J. Appl. & Pure Math. Vol. **4***(2022), No. 1 - 2, pp. 51 - 61*
https://doi.org/10.23091/japm.2022.051

4-TOTAL DIFFERENCE CORDIAL LABELING OF SOME SPECIAL GRAPHS

R. PONRAJ*, S. YESU DOSS PHILIP AND R. KALA

Abstract. Let G be a graph. Let $f : V(G) \to \{0, 1, 2, \ldots, k-1\}$ be a map where $k \in \mathbb{N}$ and $k > 1$. For each edge uv, assign the label $|f(u) - f(v)|$. f is called k-total difference cordial labeling of G if $\left|t_{df}(i) - t_{df}(j)\right| \leq 1$, $i, j \in \{0, 1, 2, \ldots, k-1\}$ where $t_{df}(x)$ denotes the total number of vertices and the edges labeled with x. A graph with admits a k-total difference cordial labeling is called k-total difference cordial graphs. In this paper we investigate the 4-total difference cordial labeling behaviour of shell butterfly graph, Lilly graph, Shackle graphs etc..
AMS Mathematics Subject Classification : 05C78.
Key words and phrases : Shell-butterfly, Lilly graph, Shackle graph.

1. Introduction

All graphs in this paper are finite, simple and undirecte. The notion of k-total difference cordial graph was introduced in [3]. 3-total difference cordial labeling behaviour of sevaral grphs like path , complete graph,comb ,armed crown, crown , wheel, star etc have been investigated in [3, 4] . Also 4-total difference cordial labeling of path , star , bistar,comb,crown,$P_n \cup K_{1,n}$,$S(P_n \cup K_{1,n})$,$P_n \cup B_{n,n}$ etc.,have been invetigated in [5, 6, 7]. In this paper we investigate 4-total difference of cordial labeling of Shell butterfly graph, Lilly graph, Shackle graphs etc.,

2. k-total difference cordial graphs

Definition 2.1. Let G be a graph. Let $f : V(G) \to \{0, 1, 2, \ldots, k-1\}$ be a function where $k \in \mathbb{N}$ and $k > 1$. For each edge uv, assign the label $|f(u) - f(v)|$. f is called k-total difference cordial labeling of G if $|t_{df}(i) - t_{df}(j)| \leq 1$, $i, j \in \{0, 1, 2, \ldots, k-1\}$ where $t_{df}(x)$ denotes the total number of vertices and the edges labelled with x. A graph with a k-total difference cordial labeling is called k-total difference cordial graph.

Received December 25, 2021. Revised March 5, 2022. Accepted March 7, 2022. *Corresponding author.

3. Preliminaries

Definition 3.1. A multiple shell is defined to be a collection of edge disjoint shells that have common apex vertex. Hence a double shell consists of two edge disjoint shells with a common apex vertex. A *Shell-butterfly* graph is a double shell in which each shell has any order with exactly two pendent edges at the apex.

Definition 3.2. The *Lilly graph* $I_n : n \geq 2$ is constructed by 2 stars $2K_{1,n}$,$n \geq 2$, joining 2 path graphs $2P_n$,$n \geq 2$ with sharing of a common vertex. Let $V(I_n) = \{u_i, v_i : 1 \leq i \leq n\} \bigcup \{x_i : 1 \leq i \leq n\}$ $\bigcup \{y_i : 1 \leq i \leq n-1\}$ and $E(I_n) = \{x_n u_i, x_n y_i : 1 \leq i \leq n\} \bigcup$ $\{x_i x_{i+1} : 1 \leq i \leq n-1\} \bigcup \{x_n y_1\} \bigcup \{y_i y_{i+1} : 1 \leq i \leq n-2\}$.

Definition 3.3. *Switching a vertex s* of a graph G results in a formation of a new graph G_s, by deleting edges incident to s in G adding the edges that are obtained by joining the vertex s to the vertices which are not adjacent to s in G.

