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Combined VSI EWMA Chart with Accumulate-Combine Method for Moderate or Small Shifts

Duk-Joon Chang†

Department of statistics, Changwon National University, Changwon, Korea

Abstract

In a multivariate normal production process $N_p(\underline{\mu}, \Sigma)$, a chart combining three EWMA charts with accumulate-combine method for $\underline{\mu}$, variance components of Σ, and off-diagonal elements of Σ, into a EWMA (exponentially weighted moving average) chart is considered, which is called a combined EWMA chart. Through simulation work, the proposed combined EWMA chart's numerical performance and properties are examined. The simulation results show that the proposed combined EWMA chart, which is simultaneously monitoring all the process parameters of multivariate normal production process, works effectively in the perspective of means, variances and correlation coefficients. In addition, the combined EWMA chart is extended to VSI chart.

Keyword: accumulate-combine method, ANSW, ATS, LRT, smoothing constant

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1. Introduction

Control chart is usually used for monitoring the production process to detect quickly the shifts that may cause the quality of products to be deteriorated. Usually the quality of product is determined by some related several quality variables not a single variable. And statistical quality control procedure is one of the most powerful and effective tools deployed to improve the definite level of quality of products or services.

Traditional practice to control chart procedure is taking samples from the process with FSI (fixed sampling interval), in which the sampling interval between two sampling time point t_i and t_{i-1}, $t_i - t_{i-1}$, is fixed for any sampling time point i $(i = 1, 2, \cdots)$.

However, recent years it is more common to apply VSI (variable sampling interval) procedure instead of FSI procedure. In VSI procedure, the sampling time is a random variable and the sampling interval $t_i - t_{i-1}$ depends on the past sample information.

Process engineers generally want to detect any process changes as quick as possible. So, when the control statistics' values are near at the control limits, even though the values are not off the control limits, process engineers will try to take samples rapidly by judging there are any process changes

† Corresponding author: djchang@changwon.ac.kr

happened. The basic idea of VSI chart is that it is reasonable to take long sampling interval to the next sample point if there are no sign of process changes. So the concept of VSI control chart can be considered as very intuitive and reasonable.

Multivariate CUSUM chart monitoring mean vector $\underline{\mu}$ using accumulate-combine (A-C) method was proposed by Crosier[1] and multivariate EWMA chart about $\underline{\mu}$ of multivariate normal process using the A-C method was proposed by Lowry et. al.[2] In VSI procedures the sampling intervals are variable and samples are not taken with the fixed time interval. This irregular pattern may seem to complicate the process operation to engineers who have to control the production process.

Frequent switching is an uneasy part in application of VSI control procedure. Therefore, It needs to consider a quantity measuring the frequency of switches among several sampling intervals in VSI structure.

ANSS/ARL and ATS do not provide any information about how many switches are made, and so some quantities measuring the frequency of switching are necessary for VSI procedures.

Amin and Letsinger[3] proposed general VSI procedures and showed that ANSW (average number of switches) of CUSUM chart or EWMA chart are smaller than Shewhart chart. Amin and Hemasinha[4] and Shamma et. al.[5] studied on switching properties of VSI chart.

Reynolds and Arnold[6] investigated two sampling intervals, d_1 and d_2 $(d_1 < d_2)$, in Shewhart VSI charts in detail and showed that the use of large sampling intervals $d_2 - d_1$ is optimal.

To estimate the efficiency of VSI procedures, it is necessary to know the ANSW, the ANSW made from the beginning of the chart process until the chart signals. The ANSW can be obtained as follows

$$ANSW = (ANSS - 1) \cdot P(\mathrm{sw}).$$

And, the switching probability $P(\mathrm{sw})$ of two sampling intervals d_1, d_2 $(d_1 < d_2)$ in VSI chart is given by

$$P(\mathrm{sw}) = P(d_1) \cdot P(d_2|d_1) + P(d_2) \cdot P(d_1|d_2).$$

EWMA chart detects any process changes by giving large weights, smoothing constant λ $(0 < \lambda \le 1)$, to recent sample information and giving small weights to past sample information. When the smoothing constant $\lambda = 1$ in EWMA chart, EWMA chart is equal to basic Shewhart chart.

