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CHARACTERIZATION OF HOMOMORPHISM ON IMPLICATIVE ALGEBRAS

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Abstract. The concept of hom-set in an implicative algebra as an implicative algebra is introduced. The idea of sub implicative algebra in hom-set, 1-commutative algebra of hom-set are investigated. Different characterization theorems are proved.

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1. Introduction

Abbott[1] introduced pre-implicative algebras and implication algebra based on ortho semi lattices which generalize the concept of implication algebra. The concepts of $B-$almost distributive fuzzy lattice interms of its principal ideal fuzzy lattice initiated by Berhanu ,and et al in [2].
The ideas of properties(of homomorphism in implication algebra with different homomorphism theorems must be included), ideals and filters on implication algebra with different properties theorems introduced by T.Gerima, and et al in [3],and the homomorphism in BF- algebra with different properties discussed by Joemar in [4].
Roh, and et al in [5] introduced the concept of $\otimes$-closed set, and a $\otimes$-homomorphism in lattice implication algebras and some properties, and the concepts of homomorphism in lattice implication algebra with different characterization was introduced by Xu in[6].
In this paper the concept of characterazation of homomorphism on implicative algebras are introduced,and different characterizations are investigated.

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2. preliminaries

Definition 2.1. [4] An algebra $(A, \rightarrow,', 0, 1)$ of type $(2, 1, 0, 0)$ is called implicative algebra if it satisfies the following conditions ,for all $x, y, z \in A$:

(1) $x \rightarrow (y \rightarrow z) = y \rightarrow (x \rightarrow z)(Exchangerule)$.
(2) $1 \rightarrow x = x$.
(3) $x \rightarrow 1 = 1$.
(4) $x \rightarrow y = y^{'} \rightarrow x^{'}$.
(5) $(x \rightarrow y) \rightarrow y = (y \rightarrow x) \rightarrow x$.
(6) $0^{'} = 1$

Lemma 2.2. [4] *Let A be an implicative algebra. Then the following holds for all* $x \in A$:

(1) $x \rightarrow x = 1$.
(2) $1^{'} = 0$.

In an implicative algebra $(A, \rightarrow,', 0, 1)$, the relation $\leq$ on A is defined as $x \leq y$ if and only if$x \rightarrow y = 1$.

Definition 2.3. Let $(A, \vee, \wedge,', \rightarrow, \otimes, 0, 1)$ and $(A. * . \vee .\wedge, .'', \rightarrow . \otimes .0.1)$. be implicative algebras and let $f : A \rightarrow A^*$ be a mapping . Then f is called the implication homomorphism if for all $x, y \in A$, We have $f(x \rightarrow y) = f(x) \rightarrow f(y)$and f is called the lattice implication homomorphism if for all $x, y \in A$,$f(x \vee y) = f(x) \vee f(y)$;
$f(x \wedge y) = f(x) \wedge f(y)$ and $f(x^{'}) = f(x)^{'}$.
In an implicative algebra $(A, \rightarrow,', 0, 1)$, we define $\vee$ and $\wedge$ on A by:

(1) $x \vee y = (x \rightarrow y) \rightarrow y = (y \rightarrow x) \rightarrow x$.
(2) $x \wedge y = y \rightarrow (y \rightarrow x) = x \rightarrow (x \rightarrow y)$.

3. Main results

3.1. Hom set as implicative algebras.

Definition 3.1. Let $Hom(A, B)$ be the set of all homomorphisms of implicative algebras A and B. Then $(Hom(A, B), \rightarrow,', 1)$ is an implicative algebra if it satisfies the following conditions fo all $f, g, h \in Hom(A, B)$:

(1) $f \rightarrow (g \rightarrow h) = g \rightarrow (f \rightarrow h)$.
(2) $1 \rightarrow f = f$.
(3) $f \rightarrow 1 = 1$.
(4) $f \rightarrow g = g^{'} \rightarrow f^{'}$.
(5) $(f \rightarrow g) \rightarrow g = (g \rightarrow f) \rightarrow f$.
(6) $0^{'} = 1$, where $0(a) = 0$ and $1(a) = a$.

