge sharing and development in the collaborative learning process using in-depth learning analytics methods such as time-sequential analysis.

Most research adopted sequential analysis approaches to unfold various patterns of different learning forms. For example, college students' learning behavioral patterns and temporal changes in online interact platforms were reported by applying a sequential analysis to identify significant transitions between interactions with content, peers, and instructors (Cheng et al., 2017). Also, through collecting the discussion activities of teachers and students, the learning behavior transitions were identified to show the effective problem-solving strategy combined with the flipped classroom (Chiang, 2017). Moreover, sequential analysis was adopted into an evidence-centered design framework to analyze students' problem-solving behavior patterns (Hu et al., 2017), which provided a micro-level approach for problem-solving competency assessment. Tsuei (2017) used the sequential analysis to identify several behavior patterns of peer-tutoring, which supported the effectiveness of helping tools in enhancing children's tutoring actions in a face-to-face synchronous peer-tutor system. In the synchronous system, the scenario of a face-to-face interaction provided greater opportunities to apply time-sequential analysis to learning in the actual world. However, much of the prior study focused on students' collaborative learning habits, and there is currently a noticeable paucity of research on the growth of teachers' collaboration. The focus of this study was on teachers' knowledge development in collaborative learning in reality and in complex debate circumstances.

Research Questions

With the popularity of applying teachers' collaborative learning, various research on specific mathematical content were involved in teachers' professional development, but the evidence for generating this development has been hitherto lacking, especially in the process of collaborative learning. Teacher learning in communities have an impact on teachers' knowledge, but not to an extent to bring about such an effect (Vrikki et al., 2017). Besides, in the light of secondary mathematics, the centroid of a triangle is often considered as the theme of inquiry activity in the classroom, whose definition and properties were often misunderstood by secondary mathematics teachers (Shin, 2016). Therefore, the purpose of this study is to explore the teachers' collaborative learning process while discussing specific mathematical content such as centroid of a triangle. More specifically, answers to the following research questions are sought in this research.

A. How different is teachers' knowledge for instruction about the centroid before and after collaborative learning on characteristics of a video-taped lesson?
B. What features in discourses are within teachers' knowledge for teaching of the centroid during the collaborative learning process?

III. METHODS

There is abundant evidence that analyzing video-taped lessons has a positive effect on teachers' understanding of the teaching-learning process and on their overall professional development (Alsawaie & Alghazo, 2010). With video-based learning, this study used content analysis and time-sequential analysis to resolve the research questions.

Design of Experiment

Participants. Four mathematics teachers with about an average of ten years of teaching experience participated in this study (1 male, 3 female). They all enrolled in master's courses of mathematics education and have taught mathematics in middle schools. The participants have experiences of collaborative learning, who are affiliated with a respective learning community, with regularly participating in the community to develop their knowledge of teaching mathematics.

Experiment Procedures. The experiment was implemented in two parts. In the first part, teachers were provided with a video-taped lesson themed as the centroid using the inquiry activity, which gave the definition of a centroid as the intersect point of three mid-lines (the specific introduction is included in the Appendix). After watching the video-taped lesson, the teachers were required to write personal reports (pre-report). In the second part, teachers could have a 40-minute discussion about the video-taped lesson, with the process being recorded both in audio and video formats. Equally, teachers wrote reports after the discussion (post-report). The concrete procedures included: (i) watch the video-taped lesson, (ii) write the pre-report, (iii) group discussion, and (iv) write the post-report. The pre- and post-reports written by the teachers were about their judgments or opinions on the instructor's teaching in the video, which would present their ideas or knowledge based on their previous awareness.