Definition 3.4. The *Torch graph* O_n is the graph with
$V(O_n) = \{u_i : 1 \leq i \leq n+4\}$ and $E(O_n) = \{u_i u_{n+1} : 2 \leq i \leq n-2\} \bigcup$ $\{u_1 u_i : n \leq i \leq n+4\} \bigcup \{u_{n-1}u_n, u_n u_{n+2}, u_n u_{n+4}, u_{n+1}u_{n+3}\}$.
Clearly $|V(O_n)| + |E(O_n)| = 3n + 7$

Definition 3.5. The *Shell-butterfly graph* S_n is the graph with
$V(S_n) = \{u_0, v_0, w_0, u_i, v_i : 1 \leq i \leq m, 1 \leq i \leq n\}$ and
$E(S_n) = \{u_0 v_0, u_0 w_0, u_0 u_i, u_0 v_i : (1 \leq i \leq m), (1 \leq i \leq n)\}$.

Definition 3.6. Let P_n be the path $u_1 u_2 \dots u_n$. The pentagonal snake is the graph with
$V(PS_n) = V(P_n) \bigcup \{v_i, w_i, x_i : 1 \leq i \leq n-1\}$ and
$E(PS_n) = \{u_i u_{i+1} : 1 \leq i \leq n-1\} \bigcup \{u_i v_i, v_i w_i, w_i x_i, x_i u_{i+1} : 1 \leq i \leq n-1\}$.

Definition 3.7. Double pentagonal snake $D(PS_n)$ is obtained by two pentagonal snakes that have common path.

4. Main Results

Theorem 4.1. The *Shell-butterfly* graph S_n is 4-total difference cordial for all $n \geq 4$.

Proof. Take the vertex set and edge set as in definition 3.5 Clearly $|V(S_n)| + |E(S_n)| = 3m + 3n + 3$.
Case 1. $m \leq 3, n \leq 3$. In this case we assign the labels to the vertices as in table 1

Case 2. $n \equiv 0 \pmod 4$.
Assign the to the label to the vertices as in case(1) for $1 \leq m \leq 3$ and $1 \leq n \leq 3$. Next assign the label 1 and 0 to the vertices u_n and v_n respectively.

Values of n	u_0	v_0	w_0	u_1	u_2	u_3	v_1	v_2	v_3
m,n=2	3	3	1	3	3		1	2	
m,n=3	3	3	1	3	3	1	1	2	3

Table 1

Case 3. $n \equiv 1 \pmod 4$.
Assign the to the label to the vertices $u_0, v_0, w_0, u_i, v_i : (1 \leq i \leq m-1), (1 \leq i \leq n-1$ as in case(2). Assign the label 3 and 1 to the vertices u_n and v_n.
Case 4. $n \equiv 2 \pmod 4$.
Assign the to the label to the vertices $u_0, v_0, w_0, u_i, v_i : (1 \leq i \leq m-1), (1 \leq i \leq n-1$ as in case(3). Assign the label 0 and 1 to the vertices u_n and v_n.
Case 5. $n \equiv 3 \pmod 4$.
Assign the to the label to the vertices $u_0, v_0, w_0, u_i, v_i : (1 \leq i \leq m-1), (1 \leq i \leq n-1$ as in case(4). Assign the label 1 and 3 to the vertices u_n and v_n.
The table 2 shows that the above labeling pattern is 4-total difference cordial labelling .

Values of n	$t_{df}(0)$	$t_{df}(1)$	$t_{df}(2)$	$t_{df}(3)$
n is odd	$\frac{3m+3n+2}{4}$	$\frac{3m+3n+6}{4}$	$\frac{3m+3n+2}{4}$	$\frac{3m+3n+2}{4}$
n is even	$\frac{3m+3n+4}{4}$	$\frac{3m+3n+4}{4}$	$\frac{3m+3n}{4}$	$\frac{3m+3n+4}{4}$