Example 3.2. Let $A = \{0, a, b, c, 1\}$ be a set. Define the partial order relation on A $0 < a < b < c < 1$ and define $"'"$and $" \rightarrow "$ by the table below:

$\rightarrow$	0	a	b	c	1
0	1	1	1	1	1
a	c	1	1	1	1
b	b	c	1	1	1
c	a	b	c	1	1
1	0	a	b	c	1

x	x'
0	1
a	c
b	b
c	a
1	0

Define a map $f : A \rightarrow A$ by $f(a) = 1, \forall\ a \in A$ and a map $g : A \rightarrow A$ by $g(a) = a, \forall\ a \in A$.
Let $a_1, a_2 \in A$. Then $f(a_1 \rightarrow a_2) = 1$ and $f(a_1) \rightarrow f(a_2) = 1 \rightarrow 1 = 1$.
Hence $f(a_1 \rightarrow a_2) = f(a_1) \rightarrow f(a_2)$.
$g(a_1 \rightarrow a_2) = a_1 \rightarrow a_2$ by definition above and $g(a_1) \rightarrow g(a_2) = a_1 \rightarrow a_2$.
Hence $g(a_1 \rightarrow a_2) = g(a_1) \rightarrow g(a_2)$. Thus $f, g \in Hom(A, B)$.
$(f \rightarrow g)(a_1 \rightarrow a_2) = f(a_1 \rightarrow a_2) \rightarrow g(a_1 \rightarrow a_2)$
$= 1 \rightarrow (a_1 \rightarrow a_2) = a_1 \rightarrow a_2$, and $(f \rightarrow g)(a_1) \rightarrow (f \rightarrow g)(a_2)$
$= (f(a_1) \rightarrow g(a_1)) \rightarrow (f \rightarrow g)(a_2)$
$= (1 \Longrightarrow a_1) \rightarrow (1 \rightarrow a_2)$
$= a_1 \rightarrow a_2$.
Hence $(f \rightarrow g)(a_1 \Longrightarrow a_2) = (f \rightarrow g)(a_1) \rightarrow (f \rightarrow g)(a_2), forall\ \ a_1, a_2 \in A$.
Thus $f \rightarrow g \in Hom(A, B)$.
Therefore $Hom(A, B)$ is closed under " $\rightarrow$ ".
To show $(Hom(A, B), \rightarrow, ', 1)$ is implicative algebra.
Let $f, g, h \in Hom(A, B)$ defined by $f(a) = 1, g(a) = a$ and $h(a) = 0$,for all $a \in A$.

(1) $f(a) \rightarrow (g(b) \rightarrow h(c))$
$= f(a) \rightarrow (g(b) \rightarrow 0)$
$= f(a) \rightarrow b = 1 \rightarrow b = b$.
Hence $f(a) \rightarrow (g(b) \rightarrow h(c)) = b$.
$g(b) \rightarrow (f(a) \rightarrow h(c))$
$= g(b) \rightarrow (1 \rightarrow 0)$
$= b \rightarrow 0 = b$.
Hence $g(b) \rightarrow (f(a) \rightarrow h(c)) = b$.
Therefore $f \rightarrow (g \rightarrow h) = g \rightarrow (f \rightarrow h)$, for all $a, b, c \in A$.
(2) $1 \rightarrow g(b) = 1 \rightarrow b = b$, for all $b \in A$.
(3) $g(b) \rightarrow 1 = b \rightarrow 1$,for all $b \in A$.
(4) $f(a) \rightarrow g(b) = 1 \rightarrow b = b$, for all $a, b \in A$.
$g'(b) \rightarrow f^{'}(a) = g''(b) \vee f'(a)$
$= f'(a) \vee g''(b)$
$= f'(a) \vee g(b)$
$= f(a) \rightarrow g(b)$.
(5) $(f(a) \rightarrow g(b)) \rightarrow g(b)$
$= (1 \rightarrow b) \rightarrow b = b \rightarrow b = 1$.
Hence $(f(a) \rightarrow g(b)) \rightarrow g(b) = 1$.
$(g(b) \rightarrow f(a)) \rightarrow f(a)$

$= (b \to 1) \to 1$
$= 1 \to 1 = 1.$
Hence $(g(b) \to f(a)) \to f(a) = 1$,for all $a, b \in A$.
Thus $(f \to g) \to g = (g \to f) \to f$.