Instrument

Framework for Analyzing Teachers' Reports. The video is a lesson themed on the centroid using a principle inquiry activity; it is also a required framework on geometric pedagogy for analyzing teachers' reports. The Geometry Assessments for Secondary Teachers (GAST) is adopted, for assessing teachers' knowledge for teaching geometry (Margaret et al., 2017). The blueprint of the framework (Table 1) involved in teaching geometry at the secondary level was implemented and three categories of knowledge for teaching geometry sub-domains were used to analyze the depth of knowledge. The authors in this study evaluated the reports by classifying their opinions founded upon Table 1 in order to analyze teachers' knowledge for teaching on the centroid. If there are opinions on the geometry content typically found in the secondary geometry curricula and taught in secondary schools, these opinions were sorted as the knowledge of school geometry; the ideas also included post-secondary geometry which belonged to the knowledge sphere of advanced geometry; and the pedagogy and practices in teaching geometry were classified as geometry pedagogical content knowledge.

Table 1. Framework for analyzing teachers' reports (Margaret et al., 2017)

Sub-domains	Contents
Section 1: Knowledge of school geometry	a. Can recognize and describe appropriate demonstrations, interpretations, analogies, and justifications to develop mathematical skills and procedures. b. Can recognize and describe appropriate definitions, representations, examples, distinguishing examples, non-examples, counterexamples, and the necessity and sufficiency to develop mathematical concepts. c. Can recognize and describe meaningful connections (lateral, upward, downward) within and among mathematical content. d. Can recognize and construct a meaningful mathematical model of real situations.
Section 2: Knowledge of advanced geometry	a. Can solve non-routine problems, including real applications, in geometry. b. Can analyze and construct synthetic, transformational, and analytical proof and recognize valid and invalid arguments (e.g., reasoning by converse). c. Can analyze and justify geometric formulae.
Section 3: Geometry pedagogical content knowledge	a. Can recognize and describe strategies and activities that promote student reasoning and problem solving (e.g., questioning, posing a problem). b. Can anticipate, recognize, describe, assess, and address correct and incorrect elements of student responses (e.g., skills, concepts, reasoning). c. Can recognize, describe, and assess critical student prerequisite knowledge. d. Can recognize and construct assessment tasks at various cognitive levels. e. Can recognize and describe the advantages and limitations of using digital technologies (e.g., interactive geometry software, graphing calculators). f. Can recognize and describe advantages and limitations of using physical models (e.g., solids, paper folding) and tools (e.g., compasses, straightedges).

Table 2. Coding scheme for analyzing teachers' discourses (Vrikki et al., 2017; Warwick et al., 2016)

Codes	Descriptions	Examples
D1	Requesting information, opinion or clarification	If the plenary is to apply to different things, what would be a good thing to do?
D2	Building on ideas	Yes, I think that Ben, you hope, would now able to explain why both are a whole.
D3	Providing evidence or reasoning	It's tricky because there's a spread of abilities and they have different learning intentions almost.
D4	Challenging ideas or re-focusing talk	Yeah, but in the first lesson like that we shared too much? Remember when we said that we would put up some results on the board that, for discussion, and…

Coding Scheme for Analyzing Teachers' Discourses. Vrikki and Warwick (Vrikki et al., 2017; Warwick et al., 2016) reported a reliable coding scheme revealing that learning in a group had an impact on teachers' individual learning processes with descriptions and interpretations. The coding scheme was adopted for analyzing the discourse of each teacher in this study. Table 2 presented the scheme, which included four mutually exclusive categories with examples for coding.

The teachers' discussion was divided and selected five episodes based on the main issues for teaching the centroid in the discussion listed in Table 3. Three coders were trained to analyze the teachers' discourse before the experiment. The inter-coder reliability coefficient Kappa was 0.95 (>0.8).