When VSI chart is designed, an initial value d_0 has to be determined from between the time 0 and the time point when the first sample is selected. Some charts design to $d_0 = d_1$ in order to select the first sample at the time point the process control started.

2. Control Statistic for Multivariate Normal Process with Accumulate-Combine Method

In this work, we assume that the product process has p quality variables $\underline{X}' = (X_1, X_2, \cdots, X_p)$ and $\underline{X}$ has a multivariate normal distribution, $N_p(\underline{\mu}, \Sigma)$. The distribution of $\underline{X}$ has a set of process parameters $\underline{\theta} = (\underline{\mu}, \Sigma)$ and $\underline{\mu}$ is the mean vector and Σ is the dispersion matrix of $\underline{X}$. We also assume that $\underline{\theta}_0 = (\underline{\mu}_0, \Sigma_0)$ is a known targe value for $\underline{\theta}$ where

$$\underline{\mu}_0 = (\mu_{10}, \mu_{20}, \cdots, \mu_{p0})'$$

and

$$\Sigma_0 = \begin{bmatrix} \sigma_{10}^2 & \rho_{120}\sigma_{10}\sigma_{20} & \cdots & \rho_{1p0}\sigma_{10}\sigma_{p0} \\ & \sigma_{20}^2 & \cdots & \rho_{2p0}\sigma_{20}\sigma_{p0} \\ & & \ddots & \vdots \\ & Sym & & \sigma_{p0}^2 \end{bmatrix},$$

and the known target covariance of quality characteristics X_r and X_s is $\sigma_{rs0} = \rho_{rs0}\,\sigma_{r0}\,\sigma_{s0}$ $(r, s = 1, 2, \cdots, p)$. At the sampling time point i $(i = 1, 2, \cdots)$, we obtain a successive random vectors $\underline{X}_1, \underline{X}_2, \underline{X}_3, \cdots$ where $\underline{X}'_i = (\underline{X}_{i1}', \underline{X}_{i2}', \cdots, \underline{X}_{in}')$ is a sample of observations and $\underline{X}_{ij}' = (X_{ij1}, X_{ij2}, \cdots, X_{ijp})$. Thus, random vector $\underline{X}_i$ is an $np \times 1$ vector.

Chang and Shin[7] showed that multivariate chart with combine-accumulate (C-A) method based on LRT (likelihood ratio test) statistic $TV_i \quad (= tr(A_i \Sigma_0^{-1}) - n\ln|A_i| + n\ln|\Sigma_0| + np\ln n - np)$ for testing $H_0 : \Sigma = \Sigma_0$ vs $H_1 : \Sigma \neq \Sigma_0$ is not effective for detecting dispersion matrix $\Sigma_{p\times p}$.

And they also found that when the correlation coefficient components have been changed, the control statistic TV_i based on C-A approach has larger ANSS even though there are some process changes happened than in-control state, and so that the control statistic TV_i is improper in terms of controlling Σ.

To overcome the weakness, we proposed the combined three accumulate-combined multivariate EWMA chart which combine three EWMA procedure each controlling only one distribution parameter, means, covariances, and correlation coefficients, respectively. And we found that the combined chart overcomes the shortcoming.

Chang and Heo[8] found that multivariate control chart applying to C-A method based on LRT statistic of variance-covariance matrix $\Sigma_{p\times p}$ is effective when there are some changes in variance components of $\Sigma_{p\times p}$, but not so effective when there are some changes in components of correlation coefficients.

And Chang[9] also proposed a MEWMA chart to control the components of correlation coefficients of $\Sigma_{p\times p}$, and showed the efficiency and performances of the proposed multivariate chart through simulation results.

The control statistics of three multivariate EWMA chart which monitor separately the mean vector $\underline{\mu}$, the variance and correlation coefficient components of dispersion matrix Σ are as follows.