(6) $f'(a) = 1' = 0 = h(a)$ and $f''(a) = 1'' = 0' = h'(a)$.
Imply that $f(a) = 1 = 0' = h'(a)$, for all $a \in A$.
Hence $(Hom(A, B), \to,', 0, 1)$ is an implicative algebra.

Lemma 3.3. *Let*$(Hom(A, B), \to,', 1)$ *be an implicative algebra of type* $(2, 1, 0)$. *Then*

(1) $f \to f = 1$, *for all* $f \in Hom(A, B)$.
(2) $1' = 0$.

Lemma 3.4. *Let* $(Hom(A, B), \to,', 1)$ *be an implicative algebra. Then* $g^{'} = g \to 0$, *for all* $g, 0 \in Hom(A, B)$.

Proof. Let $g \in Hom(A, B)$. Then $g^{'} = 1 \to g^{'}$
$= (g^{'})^{'} \to 1^{'}$
$= g \to 0$, since $1^{'} = 0" = 0$.
Hence $g^{'} = g \to 0$, for all $g \in Hom(A, B)$. □

Theorem 3.5. *If A is an implicative algebra and B is an associative implicative algebra, then* $(Hom(A, B), \to, 1, 1)$ *is an implicative algebra.*

Proof. Let A be an implicative algebra and let B be an associative implicative algebra. Then we have to show $(Hom(A, B), \to,', 1)$ is an implicative algebra.
Now let $f, g \in Hom(A, B)$ and $a, b \in A$. Then $(f \to g((a \to b)$
$= f(a \to b) \to g(a \to b)$
$= (f(a) \to f(b)) \to (g(a) \to g(b))$, since $f, g \in Hom(A, B)$, for all $a, b \in A$.
$=f(a) \to (f(b) \to (g(a) \to g(b)))$
$= f(a) \to (g(a) \to (f(a) \to g(b)))$, by exchange rule.
$=(f(a) \to g(a)) \to (f(b) \to g(b))$, since B is associative implicative algebra
$=(f \to g)(a) \to (f \to g)(b)$.
Hence $f \to g \in Hom(A, B)$, for all $f, g \in Hom(A, B)$.
Since B is an associative implicative algebra, it is enough to show definition 3.1 holds. Then we have to show for all $f, g, h \in Hom(A, B)$, $(Hom(A, B), \to,', 1)$ is an implicative algebra.

(1) Let $f, g, h \in Hom(A, B)$ and $a \in A$. Then $(f \to (g \to h))(a)$
$= f(a) \to (g(a) \to h(a))$
$= g(a) \to (f(a) \to h(a))$, by exchange rule.
$=(g \to (f \to h))(a)$.
Hence $f \to (g \to h) = g \to (f \to h)$,for all $f, g, h \in Hom(A, B)$.
(2) $1 \to f = f$, for all $f \in Hom(A, B)$.
(3) $f \to 1 = g(a) = 1$, for all $f, g(a) = 1 \in Hom(A, B)$, and $a \in A$.

(4) $f \to g = f^{'} \vee g$
$= g \vee f^{'} = g^{''} \vee f^{'}$
$= g^{'} \to f^{'}$

(5) $((f \to g) \to g)(a)$
$= (f \to g)(a) \to g(a)$
$= (f(a) \to g(a)) \to g(a)$
$= f(a) \to (g(a) \to g(a))$, since B is associative implicative algebra.
$=f(a) \to 1 = 1$, since $g(a) \to g(a) = 1$.
Hence $)(f \to g(\to g)(a) = 1$, for all,$a \in A$.
$((g \to f) \to f)(a) = (g \to f)(a) \to f(a)$
$= (g(a) \to f(a)) \to f(a)$
$= g(a) \to (f(a) \to f(a))$, since B is associative implicative algebra.
$=g(a) \to 1 = 1$, since $f(a) \to f(a) = 1$.
Hence $((g \to f) \to f)(a) = 1$, for all,$a \in A$.
Therefore, $(f \to g) \to g = (g \to f) \to f$, for all,$f, g \in Hom(A, B)$.

(6) Let $f, g \in Hom(A, B)$, $f(a) = 0$, and $g(a) = 1$, for all $a \in A$. Then $f^{'}(a) = 0^{'} = 1 - g(a)$ and $g^{'}(a) = 1^{'} = 0 = f(a)$, for all $a \in A$.
Hence $0^{'} = 1$.
Therefore $(Hom(A, B), \to,', 1)$ is an implicative algebra.