Table 3. Descriptions of five episodes in teachers' discussion

Episodes	Time	Descriptions
Intro	00:00:00-05:44:24	Introducing on the video-taped lesson.
1	05:44:25-11:53:57	Considering students' inquiring the centroid activity.
2	11:53:57-18:32:59	Talking about themselves' understanding of centroid and the relevant theorems.
3	18:59:11-21:07:53	Providing evidence or reasoning for themselves' understanding of centroid and the relevant theorems.
4	23:40:02-26:59:62	Deliberating on how to deal with the inquiry results given by students for the theorem of centroid.
5	27:05:58-31:58:09	Talking about themselves' attitude to the students' group activity.
End	31:58:09-40:00:00	Having a free talk about personal experiences with lesson implementing.

In order to analyze the whole process and each episode, GSEQ was employed. The GSEQ method is a computer program for analyzing sequential observational data for computing varied simple and contingency table statistics including joint frequencies, adjusted residuals, chi-squares, for 2×2 tables, Yule's Q, and odds ratios. Adjusted residuals were computed to determine the time-sequential patterns of activation (z-score≥1.96) of the target behavior (Hou, 2012). Z-scores were computed for each possible event pairing while considering the differences in relative and observed frequencies of given and target events (Jeong, 2003). When the z-score is greater than 1.96 (z-score≥1.96), it showed the significant probability of a behavior sequence ($p<0.05$). Related to the research aim, the summary of the relationship between methods and instruments in this study is shown in Figure 1.

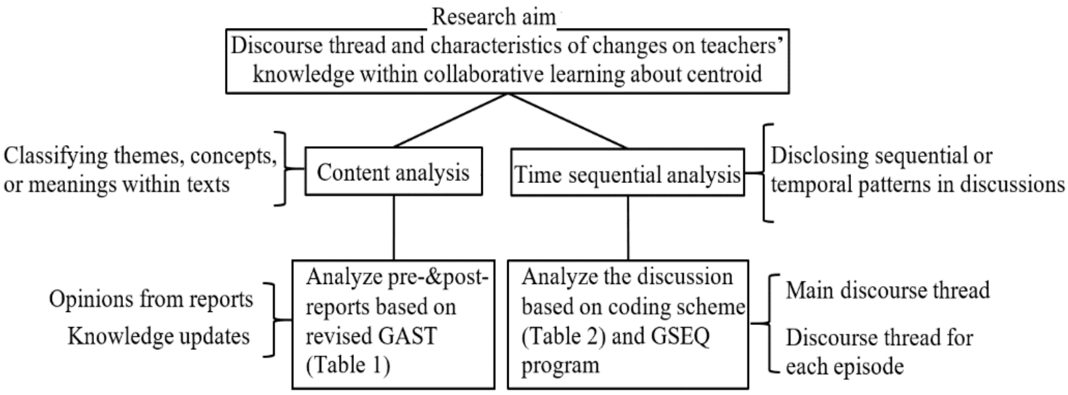

Figure 1. Research methods

IV. RESULTS

Analysis of Teachers' Reports

According to Table 1, the authors classified teachers' opinions on the pre- and post-reports. The following are the specific examples.

> There is no corresponding feedback to the statement that the student accounted for half of the area. (Section 3-b)

The comment above was about the teacher in the video who addressed students' responses, and it indicated that the teacher in the video did not give any assessment to the student. Accordingly, it was classified into Section 3-b. In addition, the opinion below belonged to Section 3-f because it commented on the students' paper folding activity and pointed out a limitation of the physical model.

> Students fold in half, and then hand over, after folding, it may not be flat, and the center of gravity will be affected. (Section 3-f)

After classifying the viewpoints of teachers' reports, the number of teachers' views in each section was counted and recorded in Table 4. Compared with the number of views teachers wrote, it indicated two types of updates even though teachers had some common opinions on the video-taped lesson.