Table 2

A 4-total difference cordial labeling of S_7 is shown in Figure 1

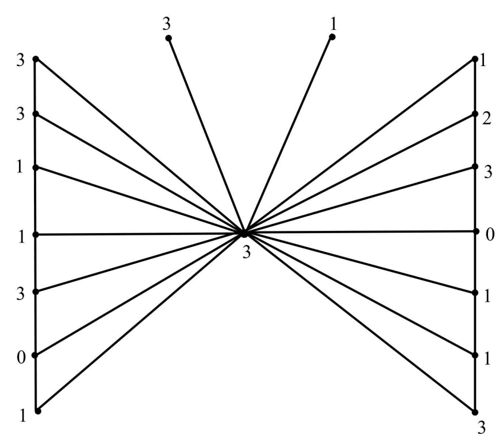

Figure 1

□

Theorem 4.2. Let G_n be the graph with $V(G_n) = \{x_i : 1 \leq i \leq 2n\} \bigcup \{y_i : 1 \leq i \leq n+1\}$ and $E(G_n) = \{x_i x_{i+1} : 1 \leq i \leq 2n\}$ i is odd $\bigcup \{x_{2i-1}y_i : 1 \leq i \leq n\} \bigcup \{x_{2i}y_i : 1 \leq i \leq n\} \bigcup \{x_{2i-1}y_{i+1} : 1 \leq i \leq n\} \bigcup \{x_{2i}y_{i+1} : 1 \leq i \leq n\}$. Then G_n is 4-total difference cordial.

Proof. Clearly $|V(G_n)| + |E(G_n)| = 8n + 1$.
Case 1. n is odd. Define

$$
\begin{aligned}
h(x_i) &= 3, \quad i = 2n+1, n = 0, 1, 2, \ldots \\
h(x_i) &= 1, \quad i \equiv 2 \pmod 4 \\
h(x_i) &= 2, \quad i \equiv 3 \pmod 4 \\
h(y_i) &= 3, \quad i = 1, 2, \ldots, n-1 \\
h(y_n) &= 1,
\end{aligned}
$$

Clearly $t_{df}(0) = t_{df}(1) = t_{df}(2) = \frac{8n}{4}$, $t_{df}(3) = \frac{8n+4}{4}$.
Case 2. n is even. Define

$$
\begin{aligned}
h(x_i) &= 3, \quad i = 2n+1, n = 0, 1, 2, \ldots \\
h(x_i) &= 1, \quad i \equiv 2 \pmod 4 \\
h(x_i) &= 2, \quad i \equiv 0 \pmod 4 \\
h(y_i) &= 3, \quad i = 1, 2, \ldots, n
\end{aligned}
$$

Clearly $t_{df}(0) = t_{df}(1) = t_{df}(2) = \frac{8n}{4}$, $t_{df}(3) = \frac{8n+4}{4}$.

□

Theorem 4.3. The graph G obtained by switching of an end vertex in path P_n is a 4-total difference cordial.

Proof. Let $u_1 u_2 \ldots u_n$ be the path P_n and G_{u_1} be the graph obtained by switching of the vertex u_1 in the path P_n.
Case 1. $n \equiv 0 \pmod 4$.
Assign the label 3 to the vertex u_1. Fix the label $3, 1, 2, 1, 3, 3$ and 3 to the vertices u_2,u_3,u_4,u_5,u_6,u_7 and u_8. Next assign the labels $0, 2, 2$ and 3 to the vertices v_9,v_{10},v_{11} and v_{12}. Similarly assign the labels $0, 2, 2$ and 3 to the vertices v_{13},v_{14},v_{15} and v_{16}. Proceding like this until we reach the vertex u_n. Clearly the vertex u_n receive the label 0 when $n \equiv 1 \pmod 4$,2 when $n \equiv 2, 3 \pmod 4$,3 when $n \equiv 0 \pmod 4$.
Case 2. $n \equiv 1 \pmod 4$.
Assign the label 3 to the vertex u_1. Fix the labels $2, 1, 3$ and 3 to the vertices u_2,u_3,u_4 and u_5. Assign the labels $0, 2, 2$ and 3 to the vertices u_6,u_7,u_8 and u_9. Next assign the labels $0, 2, 2$ and 3 to the vertices v_{10},v_{11},v_{12} and v_{13}. Similarly assign the labels $0, 2, 2$ and 3 to the vertices v_{14},v_{15},v_{16} and v_{17}. Proceding like this until we reach the vertex u_n. Clearly the vertex u_n receive the label 0 when $n \equiv 2 \pmod 4$,2 when $n \equiv 0, 3 \pmod 4$,3 when $n \equiv 1 \pmod 4$.
Case 3. $n \equiv 2 \pmod 4$.
Assign the label 3 to the vertex u_1. Fix the labels $3, 2, 1, 3$ and 3 to the vertices u_2,u_3,u_4,u_5 and u_6. Assign the labels $0, 2, 2$ and 3 to the vertices u_7,u_8,u_9 and