□

Definition 3.6. Let S be a non-empty subset of an implicative algebra A. Then S is called a subimplicative algebra of A if $a \to b \in S$, for all, $a, b \in S$.

Definition 3.7. Let M and θ be subsets of an implicative algebra A,and $Hom(A, B)$ respectively, where B is an associative implicative algebra.
Define $M^{\perp\perp} = \{f \in Hom(A, B)|f(a) = 1 \ \ \forall a \in M\}$and
$\theta^{\perp\perp} = \{a \in A|f(a) = 1, \ \ \forall \ \ f \in Hom(A, B)\}$. We say $M^{\perp\perp}$ and $\theta^{\perp\perp}$ be dual orthogonal subsets of M and θ respectively.

Theorem 3.8. *Let A be an implicative algebra,B be an associative implicative algebra,$M \subseteq A$ and $\theta \subseteq Hom(A, B)$. Then $M^{\perp\perp}$ and $\theta^{\perp\perp}$ are sub implicative algebras of $Hom(A, B)$ and A respectively.*

Proof. Let $f \to g$,$h \to k \in M^{\perp\perp}$ for all $f, g, h \in Hom(A, B)$.Then $(f \to g)(a) = 1$ for all $a \in M$ and
$(h \to k)(a) = 1$ for all $a \in M$ by theorem 3.5 $(Hom(A, B), \to,', 1)$ is an implicative algebra.

(1) $((f \to h) \to (g \to k))(a)$
$= (f \to h)(a) \to (g \to k)(a)$
$= (f(a) \to h(a)) \to (g(a) \to k(a))$
$= f(a) \to (h(a) \to (g(a) \to k(a))$ by associative property.
$=f(a) \to (g(a) \to (h(a) \to k(a)))$ by exchange rule.
$=(f(a) \to g(a)) \to (h(a) \to k(a))$ by associative property.
$=(f \to g)(a) \to (h \to k)(a)$

$= 1 \to 1 = 1$, since $(f \to g)(a) = 1$ and $(h \to k)(a) = 1$.
Hence $((f \to h) \to (g \to k))(a) = 1$,for all $a \in M$. Which imply that $(f \to h) \to (g \to k) \in M^{\perp\perp}$.
Therefore $M^{\perp\perp}$ is a sub implicative algebra of $Hom(A, B)$.

(2) Let $a, b, c, d \in A$ with $a \to b, c \to d \in \theta^{\perp\perp}$. Then $f(a \to b) = 1$ and $f(c \to d) = 1$,for all $f \in Hom(A, B)$.
Since B is associative implicative algebra.
$f((a \to c) \to (b \to d))$
$= f(a \to c) \to f(b \to d)$,forall $f \in Hom(A, B)$.
$=(f(a) \to f(c)) \to (f(b) \to f(d))$
$= f(a) \to (f(c) \to (f(b) \to f(d))$, since B is associative property of B.
$=f(a) \to (f(b) \to (f(c) \to f(d)))$ by exchange rule.
$=(f(a) \to f(b)) \to (f(c) \to f(d))$,associative property in B.
$=f(a \to b) \to f(c \to d)$, since $f \in hom(A, B)$
$=1 \to 1 = 1$,because $f(a \to b) = 1$ and $f(c \to d) = 1$.
Hence $f((a \to c) \to (b \to d)) = 1$; Consequently , $(a \to c) \to (b \to d) \in \theta^{\perp\perp}$, for all $f \in Hom(A, B)$.
Therefore $\theta^{\perp\perp}$ is a sub implicative algebra of A.

□

3.2. 1−Commutative implicative algebra.

Definition 3.9. An implicative algebra $(A, \to, ', 1)$ is said to be 1− commutative if $a \to (1 \to b) = b \to (1 \to a)$,forall $a, b \in A$.