Table 4. Number of each teacher's updated viewpoints after the discussion

Teacher		Section 1				Section 2			Section 3					
		a	b	c	d	a	b	c	a	b	c	d	e	f
Teacher 1	pre-	1	2	1	0	0	1	0	2	2	1	0	0	0
	post-	0	0	1	0	0	1	0	3	0	0	1	1	1
	updated			1			1		3			1	1	1
Teacher 2	pre-	0	4	0	1	0	0	0	1	1	0	0	0	3
	post-	0	1	1	0	0	1	0	1	1	0	1	0	1
	updated		1	1			1		1	1		1		1
Teacher 3	pre-	0	0	1	0	0	1	1	7	5	0	0	1	3
	post-	0	0	0	0	0	1	1	2	3	0	0	0	1
	updated						1	1	2	3				1
Teacher 4	pre-	0	0	0	0	0	1	0	1	0	0	0	0	6
	post-	0	0	0	0	0	0	0	1	2	0	0	0	0
	updated								1	2				

One type was that opinions or comments existed in both pre- and post- reports in the same section but to different extents. Teacher 1's viewpoint about Section 2-b was an instance as follows.

> Teacher 1's pre-report: It could not explore the true meaning of the centroid, because students just only knew that the triangle would stand up if a finger was put at the intersection of three mid-lines. (Section 2-b)
>
> Teacher 1's post-report: … the logic of the 'mid-line divided it into two triangles with equal area, so the intersection of the three mid-lines is the centroid' is not correct. (Section 2-b)

There were similar opinions on the meaning of the centroid in two reports from Teacher 1, which belonged to the recognition of invalid arguments in the knowledge of advanced geometry referring to Section 2-b. However, the pre-report focused more on students' behaviors while the post-report showed the understanding of the mathematical content involving the nature of the centroid. Teacher 1's understanding of advanced geometry linked to the centroid was updated after discussion, as she recognized the invalid arguments, based on her further comprehension of the mathematical content in the post-report.

The other type was that opinions noted in the post-report, but nothing proposed in the pre-report. For example, there was no viewpoint in Section 2-b from the Teacher 2's pre-report, but Teacher 2 mentioned that she learned the knowledge about the centroid in the post-report shown as follows.

Teacher 2's post-report: I recognized that I did not know about the centroid very well after Teacher 3's statement. I learned it (i.e., the centroid is the point where the rotational force is zero like the fulcrum in the principle of levers.) was not a good explanation and that the reason why the centroid is defined as the intersection of mid-lines is because one of the mid-line divides the area into half. (Section 2-b)

It indicated that Teacher 2 had some gains to perceive the invalid argument related to the centroid through the discussion. Also, in Section 3-d, Teacher 1 extended the range of centroid in the post-report.

Teacher 1's post-report: … if setting the speech for the centroid of a circle or square at the ending of this lesson, it might be better than setting it as homework, because finding the centroid of a square is too hard for the students without teacher's guide. (Section 3-d)

Teacher 1's comment above was beyond the triangle level and enlarged to the circle and quadrilateral level, and she gave the specific proposals of setting the lesson's ending after the discussion. It showed Teacher 1's recognition on the content at different cognitive levels. Teacher 1's learning about constructing her own lesson for the centroid was generated during the collaborative process, which affected and motivated Teacher 1 to acquire the knowledge for teaching geometry.

Analysis of Teachers' Discourses

Based on the transcripts of the teachers' discussion, the coders provided teachers' discourses with the four elements in Table 2 for examples as follows.

Teacher 1: If developing this, the paper needs to be put on the finger. Let's talk about this one. (D1)

Teacher 1: Children were just doing that but didn't feel interested, even though they were still doing the task. (D2)

Teacher 3: Because in physics, the centroid is the point where the rotational force is zero like the fulcrum in the principle of levers. (D3)

Teacher 4: Really? I think this method is not bad …. (D4)

Based on sequential analysis rules (Hou, 2012), the adjusted residuals were determined by a series of matrix operations, and the frequency of the discourse sequence was statistically significant ($p<0.05$) if the z-score of the sequence was greater than or equal to 1.96. Table 5 listed teachers' discourse sequences. The first column listed the initial discourses, and the first row listed the consecutive discourses. The significant sequences (z-score≥1.96) were listed in the cells of Table 5 such as D1→D2, D2→D3, D4→D4.