u_{10}. Next assign the labels $0, 2, 2$ and 3 to the vertices v_{11},v_{12},v_{13} and v_{14}. Similarly assign the labels $0, 2, 2$ and 3 to the vertices v_{15},v_{16},v_{17} and v_{18}. Proceding like this until we reach the vertex u_n. Clearly the vertex u_n receive the label 0 when $n \equiv 3 \pmod 4$,2 when $n \equiv 0, 1 \pmod 4$,3 when $n \equiv 2 \pmod 4$.

Case 4. $n \equiv 3 \pmod 4$.

Assign the label 3 to the vertex u_1. Fix the labels $1, 2, 3, 1, 3$ and 3 to the vertices u_2,u_3,u_4,u_5,u_6 and u_7. Assign the labels $0, 2, 2$ and 3 to the vertices u_8,u_9,u_{10} and u_{11}. Next assign the labels $0, 2, 2$ and 3 to the vertices v_{12},v_{13},v_{14} and v_{15}. Similarly assign the labels $0, 2, 2$ and 3 to the vertices v_{16},v_{17},v_{18} and v_{19}. Proceding like this until we reach the vertex u_n. Clearly the vertex u_n receive the label 0 when $n \equiv 3 \pmod 4$,2 when $n \equiv 0, 1 \pmod 4$,3 when $n \equiv 2 \pmod 4$.

The table 3 shows that this vertex labeling is a 4-total difference cordial labeling. □

Values of n	$t_{df}(0)$	$t_{df}(1)$	$t_{df}(2)$	$t_{df}(3)$
$n \equiv 0 \pmod 4$	$\frac{3n-4}{4}$	$\frac{3n-4}{4}$	$\frac{3n-4}{4}$	$\frac{3n-4}{4}$
$n \equiv 1 \pmod 4$	$\frac{3n}{4}$	$\frac{3n-4}{4}$	$\frac{3n}{4}$	$\frac{3n}{4}$
$n \equiv 2 \pmod 4$	$\frac{3n-6}{4}$	$\frac{3n-2}{4}$	$\frac{3n-6}{4}$	$\frac{3n-2}{4}$
$n \equiv 3 \pmod 4$	$\frac{3n-5}{4}$	$\frac{3n-1}{4}$	$\frac{3n-5}{4}$	$\frac{3n-5}{4}$

TABLE 3

Theorem 4.4. The *Lilly graph* I_n is 4-total difference cordial.

Proof. Take the vertex set and edge set as in definition 3.2. Clearly $|V(I_n)| + |E(I_n)| = 8n-3$. Assign the label 3 to the path vertices $x_1, x_2, \ldots, x_n, y_1, y_2, \ldots, y_{n-1}$. Next we assign the label 1 to the vertices $u_1, u_2, \ldots, u_n, v_1, v_2, \ldots, v_{n-1}$ and assign the label 3 to the vertex v_n.

Clearly $t_{df}(0) = t_{df}(1) = t_{df}(2) = 2n-1$, $t_{df}(3) = 2n$.

A 4-total difference cordial labeling of I_7 is shown in Figure 2

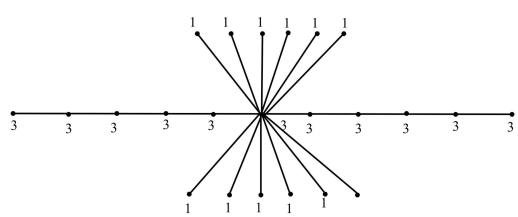

FIGURE 2

□

Theorem 4.5. Switching of an pendent vertex in I_n is a 4-total difference cordial.