Theorem 3.10. *If A is an implicative algebra, and B is a 1−commutative associative implicative algebra, then $(Hom(A, B), \to, ', 1)$ is a 1−commutative implicative algebra.*

Proof. Let $f, g \in Hom(A, B)$ and $a, b \in A$. Then $(f \to g)(a \to b) = f(a \to b) \to g(a \to b)$
$= (f(a) \to f(b)) \to (g(a) \to g(b))$, since $f, g \in Hom(A, B)$
$= f(a) \to (f(b) \to (g(a) \to g(b)))$, since B is associative implicative algebra.
$=f(a) \to (g(a) \to (f(b) \to g(b)))$ by exchange rule.
$=(f(a) \to g(a)) \to (f(b) \to g(b))$
$= (f \to g)(a) \to (f \to g)(b)$.
Hence $(f \to g)(a \to b) = (f \to g)(a) \to (f \to g)(b)$.
Therefore $f \to g \in Hom(A, B)$,for all $f, g \in Hom(A, B)$. We have to show $(Hom(A, B), \to, ', 1)$ is a 1−commutative implicative algebra.
Let $f, g, h \in Hom(A, B)$, for all $a \in A$. Then

(1) $f \to (g \to h))(a)$
$= f(a) \to (g(a) \to h(a))$
$= (g(a) \to h(a)) \to (1 \to f(a))$, 1− commutative property.
$=g(a) \to (h(a) \to (1 \to f(a))$, associative property in B.
$=g(a) \to ((1 \to f(a)) \to (1 \to h(a)))$
$= g(a) \to (f(a) \to h(a))$, since $1 \to f(a) = f(a)$ and $1 \to h(a) = h(a)$.

$=(g \to (f \to h)(a)$.

We get $(f \to (g \to h))(a) = (g \to (f \to h))(a)$, for all $a \in A$.

Hence $(f \to (g \to h) = g \to (f \to h)$.

(2) For $f \in Hom(A, B)$, we have $1 \to f = f$ hold by definition 3.1.

(3) Let $f \in Hom(A.B)$. Then $f \to 1 = 1$ by definition 3.1.

(4) Let $f, g \in Hom(A, B)$ and $f \to g \in Hom(A, B)$. Then $f \to g = f^{'} \vee g$
$= f^{'} \vee g$,by involution property
$=g^{''} \vee f^{'}$, since $g = g^{''}$
$=g^{'} \to f^{'}$.

(5) Let $f, g \in Hom(A, B)$, for all $a \in A$.
$((f \to g) \to g)(a)$
$= (f \to g)(a) \to g(a)$
$= g(a) \to (1 \to (f \to g)(a))$, $1-$ commutative property.
$=g(a) \to (1 \to (f(a) \to g(a)))$
$=g(a) \to (g(a) \to (1 \to f(a))$,$1-$ commutative property
$=(g(a) \to g(a)) \to (1 \to f(a))$, associative property of B.
$=1 \to f(a)$, since $g(a) \to g(a) = 1$ and $1 \to f(a) = f(a)$
$=f(a) \to (1 \to 1)$, $1-$ commutative
$=f(a) \to 1 = 1$.
Hence $((f \to g) \to g)(a) = 1$.
Again $((g \to f) \to f)(a)$
$= (g \to f)(a) \to f(a)$
$= f(a) \to (1 \to (g \to f)(a))$,$1-$ commutative property.
$=g(a) \to (1 \to (f(a) \to g(a)))$
$=g(a) \to (f(a) \to g(a))$, since $1 \to (f(a) \to g(a)) = f(a) \to g(a)$.
$=g(a) \to (g(a) \to (1 \to f(a)))$,$1-$ commutative property.
$=(g(a) \to g(a)) \to (1 \to f(a))$, associative property.
$=1 \to f(a) = f(a) \to (1 \to 1)$,since $g(a) \to g(a) = 1$, and $1 \to f(a) = f(a)$
$=f(a) \to 1 = 1$.
Hence $((f \to g) \to g)(a) = 1$, for all $a \in A$.„,(1)
Again $((g \to f) \to f)(a)$
$= (g \to f)(a) \to f(a)$
$= f(a) \to (1 \to (g \to f)(a))$,$1-$commutative property.
$=f(a) \to (1 \to (g(a) \to f(a)))$
$= f(a) \to (g(a) \to f(a))$
$= f(a) \to (f(a) \to (1 \to g(a)))$,$1-$ commutative property
$=f(a) \to (f(a) \to g(a))$
$= (f(a) \to f(a)) \to g(a)$, associative property
$=1 \to g(a)$
$= g(a) \to (1 \to 1)$
$= g(a) \to 1 = 1$.