Table 5. A developmental progression for volume measurements (Sarama & Clements, 2009)

initial discourses \ consecutive discourses	D1	D2	D3	D4
D1	-0.88	2.33*	-2.04	-0.44
D2	1.31	-0.78	2.51*	-3.88
D3	-0.65	0.48	-1.19	1.67
D4	-0.39	-2.56	-0.28	5.97*

All significant discourse sequences in Table 5 were diagrammed in Figure 2. The arrows in the diagram represent significant transitions from one discourse to another, suggesting that the teachers have consistent discourse threads with characteristics referring to the z-score (≥1.96). From the transition diagram, it is seen as representations of a discussion with members' reflection that led to change the related knowledge among the teachers involved.

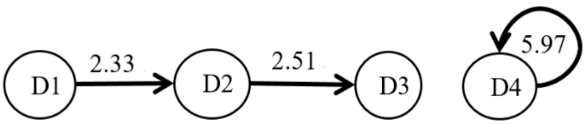

Figure 2. Transfer state diagram of teachers' discourses

In Figure 2, the transition confirms the typical logic of discourses with requesting, building on ideas, and providing evidence, shown as the following transition route: D1→D2→D3. The route D3→D4 is not shown in this diagram since the z-score (=1.67) is not larger than 1.96. However, the route D4→D4 (z-score=5.97) is illustrated in that challenging ideas or re-focusing talk can be strongly performed during the discussion.

Focusing on the process of the teachers' discussion, the main discourse thread (D1→D2→D3) from the time-sequential analysis showed that teachers could proffer their own reasonable opinions about the video-taped lesson based upon their background knowledge. From the recorded transcripts, this thread could be found as follows.

Teacher 1: Let's talk about the properties! (D1)

Teacher 3: It's really awful. It would be better if having tried it once, but this one was …actually it could be confirmed simply, but it became like this. (D2, D3)

In addition, in the main thread, D4→D4 displayed teachers' debate scenes for challenging ideas or re-focusing talk during the discussion, showing the transcripts as follows.

Teacher 4: Wouldn't it be best if the groups didn't do that as quite systematically? Actually, it would be better to give students more time, but it's impossible obviously. (D4)

Teacher 1: Let's talk about this question later. I just thought it would be better if studying deeper about the centroid. (D4)

As just noted, teachers' discourses are followed as the thread of requesting, building on ideas and providing evidence (D1→D2→D3, shown in Figure 2), asking for more information by repeating challenges or re-focusing the talk (D4→D4, shown in Figure 2). For each episode, the transition diagrams were plotted in Figure 3. The threads of Episode 1 were fairly consistent with the main threads in Figure 2. Episodes 3 and 5 could not be drawn as transfer state diagrams because there were no z-scores larger than 1.96. Some z-scores (1.96 for D3→D1 in Episode 2; 2.49 for D3→D4 in Episode 4) were not prominent and not recorded in the main thread in Figure 1. However, it still caught our attention as there might be two kinds of discourse threads after D3, with the further descriptions elaborated as the following specifically.

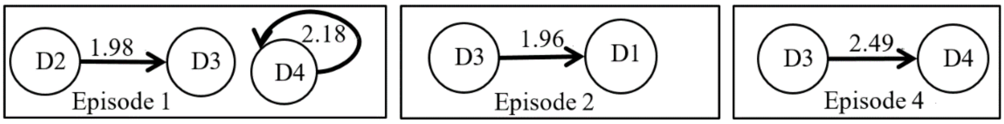

Figure 3. Transfer state diagrams of teachers' discourses in episodes

In Episode 2, teachers expressed their understanding about the method and explanation of discussing the centroid in the video-taped lesson. Teacher 3 elaborated her own viewpoint about the centroid with agreement on the explanation of the method, but Teacher 4 queried the method proposed in the lesson and requested more confirmation, shown as detailed as follows.

