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통합 베이즈 총변이 정규화 방법과 영상복원에 대한 응용

류재홍*

An Unified Bayesian Total Variation Regularization Method and Application to Image Restoration

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요 약

본 논문은 통합 베이즈 티코노프 정규화 방법을 총변이 정규화에 대한 해법으로 제시한다. 통합된 방법은 총변이 항을 가중된 티코노프 정규화 항으로 변형하여 정규화 모수를 구하는 공식을 제시한다. 정규화 모수를 구하고 이를 바탕으로 새로운 가중인수를 구하는 것을 복원된 영상이 수렴하기까지 반복한다. 실험결과는 영상 복원 문제에 대하여 제안하는 방법의 효능을 보여준다.

ABSTRACT

This paper presents the unified Bayesian Tikhonov regularization method as a solution to total variation regularization. The integrated method presents a formula for obtaining the regularization parameter by transforming the total variation term into a weighted Tikhonov regularization term. It repeats until the reconstructed image converges to obtain a regularization parameter and a new weighting factor based on it. The experimental results show the effectiveness of the proposed method for the image restoration problem.

키워드
Bayesian Tikhonov Regularization, Total Variation Regularization, Iteratively Reweighted L2 Norm, Image Restoration
바에즈 티코노프 정규화, 총변이 정규화, 반복 가중 L2 놈, 영상 복원

Ⅰ. Introduction

Image restoration is an example of inversion problems in the ill-posed systems[1,2]. Image restoration is the process to recover an original image from distorted one by using an appropriate degradation model[3-5]. In the linear degradation model, we assume that a given input image f is blurred by a Point Spread Function(: PSF) h and further distorted by a Gaussian noise η. This can be written in the form

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$$g(x,y) = h(x,y) \star f(x,y) + \eta(x,y) \quad (1)$$

where symbol $\star$ denotes convolution.

Figure 1 shows the degradation process of the satellite image and the restoration result by Wiener filtering[6].

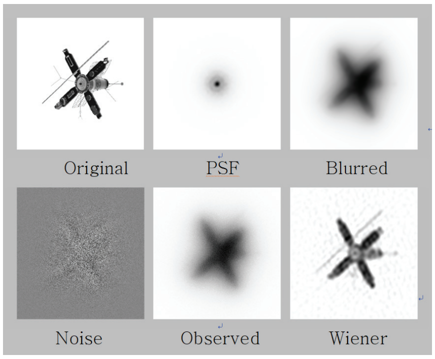

Fig. 1 Satellite image data.

In the Fourier transform, we have

$$G(u,v) = H(u,v)F(u,v) + N(u,v). \quad (2)$$

In the inverse filtering, we have

$$\hat{F}(u,v) = \frac{G(u,v)}{H(u,v)} = F(u,v) + \frac{N(u,v)}{H(u,v)}. \quad (3)$$

This formula shows that if $H(u,v)$ is zero or very small, the second term $N(u,v)/H(u,v)$ is amplified. Thus we need the regularization as a remedy solving ill-posed inverse problem.

Total Variation(:TV) regularization is well known for removing noise while recovering edge information[7,8].

In this paper, the unified Bayesian Tikhonov regularization is formulated for TV regularization problem and applied to the image restoration problems. In section II, TV regularization is reformulated as weighted Tikhonov regularization. In section III, the unified Bayesian method for weighted Tikhonov regularization is developed in the frequency domain. In section IV, a new method for TV regularization is suggested. In section V, experimental results show the effectiveness of the proposed method followed by the conclusion and reference sections[9-13].

II. Total Variation Regularization from Tikhonov Regularization

In the Tikhonov regularization for an ill-posed problem $\boldsymbol{Hf} = \boldsymbol{g}$, we seek a limiting vector $\hat{\boldsymbol{f}}_\lambda$ to fit data $\boldsymbol{g}$ in least squares sense with l^2 norm penalty term for large normed solution in the cost function as

$$J(\boldsymbol{f}) = \frac{1}{2}(\| \boldsymbol{Hf} - \boldsymbol{g} \|^2 + \lambda \| \boldsymbol{Cf} \|^2). \quad (4)$$

Here, $\boldsymbol{H}$ and $\boldsymbol{C}$ are the block circulant matrices of a PSF h and a smoothing operator p respectively. Both $\boldsymbol{H}$ and $\boldsymbol{C}$ matrices have the dimension, MN by MN. That is, $m = n = MN$ with image dimension M by N. λ is the Tikhonov regularization parameter.