Proof. Let S be the switching vertex of I_n. Clearly $|V(I_n)_s|+|E(I_n)_s| = 12n-7$.
Case 1. When u_1 is a switching vertex. That is $s = u_1$.
Assign the label 3 to the vertex x_1. Next assign the label 2 to the vertices $u_2, u_3, \ldots, u_n$. Next assign the label 1 to the vertices $v_1, v_2, \ldots, v_n$. We now consider the path vertices. Assign the label 3 to the vertices $y_1, y_2, \ldots, y_{n-1}$. Fix the label 3 to the vertices x_n and x_{n-1}. Now assign the label 0 and 3 to the vertices x_1 and x_2. Next assign the label 0 and 3 to the vertices x_3 and x_4. Continue in this process untill we reach the vertex x_{n-2}. The vertex x_{n-2} receive the label 0 when n is odd or 3 when n is even.

Clearly $t_{df}(0) = t_{df}(1) = t_{df}(3) = \frac{12n-8}{4}$, $t_{df}(2) = \frac{12n-4}{4}$.
Case 2. When x_1 is a switching vertex. That is $s = x_1$.
Assign the label 3 to the vertex x_1. Next assign the label 2 to the vertices $u_1, u_2, \ldots, u_n$. Next assign the label 1 to the vertices $v_1, v_2, \ldots, v_n$. We now consider the path vertices. Assign the label 3 to the vertices $y_1, y_2, \ldots, y_{n-1}$. Fix the label $1, 3$ and 0 to the vertices x_2 x_3 and x_{n-1}. Now assign the label 0 and 3 to the vertices x_4 and x_5. Next assign the label 0 and 3 to the vertices x_6 and x_7. Continue in this process untill we reach the vertex x_{n-2}. The vertex x_{n-2} receive the label 0 when n is odd or 3 when n is even.
The table 6 shows that this vertex labeling is a 4-total difference cordial labeling. □

Values of n	$t_{df}(0)$	$t_{df}(1)$	$t_{df}(2)$	$t_{df}(3)$
n is odd	$\frac{12n-8}{4}$	$\frac{12n-8}{4}$	$\frac{12n-8}{4}$	$\frac{12n-4}{4}$
n is even	$\frac{12n-8}{4}$	$\frac{12n-8}{4}$	$\frac{12n-8}{4}$	$\frac{12n-4}{4}$

TABLE 4

Theorem 4.6. The graph $P_m \times P_4$ is a 4-total difference cordial.

Proof. Let u_{i1},u_{i2},u_{i3},u_{i4} be the vertices in the i^{th} row. Clearly $|V(P_m \times P_4)| + |E(P_m \times P_4)| = 11m - 4$.
Case 1. $n \equiv 0 \pmod 4$.
Assign the labels $1, 3, 3$ and 1 to the vertices u_{11},u_{12},u_{13} and u_{14}. Next assign the labels $2, 3, 3$ and 3 to the vertices u_{21},u_{22},u_{23} and u_{24}. Assign the labels $1, 3, 3$ and 1 to the vertices u_{31},u_{32},u_{33} and u_{34}. Next assign the labels $3, 3, 2$ and 3 to the vertices u_{41},u_{42},u_{43} and u_{44}. Clearly $t_{df}(0) = t_{df}(1) = t_{df}(2) = t_{df}(3) = \frac{11m-4}{4}$.
Case 2. $n \equiv 1 \pmod 4$.
Assign the labels to the vertices as in case.1. Next assign the labels $3, 1, 3$ and 3 to the vertices u_{51},u_{52},u_{53} and u_{54}. Clearly $t_{df}(0) = t_{df}(2) = t_{df}(3) == \frac{11n-3}{4}$, $t_{df}(3) = \frac{11n-7}{4}$.
Case 3. $n \equiv 2 \pmod 4$.
Assign the labels to the vertices as in case.2. Next assign the labels $3, 3, 2$

and 3 to the vertices u_{61},u_{62},u_{63} and u_{64}. Clearly $t_{df}(0) = t_{df}(3) = \frac{11m-2}{4}$, $t_{df}(1) = t_{df}(2) = \frac{11m-6}{4}$.

Case 4. $n \equiv 3 \pmod 4$.

Assign the labels to the vertices as in case.3. Next assign the labels $1, 3, 1$ and 3 to the vertices u_{71},u_{72},u_{73} and u_{74}. Clearly $t_{df}(0) = t_{df}(1) = t_{df}(3) = \frac{11m-5}{4}$, $t_{df}(2) = \frac{11m-1}{4}$. □

Theorem 4.7. The *pentagonal snake* PS_n is 4-total difference cordial.