Teacher 3: The explanation could be correct for the triangle. However, the intersection of line that divides the area of a polygon into half is not the centroid of the polygon. The centroid is the point where the rotational force is zero like the fulcrum in the principle of the levers. (D3)

Teacher 4: In a polygon, the meaning is not this one, right? (D1)

The challengeable discourses demonstrated the process of sharing and building knowledge from teachers' discourses, which exactly conformed to what Teacher 2 wrote in the post-report that the invalid argument was obtained from Teacher 3's statements in Episode 2. Although Teacher 3 gave the further statement on a general centroid followed this episode, she admitted the idea of dividing the area into half was misleading and misunderstanding for the situation of a general centroid. With Teacher 2's report and the discourses in this episode, it emerged clearly the knowledge sharing and constructing in teachers' collaborative learning.

Besides the discourse thread of requesting information after getting the sign of a certain idea, challenging or re-focusing the topic could also be a productive thread to get satisfying and convincing explanations, as the example below from Episode 4.

Teacher 3: … I think they were similar triangles because I would let students write down the specific length and point it out, but the teacher didn't mention that …. (D3)

Teacher 1: (Interrupting Teacher 3) She mentioned it. (D4)

Teacher 2: (Almost at the same time with Teacher 1) The teacher in the video actually mentioned that there could be certain errors, not necessarily just as the ratio of 2:1. (D4)

Teacher 3 elucidated her views on the teacher's guiding and gave the specific reasons such as she deemed that the teacher did not signal some details of similar triangles. However, Teacher 1 directly interrupted Teacher 3's comment and pointed out the inaccuracies of the evidence provided by Teacher 3, which displayed that Teacher 1 challenged Teacher 3's reasoning. Then, Teacher 2 provided the explicit information with a quotation of the words from the teacher who was observed in the video-taped lesson. By manifesting the specific evidence of the teacher's awareness of the property of similar triangles, it also demonstrated Teacher 2's challenging to Teacher 3's explanation.

V. DISCUSSION AND CONCLUSIONS

Evidence demonstrating teachers' learning processes through collaboration is not sufficient enough, although collaborative learning is acknowledged by many countries. Most studies implemented unitary analysis methods or pressed the outcomes and effects of collaborative learning. In this study, content analysis and time-sequential analysis were used for excavating the teachers' reports and discourses. Specifically, content analysis was employed to analyze the teachers' updated knowledge for instruction upon the centroid from the pre- and post-reports based on the framework of GAST. With the technology of time-sequential analysis, the discourse threads were presented in the perspective of temporality, which to some extent revealed that teachers caught information and moved on the discussion to expand their knowledge for teaching of the centroid.

In addressing the first research question, we analyzed the teachers' reports written before and after the discussion. Based on the framework of GAST, it showed that the teachers' understanding of teaching the centroid was indeed updated after the collaborative discussion, especially in Section 2 and Section 3 related to the knowledge of advanced geometry and the pedagogical content of geometry. Teacher 1 and Teacher 2 had apparently changed after collaborative learning. Teacher 1 was inspired by ideas on knowledge of pedagogical content in geometry, such as the extended teaching design for lesson ending, which was rather educational and clear evidence for the influence on the collaboration. Teacher 2 did not express lots of ideas during the discussion but wrote what she learned through the discussion in the post-report. The direct statement of her own insufficiency in advanced geometry is shown by the effect the discussion had upon her. From the results of Episode 2, one viewpoint in the report of Teacher 2 after the discussion was found to be consistent with that provided by Teacher 1 before the discussion. It is acceptable to speculate that teachers' discussion promoted the production of shared knowledge and creative ideas (Mercer, 2000).