Imply that $((g \to f) \to f)(a) = 1 ...(2)$
Thus by 1 and 2 $(f \to g) \to g = (g \to f) \to f$,for all $f, g \in Hom(A, B)$.

(6) Let $f, g \in Hom(A, B)$ such that $f(a) = 0$, and $g(a) = 1$,for all $a \in A$.
Then $f(a)^{'} = 0^{'} = 1 = g(a)$ and $g(a)^{'} = 1^{'} = 0 = f(a)$.
Hence $0^{'} = f(a)^{'} = g(a) = 1$.
Therefore , $(H0m(A, B), \to, ', 1)$ is a $1-$commutative implicative algebra.

□

Theorem 3.11. *Let A be am implication algebra and B be a $1-$ commutative associative implicative algebra with $M \subseteq A$ and $\theta \subseteq Hom(A, B)$. Then $M^{\perp\perp}$ and $\theta^{\perp\perp}$ are subimplication algebras of $Hom(A, B)$ and A respectively.*

Proof. (1) Let $f, g, h, k \in Hom(A, B)$ and $f \to g$,$h \to k \in M^{\perp\perp}$. Then $(f \to g)(a) = 1$ and $(h \to k)(a) = 1$,for all $a \in M$ by theorem 3.8 $(Hom(A, B), \to, ', 1)$ is a $1-$commutative implicative algebra.
Now $((f \to h) \to (g \to k))(a)$
$= (f \to h)(a) \to (g \to k)(a)$
$= (g \to k)(a) \to (1 \to ((f \to h)(a))$
$= (g(a) \to k(a)) \to (1 \to (f(a) \to h(a))$
$= (g(a) \to k(a)) \to (f(a) \to h(a))$
$= (k(a) \to (1 \to g(a)) \to (h(a) \to (1 \to f(a))$
$= (k(a) \to g(a)) \to (h(a) \to f(a))$
$= k(a) \to (g(a) \to (h(a) \to f(a)))$ by associative property of B.
$=K(a) \to (h(a) \to (g(a) \to f(a)))$ by exchange rule.
$=(k(a) \to h(a)) \to (g(a) \to f(a))$
$= (h(a) \to (1 \to k(a))) \to (f(a) \to (1 \to g(a)))$
$= (h(a) \to k(a)) \to (f(a) \to g(a))$
$= (h \to k)(a) \to (f \to g)(a)$
$= 1 \to 1 = 1$.
Hence $((f \to h) \to (g \to k))(a) = 1$,for all $a \in M$.
Therefore $(f \to h) \to (g \to k) \in M^{\perp\perp}$.
So that $M^{\perp\perp}$ is a sub implicative algebra of $Hom(A, B)$.

(2) Let $a \to b$,$c \to d \in \theta^{\perp\perp}$ for all $a, b, c, d \in A$. Then $f(c \to d) = 1$, and $f(c \to d) = 1$,for all $a, b, c, d \in A$. Then $f(a \to b) = 1$, and $f(c \to d) = 1$,for all $f \in Hom(A, B)$.
Now $f((a \to c) \to (b \to d))$
$= f(a \to c) \to f(b \to d)$, since $f \in Hom(A, B)$.
$=(f(a) \to f(c)) \to (f(b) \to f(d))$
$= f(a) \to (f(c) \to (f(b) \to f(d)))$,associative property of B.
$=f(a) \to (f(b) \to (f(c) \to f(d)))$ by exchange rule.
$=(f(a) \to f(b)) \to (f(c) \to f(d))$ by associative property of B.
$=f(a \to b) \to f(c \to d)$,since $f \in Hom(A, B)$.
$=1 \to 1 = 1$.

Hence $f((a \to c) \to (b \to d)) = 1$.
Thus $(a \to c) \to (b \to d) \in \theta^{\perp\perp}$,for all $f \in Hom(A, B)$.
Therefore $\theta^{\perp\perp}$ is a sub implicative algebra.

□

4. Conclusions

In this paper the characterization of homomorphisms in an implicative algebra as implicative algebras have been introduced. The concepts of subimplicative algebra, $1-$commutative algebra are discussed. Different characterization of theorems are proved.

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