Proof. Take the vertex set and edge set as in definition 3.6. Clearly $|V(PS_n)| + |E(PS_n)| = 9n - 8$.

Assign the label 3 to the all the path vertices $u_1 u_2 \dots u_n$. Fix the label 1 and 3 to the vertices w_1 and w_2. Next assign the labels $1, 2, 0$ and 3 to the vertices w_3, w_4, w_5 and w_6. Similarly assign the labels $1, 2, 0$ and 3 to the vertices w_7, w_8, w_9 and w_{10}. Proceeding like this until we reach the vertex w_{n-1}. Clearly the vertex v_{n-1} receive the label 1 or 2 or 0 or 3 according as $n \equiv 3 \pmod 4$ or $n \equiv 0 \pmod 4$ or $n \equiv 1 \pmod 4$ or $n \equiv 2 \pmod 4$.

Consider the vertices $v_i : (1 \le i \le n-1)$. Fix the label 3 to the vertices v_1. Next assign the labels $1, 3, 3$ and 2 to the vertices v_2, v_3, v_4 and v_5. Similarly assign the labels $1, 3, 3$ and 2 to the vertices v_6, v_7, v_8 and v_9. Proceeding like this until we reach the vertex v_{n-1}. Clearly the vertex v_{n-1} receive the label 1 or 3 or 2 according as $n \equiv 2 \pmod 4$ or $n \equiv 0, 3 \pmod 4$ or $n \equiv 1 \pmod 4$.

Consider the vertices $x_i : (1 \le i \le n-1)$. Fix the label 1 to the vertices x_1. Next assign the labels $2, 1, 1$ and 3 to the vertices x_2, x_3, x_4 and x_5. Similarly assign the labels $2, 1, 1$ and 3 to the vertices x_6, x_7, x_8 and x_9. Proceeding like this until we reach the vertex x_{n-1}. Clearly the vertex x_{n-1} receive the label 2 or 1 or 3 according as $n \equiv 2 \pmod 4$ or $n \equiv 0, 3 \pmod 4$ or $n \equiv 1 \pmod 4$. The table 5 shows that this vertex labeling is a 4-total difference cordial labeling.

Values of n	$t_{df}(0)$	$t_{df}(1)$	$t_{df}(2)$	$t_{df}(3)$
$n \equiv 0 \pmod 4$	$\frac{9n-8}{4}$	$\frac{9n-8}{4}$	$\frac{9n-8}{4}$	$\frac{9n-8}{4}$
$n \equiv 1 \pmod 4$	$\frac{9n-9}{4}$	$\frac{9n-5}{4}$	$\frac{9n-9}{4}$	$\frac{9n-9}{4}$
$n \equiv 2 \pmod 4$	$\frac{9n-6}{4}$	$\frac{9n-10}{4}$	$\frac{9n-10}{4}$	$\frac{9n-6}{4}$
$n \equiv 3 \pmod 4$	$\frac{9n-11}{4}$	$\frac{9n-7}{4}$	$\frac{9n-7}{4}$	$\frac{9n-7}{4}$

TABLE 5

A 4-total difference cordial labeling of PS_5 is shown in Figure 3

□

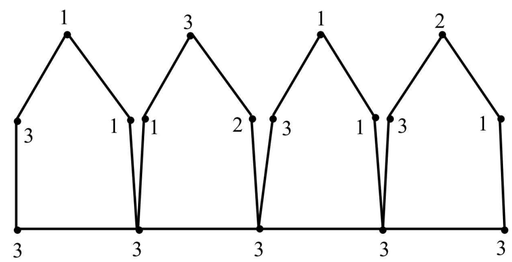

FIGURE 3

Theorem 4.8. The *Double pentagonal snake* $D(PS_n)$ is 4-total difference cordial.