Considering the second research question, Vrikki et al. (2017) proposed the discourse features, including requesting information, giving reasons, providing evidence, making supportive comments, and challenging ideas, when teachers were engaged in dialogic interactions. However, the factor of temporality was not noticed (Chen et al., 2017). This study highlighted the generated threads during the discussion by employing the time-sequential analysis. The main discourse thread D1→D2→D3 was uncovered by the sequential analysis, which elaborated the typical logic of discourses with requesting, building on ideas and providing evidence, and D4→D4 showed the importance of challenging ideas or re-focusing talk during the discussion. Moreover, there could be two possible consecutive discourses after D3, which provided two paths after someone giving evidence: D3→D1 is requesting more information to make sense; D3→D4 is challenging or re-focusing the talk.

Through collaborative learning, teachers had an advantageous experience for their knowledge sharing and building. Based on teachers' reports, the updated knowledge was clearly illustrated by two types. Learning in collaboration provided opportunities for teachers to gain deeper geometry pedagogical knowledge, broaden their thinking range of the related lessons, which was a benefit for their professional development, and it also motivated teachers' creativity for preparing the geometry lessons. The discourse threads showed the logic of teachers via discussion when talking about the video-taped lesson. Besides the general thread, challenging and requesting more information played a crucial role in the threads. In the situation like a debate, the thread of challenges or re-focusing after reasoning could clarify some mistakes of evidence, and the direct expression and challenges helped to create a discussion atmosphere and to unify the views of topics for knowledge construction. Through these kinds of discourses in collaborative discussion, teachers could get more inspirations and experienced greater brainstorming for expanding knowledge of the centroid, recalling some critical content learned in university to deepen their understandings of the key concepts for more flexible and effective teaching.

The previous research collected rich log materials and documents, involving a considerable amount of information (Scardamalia & Bereiter, 2014). In this study, teachers' reports and discourses were linked for analyzing the development of teachers' professional knowledge during their collaborative learning, in order to disclose the detailed evidence in a micro perspective. The mixed method of applying the content analysis and time-sequential analysis is a practical exploration of combining qualitative and quantitative research methods. Focusing on the process of teachers' collaborative learning, two approaches of analyzing materials were employed in this study, including the scripts of discourses, commonly seen in qualitative research that analyzing the content of scripts, and the threads of discourses for time-sequential analysis in quantitative methods. Compared with analyses of different sources from the same process and combined the results, it could deepen or revise the understanding for the study, and enhanced the validity of qualitative research. Two approaches verified the concurrent triangulation strategy in the mixed method (Creswell & Clark, 2011), presenting the change of teachers' knowledge in forms of reports and discourse threads. This not only provides an empirical case of integration for material analysis, but offers the quantitative evidence that can be verified with qualitative research for the in-depth mining of the effectiveness of collaborative learning.

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Appendix: Flow of the video-taped lesson.

Flows	Time	Descriptions
Introduction (Midpoint-connection theorem of a triangle)	~7:00	Review previous knowledge: check students' knowledge of midpoint-connection theorem. Show learning goal: students took notes when teacher gave the goal on the board
	~8:30	Provoke learning motivation: show students the Flash material about which position should sit in the boat shaped in a triangle
Expansion (Learn about the meaning of midline, the centroid of a triangle, and property of the centroid)	~15:30	Free inquiry activity: allow a paper triangle to stand on one finger and discuss the methods
	~24:00	Deduce the meaning of the midline and centroid with the previous mathematical properties Let Students present their methods Give students the definition of midline Let students draw three midlines on one triangle and find the intersect point Located at the intersect point, allow the paper triangle to stand on one finger Give the definition of centroid and students underline the related content on the page of textbook
	~29:00	Inquiry activity about the property of centroid: present the simple example of textbook
	~35:00	Prove the property of centroid: speech the related content of textbook
	~38:30	Do the exercise of the textbook
	~43:30	Formative assessment
Review and assign homework	~45:30	Review the learning content
	~47:00	Give the homework: find the centroids of a circle and a quadrilateral