2.1 Control Statistic for $\underline{\mu}$

For monitoring mean vector $\underline{\mu}$, the vectors of EWMA's $\underline{Y}_i$ $(i = 1, 2, 3, \cdots)$ are defined as

$$\underline{Y}_i = (I - \Lambda)\underline{Y}_{i-1} + \Lambda\,\overline{\underline{X}}_i \qquad (2.1)$$

where $\underline{Y}_0 = \underline{\mu}_0$, the components of sample mean vector $\overline{\underline{X}}_i$ are p sample means at the sampling time point i, and $\Lambda = diag(\lambda_1, \cdots, \lambda_p)$ is smoothing matrix $(0 < \lambda_j \leq 1 : j = 1, 2, \cdots, p)$.

When Σ is known the vector $\underline{Y}_i$ of variance-covaiance matrix $\Sigma_{\underline{Y}_i}$ is as follows

$$\Sigma_{\underline{Y}_i} = \sum_{k=1}^{i} n \cdot [\Lambda(I-\Lambda)^{i-k}\Sigma(I-\Lambda)^{i-k}\Lambda] \qquad (2.2)$$

The MEWMA chart for monitoring the mean vector $\underline{\mu}$ gives a signal when

$$T_{1,i}^2 = (\underline{Y}_i - \underline{\mu}_0)'\Sigma_{\underline{Y}_i}(\underline{Y}_i - \underline{\mu}_0) > h_1. \qquad (2.3)$$

Since the mathematical distribution of control statistic $T_{1,i}^2$ is unknown, the upper control limit (UCL) $h_1\,(>0)$ can be obtained asymptotically through simulation work with a specified in-control state ANSS/ATS.

2.2 Control Statistic for Variances in Σ

For monitoring variance components in dispersion matrix $\Sigma_{p\times p}$, the vectors of EWMA's $\underline{Y}_i$ $(i=1,2,3,\cdots)$ is defined as

$$\underline{Y}_i = (I-\Lambda)\underline{Y}_{i-1} + \Lambda\underline{Z}_i \qquad (2.4)$$

where $\underline{Y}_0=\underline{0}$, $\underline{Z}_i=(Z_{i1},Z_{i2},\cdots,Z_{ip})$, $Z_{il}=\sum_{j=1}^{n}\{(X_{ijl}-\mu_{0l})/\sigma_{0l}\}^2-n$ $(l=1,2,\cdots,p)$, and smoothing matrix $\Lambda=diag(\lambda_1,\lambda_2,\cdots,\lambda_p)$ $(0<\lambda_j\le 1,\ j=1,2,\cdots,p)$. When the process is in-control and $\underline{Y}_0=\underline{0}$, variance-covariance matrix of $\underline{Y}_i$, $\Sigma_{\underline{Y}_i}$, is given by

$$\Sigma_{\underline{Y}_i} = \sum_{k=1}^{i}\Lambda(I-\Lambda)^{i-k}\Sigma_{\underline{Z}}(I-\Lambda)^{i-k}\Lambda \qquad (2.5)$$

where the variance-covariance matrix of $\underline{Z}$, $\Sigma_{\underline{Z}}=2nR^{(2)}$ and $R^{(2)}$ is a symmetric matrix whose (i,j)th element is the 2^{nd} power of the (i,j)th component of R, the correlation matrix of $\underline{X}=(X_1,X_2,\cdots,X_p)$.

The multivariate EWMA chart $T^2_{2,i}$ applying for accumulate-combine method to control variance components signals whenever

$$T^2_{2,i} = \underline{Y}_i'\Sigma^{-1}_{\underline{Y}_i}\underline{Y}_i > h_2 \qquad (2.6)$$

Since the exact distribution of control statistic $T^2_{2,i}$ is unknown, upper control limit (UCL) $h_2\,(>0)$ can be obtained to achieve a specified in-control ANSS/ATS through simulation work.