Proof. Let P_n be the path $u_1u_2\dots u_n$. Let $V(D(PS_n)) = V(P_n)\bigcup \{v_i, w_i, x_i : 1 \le i \le n-1\}\bigcup\left\{v_i^{'}, w_i^{'}, x_i^{'} : 1 \le i \le n-1\right\}$ and
$E(D(PS_n)) = \{u_iu_{i+1} : 1 \le i \le n-1\}\bigcup$
$\{u_iv_i, v_iw_i, w_ix_i, x_iu_{i+1} : 1 \le i \le n-1\}\bigcup$
$\left\{u_iv_i^{'}, v_i^{'}w_i^{'}, w_i^{'}x_i^{'}, x_i^{'}u_{i+1}^{'} : 1 \le i \le n-1\right\}$.
Clearly $|V(D(PS_n))| + |E(D(PS_n))| = 16n - 15$.

Fix the label 3 to the vertices $u_1u_2\dots u_n$. Fix the label $3, 3, 1, 1, 1$ and 2 to the vertices $v_1, v_1^{'}, w_1, w_1^{'}, x_1, x_1^{'}$. Next assign the label $3, 3, 3, 2, 1$ and 1 to the vertices $v_2, v_2^{'}, w_2, w_2^{'}, x_2, x_2^{'}$. Proceding like this until we reach the vertex $v_{n-1}, v_{n-1}^{'}, w_{n-1}, w_{n-1}^{'}, x_{n-1}, x_{n-1}^{'}$.
Clearly $t_{df}(0) = t_{df}(2) = t_{df}(3) = \frac{16n-16}{4}$,$t_{df}(1) = \frac{16n-12}{4}$. □

Theorem 4.9. The Torch graph O_n is a 4-total difference cordial for all n.

Proof. Take the vertex set and edge set as in definition 3.4. Clearly $|V(O_n)| + |E(O_n)| = 3n+7$. Fix the labels $3, 2, 2, 3, 3, 1$ and 3 to the vertices $u_1, u_2, u_3, u_4, u_5, u_6$ and u_7. Assign the labels $3, 2, 3$ and 0 to the vertices u_8, u_9, u_{10}, u_{11}. Next assign the labels $3, 2, 3$ and 0 to the vertices $u_{12}, u_{13}, u_{14}, u_{15}$. Continue in this process until we reach the vertex u_n. The vertex u_n receive the label $3, 2, 3$ and 0 according as $n \equiv 0 \pmod 4$,$n \equiv 1 \pmod 4$, $n \equiv 2 \pmod 4$, $n \equiv 3 \pmod 4$.
The table 6 shows that this vertex labeling is a 4-total difference cordial labeling.
A 4-total difference cordial labeling of O_3 is shown in Figure 4 □

Values of n	$t_{df}(0)$	$t_{df}(1)$	$t_{df}(2)$	$t_{df}(3)$
$n \equiv 0 \pmod 4$	$\frac{3n+8}{4}$	$\frac{3n+4}{4}$	$\frac{3n+8}{4}$	$\frac{3n+8}{4}$
$n \equiv 1 \pmod 4$	$\frac{3n+5}{4}$	$\frac{3n+9}{4}$	$\frac{3n+9}{4}$	$\frac{3n+5}{4}$
$n \equiv 2 \pmod 4$	$\frac{3n+6}{4}$	$\frac{3n+6}{4}$	$\frac{3n+10}{4}$	$\frac{3n+6}{4}$
$n \equiv 3 \pmod 4$	$\frac{3n+7}{4}$	$\frac{3n+7}{4}$	$\frac{3n+7}{4}$	$\frac{3n+7}{4}$

TABLE 6

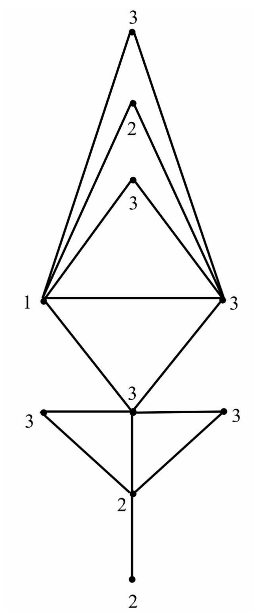

FIGURE 4

5. Discussion

Difference cordial labeling was introduced in [11] and Total product labeling was introduced in [12]. Motivated by these two concepts we have introduced the K-Total difference cordial labeling in [3]. we have been investigated the 4 - total difference cordial labeling behaviour of some graphs like shell butterfly graph, lilly graph, shackle graph, torch graph in this paper.