2.3 Control Statistic for Correlation Coefficients in Σ

For monitoring correlation coefficients in dispersion matrix $\Sigma_{p\times p}$, the multivariate EWMA vector $\underline{Y}_i$ is given by

$$\underline{Y}_i = \sum_{k=1}^{i}\Lambda(I-\Lambda)^{i-k}\underline{Z}_k + (I-\Lambda)^i\underline{Y}_0 \qquad (2.7)$$

The vectors of EWMA's $\underline{Z}_k=(Z_{k1},Z_{k2},\cdots,Z_{ks})'$ is defined such as

$\underline{Z}_k=(Z_{k12},Z_{k13},\cdots,Z_{k1p},Z_{k23},\cdots,Z_{k2p},\cdots,Z_{k,p-1,p})'$,

where the smoothing matrix

$\Lambda=diag(\lambda_1,\lambda_2,\cdots,\lambda_s)$ $(0<\lambda_j\le 1,\ j=1,2,\cdots,s)$, $s=p(p-1)/2$,

$Z_{kmu}=\left\{\sum_{j=1}^{n}(X_{kjm}-\mu_{m0})(X_{kju}-\mu_{u0})/[n\sigma_{m0}\sigma_{u0}]\right\}-\rho_{mu0}$ $(m\ne u)$.

The multivariate EWMA chart which controls the vector $\underline{\rho}=(\rho_{12},\rho_{13},\cdots,\rho_{1p},\rho_{23},\cdots,\rho_{2p},\cdots,\rho_{p-1,p})$ simultaneously, applying to accumulate-combine method, signals as soon as

$$T^2_{3,i} = \underline{Y}_i'\Sigma_{\underline{Y}_i}^{-1}\underline{Y}_i > h_3, \qquad (2.8)$$

where $\Sigma_{\underline{Y}_i}$ is a $s\times s$ variance-covariance matrix of multivariate EWMA vector $\underline{Y}_i$ in (2.7) and can refer to Chang[9] for details, and UCL h_3 in equation (2.8) is obtained through simulation.

3. Combined Control Chart and Types of Shifts

The combined EWMA chart with A-C method for simultaneously monitoring means, variance components and correlation coefficients signals whenever $T^2_{1,i}>h_1$ or $T^2_{2,i}>h_2$ or $T^2_{3,i}>h_3$. And for the combined VSI scheme in this structure, we assume that the two sampling interval d_1, d_2 $(d_1<d_2)$;

d_1 is used when $T^2_{1,i} \in (g_1, h_1]$ or
$T^2_{2,i} \in (g_2, h_2]$ or $T^2_{3,i} \in (g_3, h_3]$, (3.1)
d_2 is used when $T^2_{1,i} \in (0, g_1]$ and
$T^2_{2,i} \in (0, g_2]$ and $T^2_{3,i} \in (0, g_3]$.

The in-control ANSS, ATS and parameters h_i and $g_i (i = 1, 2, 3)$ can be obtained through simulation.

Since the exact joint probability distribution of control statistic $T^2_{1,i}$, $T^2_{2,i}$ and $T^2_{3,i}$ is not easy, the process parameters h_1, h_2 and h_3 and the performances of the proposed combined charts in (3.1) was evaluated through simulation work either when in-control state or when out-of-control state.

When the probabilities of false alarm in-control state of the proposed combined chart based on three control statistics $(T^2_{1,i}, T^2_{2,i}, T^2_{3,i})$ are α_1, α_2, α_3, respectively, the over all false alarm rate is $1-(1-\alpha_1)(1-\alpha_2)(1-\alpha_3)$.

Since it is very complicate to investigate all the possible out-of-control states at the same time, which is causing by the changes of mean vector $\underline{\mu}$ and $\Sigma_{p \times p}$ of p quality variable, we examined the numerical performances of the proposed control chart based on the most simple and ideal out-of-control state.

We examined the performances of the proposed combined control chart when the changes of μ_1 in $\underline{\mu}=(\mu_1, \mu_2, \cdots, \mu_p)$, σ_1 among $(\sigma_1, \sigma_2, \cdots, \sigma_p)$ and ρ_{12} (and ρ_{21}) among $(\rho_{12}, \rho_{13}, \cdots, \rho_{p-1,p})$ in Σ cause the process to be out-of-control state.

1) Shift in mean $M_{0.0}(\tau)$: μ_1 in $\underline{\mu}$ increases from 0 to τ where non-centrality parameter τ^2 $(= n(\underline{\mu}-\underline{\mu}_0)'\Sigma_0^{-1}(\underline{\mu}-\underline{\mu}_0))$.
2) Shift in variance $V_{1.0}(t_v)$: component σ_1 in Σ increases from 1.0 to t_v.
3) Shift in correlation coefficients $C_{0.30}(t_c)$: components ρ_{12} (and ρ_{21}) in symmetric matrix Σ changes from 0.30 to t_c.

4. Numerical Performances and Concluding Remarks

For evaluation of the combined EWMA chart's performances in (3.1), we assumed that the on-target ANSS and ATS is approximately equal to 200.0 and the sample size of each variable was $n=5$ for $p=6$, and we used $d_0 = 1.0, d_1 = 0.1, d_2 = 1.9$. For convenience we assume that target $\underline{\mu}_0 = \underline{0}$, and all diagonal elements of Σ_0 is 1.0 and off-diagonal elements of Σ_0 is 0.3. The ANSS, ATS, ANSW and the probability of switches $P(\mathrm{sw})$ between two sampling intervals of combined EWMA chart are obtained by simulation with 10,000 runs.

Numerical results in Table 1. through Table 3. show that the VSI chart is more efficient than the corresponding FSI chart. The ANSW is evaluated by $ANSW = (ANSS-1) \cdot Pr(\mathrm{sw})$.

Table 1. through Table 3. show that the combined chart based on accumulate-combine approach is effective for all types of shifts for both small and moderate shifts among all the considered charts for monitoring simultaneously both mean $\underline{\mu}$ and variance-covariance matrix $\Sigma_{p \times p}$. And the VSI chart based on accumulate-combine chart has efficient ARL performances than the corresponding FSI chart.

Hence, we conclude that the combined EWMA chart based on accumulate-combine method is effective when means $\underline{\mu}$, variances and correlation coefficients in Σ of the multivariate normal process are simultaneously monitored. And we also recommend that smaller values of smoothing constant and VSI procedure are more efficient in the perspective of ANSS, ATS and ANSW.

Chang[10] studied an optimal value of design parameter for both multivariate EWMA and CUSUM charts to monitor the dispersion matrix Σ. In practice, when we apply EWMA chart, the selection of optimal smoothing constant λ depends on the amount of process shift which needs to be detect quickly. Therefore, through numerical analysis results with various p we recommend a chart with small λ when the amount of shift is not that big.

Table 1. Numerical performances of multivariate EWMA chart based accumulate-combine method ($p=6$)