6. Limitation of Research

It is difficult to investigate the 4 - total difference cordial labeling behaviour of torus grid graph, double step grid graph, mobius grid graph presently

7. Future Research

4 - total difference cordial labeling behaviour of theta graph, plus graph, kayak paddale graph are the possible future directions of research work.

8. Conclusion

In this paper we have studied the 4 - total difference cordial labeling behaviour of shell butterfly graph, lilly graph, shackle graph, torch graph.
4 - total difference cordial labeling behaviour of some other special graphs like shadow graph,spider graph, Jahangir graph are the open problems.

References

1. J.A. Gallian, *A Dynamic survey of graph labeling*, The Electronic Journal of Combinatorics **19** (2017), #Ds6.
2. F. Harary, *Graph theory*, Addision wesley, New Delhi, 1969.
3. R. Ponraj, S. Yesu Doss Philip and R. Kala, *k-total difference cordial graphs*, Journal of Algorithms and Combutation **51** (2019), 121-128.
4. R. Ponraj, S. Yesu Doss Philip and R. Kala, 3-*total difference cordial graphs*, Global Engineering Science and research **6** (2019), 46-51.
5. R. Ponraj, S. Yesu Doss Philip and R. Kala, *Some families of* 4-*total difference cordial graphs*, J. Mathtica and Computational Science **10** (2020), 150-156.
6. R. Ponraj, S. Yesu Doss Philip and R. Kala, *Some results on* 4-*total difference cordial graphs*, International J. Math. Combin. **1** (2020), 105-113.
7. R. Ponraj, S. Yesu Doss Philip and R. Kala, 4-*total difference cordial labeling of union of some graphs*, J.Appl. and Pure Math. **2** (2020), 9-16.
8. R. Ponraj, S. Yesu Doss Philip and R. Kala, 4-*total difference cordial labeling of corona of snake graphs with* K_1, J. Math. Comput. Sci. **4** (2020), 881-890.
9. R. Ponraj, S. Yesu Doss Philip and R. Kala, 4-*total difference cordiality of some graphs*, J. Appl. and Pure Math. **2** (2020), 185-191.
10. R. Ponraj, S. Yesu Doss Philip and R. Kala, 4-*total difference cordial graphs obtained from path and cycle*, International Journal of Future Generation Communication and Networking **14** (2021), 1507-1510.
11. R. Ponraj, S. Sathish narayanan, and R. Kala, *Difference cordial labeling of graphs*, Global J. Math. Sciences : Theory and Practical **3** (2013), 192-201.
12. M. Sundaram, R. Ponraj and S. Somasundaram, *Total product cordial labeling of graphs*, Bull. Pure Appl. Sci. Sect. E Math. Stat. **25** (2006), 199-203.

R. Ponraj did his Ph.D. in Manonmaniam Sundaranar University, Tirunelveli, India. He has guided 7 Ph.D. scholars and published around 135 research papers in reputed journals. He is an authour of five books for undergraduate students. His research interest in Graph Theory. He is currently an Assistant Professor at Sri ParamakalyaniCollege, Alwarkurichi, India.

Department of Mathematics, Sri Paramakalyani College, Alwarkurichi-627412, Tamilnadu, India.
e-mail: ponrajmaths@gmail.com

S. Yesu Doss Philip did his M.Sc degree in Loyola College, Chennai and M.Phil degree at St.Xavierss College, Palayamkottai, Tirunelveli, India. He is currently a research scholar in Department of Mathematics, Manonmaniam Sundaranar University, Tirunelveli.

His research interest is in Graph Theory. He has Published eight papers in international journals.

Research Scholar, Register number: 182240120910010, Department of Mathematics, Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli-627012, Tamilnadu, India.
e-mail: jesuphilip09@gmail.com

R. Kala received her Ph.D at Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli, India. She is working as a professor of Mathematics in Manonmaniam Sundaranar University, Tirunelveli, India. She has guided 11 Ph.D. scholars and published around 140 research papers in reputed journals. Her area of interests include Graph Theory.

Department of Mathematics, Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli-627012, Tamilnadu, India.
e-mail: karthipyi91@yahoo.co.in