shifts	EWMA($\lambda=0.1$)				EWMA($\lambda=0.2$)				EWMA($\lambda=0.3$)			
	ANSS	ATS	ANSW	P(sw)	ANSS	ATS	ANSW	P(sw)	ANSS	ATS	ANSW	P(sw)
in-control	200.0	200.0	46.8	0.24	200.0	200.3	61.4	0.31	200.2	199.9	71.6	0.36
$M_{0.0}(0.5)$	187.5	184.3	43.9	0.24	191.8	190.4	58.8	0.31	194.3	192.9	69.5	0.36
$M_{0.0}(1.0)$	151.6	141.6	35.4	0.24	169.2	163.5	51.8	0.31	179.2	174.9	64.0	0.36
$M_{0.0}(1.5)$	108.1	93.0	24.9	0.23	137.5	126.9	41.7	0.31	155.3	147.0	55.2	0.36
$M_{0.0}(2.0)$	73.5	57.4	16.6	0.23	105.6	91.6	31.6	0.30	126.6	114.7	44.5	0.35
$M_{0.0}(2.5)$	50.0	35.0	10.9	0.22	77.4	62.2	22.6	0.30	98.4	84.6	34.1	0.35
$M_{0.0}(3.0)$	35.4	22.6	7.6	0.22	55.0	40.5	15.6	0.29	75.1	60.5	25.4	0.34
$M_{0.0}(3.5)$	25.9	15.5	5.5	0.22	39.9	26.9	10.9	0.28	55.8	41.7	18.3	0.33
$M_{0.0}(4.0)$	19.8	11.2	4.2	0.23	29.4	18.2	7.8	0.27	41.6	28.8	13.2	0.32
$M_{0.0}(4.5)$	15.7	8.6	3.5	0.24	22.1	12.8	5.7	0.27	31.3	20.0	9.5	0.32
$M_{0.0}(5.0)$	12.9	7.0	2.9	0.25	17.3	9.4	4.4	0.27	23.9	14.2	7.0	0.31
$V_{1.0}(1.1)$	74.6	60.7	17.4	0.24	94.0	80.6	28.2	0.30	106.5	93.6	37.3	0.35
$V_{1.0}(1.2)$	23.8	15.3	5.5	0.24	33.5	22.8	9.4	0.29	41.6	30.0	13.6	0.33
$V_{1.0}(1.3)$	11.2	6.6	2.8	0.28	14.7	8.6	4.0	0.29	18.3	11.1	5.5	0.32
$V_{1.0}(1.4)$	6.8	4.1	1.9	0.33	8.4	4.6	2.3	0.32	10.0	5.5	3.0	0.33
$V_{1.0}(1.5)$	4.7	2.9	1.5	0.39	5.6	3.1	1.7	0.36	6.4	3.5	1.9	0.36
$V_{1.0}(1.6)$	3.6	2.3	1.2	0.45	4.1	2.4	1.3	0.42	4.6	2.6	1.5	0.40
$V_{1.0}(1.7)$	2.9	1.9	1.0	0.52	3.3	2.0	1.1	0.48	3.6	2.1	1.2	0.45
$V_{1.0}(1.8)$	2.4	1.7	0.8	0.57	2.7	1.7	0.9	0.53	2.9	1.8	1.0	0.50
$V_{1.0}(1.9)$	2.1	1.5	0.7	0.63	2.3	1.6	0.8	0.59	2.5	1.6	0.8	0.56
$V_{1.0}(2.0)$	1.9	1.4	0.6	0.67	2.0	1.4	0.7	0.64	2.2	1.4	0.7	0.61
$V_{1.0}(2.1)$	1.7	1.3	0.5	0.71	1.8	1.3	0.6	0.67	1.9	1.4	0.6	0.65
$C_{0.3}(0.35)$	192.8	190.9	45.1	0.24	198.0	198.1	60.7	0.31	199.2	199.3	71.3	0.36
$C_{0.3}(0.40)$	163.4	155.1	38.1	0.24	182.4	179.7	55.8	0.31	189.5	188.5	67.6	0.36
$C_{0.3}(0.45)$	118.9	104.5	27.4	0.23	154.8	147.8	47.1	0.31	170.4	166.9	60.6	0.36
$C_{0.3}(0.50)$	78.9	62.8	17.9	0.23	121.5	110.6	36.7	0.30	145.4	138.9	51.5	0.36
$C_{0.3}(0.55)$	52.1	37.1	11.4	0.22	90.4	77.5	26.9	0.30	118.5	109.4	41.6	0.35
$C_{0.3}(0.60)$	35.7	23.3	7.6	0.22	65.8	52.2	19.1	0.30	91.7	81.2	31.9	0.35
$C_{0.3}(0.65)$	25.7	15.9	5.5	0.22	47.1	34.4	13.2	0.29	70.2	58.9	24.0	0.35
$C_{0.3}(0.70)$	19.5	11.7	4.2	0.23	34.3	23.2	9.3	0.28	53.4	42.2	17.8	0.34
$C_{0.3}(0.75)$	15.5	9.2	3.5	0.24	25.7	16.2	6.8	0.27	40.7	30.1	13.2	0.33
$C_{0.3}(0.80)$	12.8	7.7	3.0	0.25	20.2	12.0	5.2	0.27	31.5	21.8	9.9	0.32

Table 2. Numerical performances of EWMA chart based on accumulate-combine method for small shifts of the variance components in Σ $(p=6)$

shifts	EWMA($\lambda=0.1$)				EWMA($\lambda=0.2$)				EWMA($\lambda=0.3$)			
	ANSS	ATS	ANSW	P(sw)	ANSS	ATS	ANSW	P(sw)	ANSS	ATS	ANSW	P(sw)
in-control	200.0	200.0	46.8	0.24	200.0	200.3	61.4	0.31	200.2	199.9	71.6	0.36
$V_{1.0}(1.05)$	137.5	127.5	32.2	0.24	150.1	141.7	45.9	0.31	156.0	147.9	55.5	0.36
$V_{1.0}(1.10)$	74.6	60.7	17.4	0.24	94.0	80.6	28.2	0.30	106.5	93.6	37.3	0.35
$V_{1.0}(1.15)$	39.7	27.9	9.0	0.23	56.1	42.9	16.3	0.30	66.8	53.4	22.7	0.35
$V_{1.0}(1.20)$	23.8	15.3	5.5	0.24	33.5	22.8	9.4	0.29	41.6	30.0	13.6	0.33
$V_{1.0}(1.25)$	15.6	9.5	3.7	0.25	21.5	13.4	5.9	0.29	27.0	17.7	8.5	0.33
$V_{1.0}(1.30)$	11.2	6.6	2.8	0.28	14.7	8.6	4.0	0.29	18.3	11.1	5.5	0.32
$V_{1.0}(1.35)$	8.5	5.1	2.3	0.30	10.7	6.0	3.0	0.31	13.2	7.5	3.9	0.32
$V_{1.0}(1.40)$	6.8	4.1	1.9	0.33	8.4	4.6	2.3	0.32	10.0	5.5	3.0	0.33
$V_{1.0}(1.45)$	5.6	3.4	1.7	0.36	6.8	3.8	2.0	0.34	7.9	4.3	2.4	0.34
$V_{1.0}(1.50)$	4.7	2.9	1.5	0.39	5.6	3.1	1.7	0.36	6.4	3.5	1.9	0.36
$V_{1.0}(1.55)$	4.1	2.6	1.3	0.43	4.7	2.7	1.5	0.39	5.4	2.9	1.7	0.38
$V_{1.0}(1.60)$	3.6	2.3	1.2	0.45	4.1	2.4	1.3	0.42	4.6	2.6	1.5	0.40
$V_{1.0}(1.65)$	3.2	2.1	1.1	0.49	3.7	2.2	1.2	0.45	4.0	2.3	1.3	0.43
$V_{1.0}(1.70)$	2.9	1.9	1.0	0.52	3.3	2.0	1.1	0.48	3.6	2.1	1.2	0.45
$V_{1.0}(1.75)$	2.6	1.8	0.9	0.54	2.9	1.9	1.0	0.50	3.2	1.9	1.1	0.48
$V_{1.0}(1.80)$	2.4	1.7	0.8	0.57	2.7	1.7	0.9	0.53	2.9	1.8	1.0	0.50
$V_{1.0}(1.85)$	2.2	1.6	0.7	0.61	2.5	1.6	0.8	0.56	2.7	1.7	0.9	0.53
$V_{1.0}(1.90)$	2.1	1.5	0.7	0.63	2.3	1.6	0.8	0.59	2.5	1.6	0.8	0.56
$V_{1.0}(1.95)$	2.0	1.5	0.6	0.66	2.1	1.5	0.7	0.61	2.3	1.5	0.8	0.58
$V_{1.0}(2.00)$	1.9	1.4	0.6	0.67	2.0	1.4	0.7	0.64	2.2	1.4	0.7	0.61
$V_{1.0}(2.05)$	1.8	1.4	0.5	0.69	1.9	1.4	0.6	0.66	2.0	1.4	0.6	0.63
$V_{1.0}(2.10)$	1.7	1.3	0.5	0.71	1.8	1.3	0.6	0.67	1.9	1.4	0.6	0.65

Table 3. Numerical performances of EWMA chart based on accumulate-combine method for small shifts of the correlation coefficient components in Σ $(p=6)$

shifts	EWMA($\lambda=0.1$)				EWMA($\lambda=0.2$)				EWMA($\lambda=0.3$)			
	ANSS	ATS	ANSW	P(sw)	ANSS	ATS	ANSW	P(sw)	ANSS	ATS	ANSW	P(sw)
$C_{0.3}(-0.30)$	8.6	4.9	2.2	0.29	12.2	6.1	2.9	0.26	18.0	9.7	4.9	0.29
$C_{0.3}(-0.25)$	10.0	5.6	2.4	0.26	14.7	7.6	3.5	0.25	22.5	12.8	6.3	0.29
$C_{0.3}(-0.20)$	11.8	6.6	2.7	0.25	18.4	9.8	4.4	0.25	28.5	17.4	8.3	0.30
$C_{0.3}(-0.15)$	14.4	8.0	3.1	0.23	23.8	13.4	5.8	0.26	36.6	24.1	11.2	0.32
$C_{0.3}(-0.10)$	18.1	10.1	3.8	0.22	31.8	19.1	8.1	0.26	48.2	34.3	15.5	0.33
$C_{0.3}(-0.05)$	23.9	13.5	4.9	0.21	43.1	28.5	11.6	0.28	63.3	48.4	21.0	0.34
$C_{0.3}(0.0)$	33.0	19.9	6.8	0.21	59.6	43.4	16.8	0.29	82.8	67.7	28.3	0.35
$C_{0.3}(0.05)$	48.3	32.0	10.3	0.22	81.7	65.1	23.9	0.30	107.1	92.8	37.3	0.35
$C_{0.3}(0.10)$	73.2	54.6	16.2	0.23	112.3	96.8	33.6	0.30	133.8	121.5	47.2	0.36
$C_{0.3}(0.15)$	112.1	94.1	25.7	0.23	144.2	132.4	43.9	0.31	158.4	149.5	56.4	0.36
$C_{0.3}(0.20)$	153.6	141.8	35.9	0.24	171.6	164.8	52.6	0.31	179.3	174.0	64.1	0.36
$C_{0.3}(0.25)$	186.9	182.6	43.8	0.24	192.0	189.7	59.0	0.31	194.0	191.9	69.4	0.36
in-control	200.0	200.0	46.8	0.24	200.0	200.3	61.4	0.31	200.2	199.9	71.6	0.36
$C_{0.3}(0.35)$	192.8	190.9	45.1	0.24	198.0	198.1	60.7	0.31	199.2	199.3	71.3	0.36
$C_{0.3}(0.40)$	163.4	155.1	38.1	0.24	182.4	179.7	55.8	0.31	189.5	188.5	67.6	0.36
$C_{0.3}(0.45)$	118.9	104.5	27.4	0.23	154.8	147.8	47.1	0.31	170.4	166.9	60.6	0.36
$C_{0.3}(0.50)$	78.9	62.8	17.9	0.23	121.5	110.6	36.7	0.30	145.4	138.9	51.5	0.36
$C_{0.3}(0.55)$	52.1	37.1	11.4	0.22	90.4	77.5	26.9	0.30	118.5	109.4	41.6	0.35
$C_{0.3}(0.60)$	35.7	23.3	7.6	0.22	65.8	52.2	19.1	0.30	91.7	81.2	31.9	0.35
$C_{0.3}(0.65)$	25.7	15.9	5.5	0.22	47.1	34.4	13.2	0.29	70.2	58.9	24.0	0.35
$C_{0.3}(0.70)$	19.5	11.7	4.2	0.23	34.3	23.2	9.3	0.28	53.4	42.2	17.8	0.34
$C_{0.3}(0.75)$	15.5	9.2	3.5	0.24	25.7	16.2	6.8	0.27	40.7	30.1	13.2	0.33
$C_{0.3}(0.80)$	12.8	7.7	3.0	0.25	20.2	12.0	5.2	0.27	31.5	21.8	9.9	0.32

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