TV Regularization is defined with l^1 norm penalty term as following[7,8]

$$J(\boldsymbol{f}) = \frac{1}{2}(\| \boldsymbol{Hf} - \boldsymbol{g} \|^2 + \lambda(\| \boldsymbol{Df} \|) . \quad (5)$$

Here $\boldsymbol{D}$ is the gradient operator. In the isotropic TV we have the magnitude of gradient in l^2 norm. as

$$\| \boldsymbol{Df} \| = \| \sqrt{(\| \boldsymbol{D_x f} \|)^2 + (\| \boldsymbol{D_y f} \|)^2} \| . \quad (6)$$

Here, D_x and D_y are gradient operators along axises. In the anisotropic TV we get the magnitude of gradient in l^1 norm. as

$$\| Df \| = \| D_x f \| + \| D_y f \| . \tag{7}$$

Regularization term in the q-norm can be expressed in Euclidean norm as following.

$$\begin{aligned} \frac{1}{q} \| Cf \|^{q} &= \frac{2}{q} \| Cf \|^{(q-2)} \frac{1}{2} \| Cf \|^{2} \quad . \\ &= \frac{1}{2} \| (\frac{2}{q})^{1/2} (Cf)^{(q-2)/2} Cf \|^{2} \\ &= \frac{1}{2} \| W_F Cf \|^{2} \end{aligned} \tag{8}$$

Thus we can setup the TV regularization as the weighted Tikhonov regularization with q=1.

$$f_\lambda = (H^T H + \lambda C^T W_F^2 C)^{-1} H^T g . \tag{9}$$

$$W_F^2 = (\frac{2}{q}) Diag(((Cf_\lambda)^{(2-q)} + \epsilon)^{-1}) . \tag{10}$$

Here, epsilon is the small positive constant preventing division by zero.

III. Regularization Parameter Selection in the Frequency Domain

In the frequency domain, block circulant matrices H and C are diagonalized. We denote them S and T respectively. Let z and b be the column vectors of the Fourier transform of f and g respectively.

Regularized estimation vector is defined as

$$\hat{z}_\lambda = (S^H S + \lambda T^H V^2 T)^{-1} S^H b \tag{11}$$

where superscript H denotes the Hermitian or conjugate transpose.

Wight matrix V is defined as following.

$$|V_i|^{2(0)} = |v_i|^2(0) = 1$$

$$|Cf_\lambda|_i^{2-q}(k+1) = \left[\frac{|s_i|^2 |t_i|^2}{(|s_i|^2 + \lambda |t_i|^2 |v_i|^2(k))^2} |b_i|^2 \right]^{(1-q/2)}$$

$$|v_i|^2(k+1) = (\frac{2}{q})(|Cf|_i^{(2-q)}(k+1) + \eta)^{-1} . \tag{12}$$

Here, s_i and t_i are the diagonal elements of the matrices S and T respectively. b_i is the element of column vector b stacking columns of 2D Fourier transform of the degraded image g.

Square residual is defined as

$$\| \xi_\lambda \|^2 = \sum_{i=1}^{n} \frac{\lambda^2 |t_i|^4 |v_i|^4}{(|s_i|^2 + \lambda |t_i|^2 |v_i|^2)^2} |b_i|^2 . \tag{13}$$

Smoothing term is defined as

$$\| W_F Cf_\lambda \|^2 = \sum_{i=1}^{n} \frac{|s_i|^2 |t_i|^2 |v_i|^2}{(|s_i|^2 + \lambda |t_i|^2 |v_i|^2)^2} |b_i|^2 . \tag{14}$$

Then we obtain a fixed point iteration method for the regularization parameter in the weighted l^2 norm system. The basic form of unified bayesian(UB) method[4] as

$$\lambda = \frac{\gamma}{m-\gamma} \frac{\| \xi \|^2}{\| W_F Cf \|^2} , \tag{15}$$

and the extended form of UB method[5] as

$$\lambda = \frac{\gamma}{m+k-\gamma} \frac{\| \xi \|^2}{\| W_F C f \|^2}, k=0,\pm 1,...,\pm n$$

, (16)

with the number of effective parameters γ

$$\gamma = \sum_{i=1}^{n} \frac{\lambda |t_i|^2 |v_i|^2}{|s_i|^2 + \lambda |t_i|^2 |v_i|^2}. \quad (17)$$

Ⅳ. Iteratively Reweighted L2 Norm Algorithm

We can start with identity weight matrix and select the regularization parameter. Then solve for equation (9) and (10) respectively until the appropriated termination condition is satisfied. We can designate the iteratively reweighted l2 norm (: IRL2N) algorithm for the TV regularization as in the Table 1. Termination condition can be relative error on either $\hat{f}_\lambda$ or W_F^2 against the limit value.

Table 1. IRL2N Algorithm for the TV regularization

Initialize $\hat{f}_\lambda = g$, $W_F^2 = I$, k = 0
1. Save $\hat{f}_\lambda$ and W_F^2 for checking out
2. Get λ using equation (15).
3. Get $\hat{f}_\lambda$ using equation (9).
4. Get W_F^2 using equation (10.)
5. Check for termination condition. if not k=k+1 and go to step 1.

IRL2N algorithm can perform in the frequency domain. using equation (11) and (12).

Ⅴ. Experimental Results

We report the experiments with the algorithm proposed in the previous section. The algorithm has applied four methods of parameter selection: unified bayesian(:UB), Extended UB(:EUB), relative error(: EUB_RE) and GCV respectively[4,5,9]. Results of TV smoother are depicted in the figures 2 and 3. First row depicts l^2 norm and the second row shows l^1 norm regularization results. Isotropic TV shows the better results than anisotropic TV. The EUB_RE shows the benchmark results and the EUB has the best performance under the new benchmark.

Image restoration performance is measured by the figure-of-merit functions such as relative error (RE), signal to noise ratio (SNR), peak SNR (PSNR) and improvement of SNR (ISNR)[10-13]. Tables 2-5 show the image restoration performance with remarking value of regularization parameter λ.

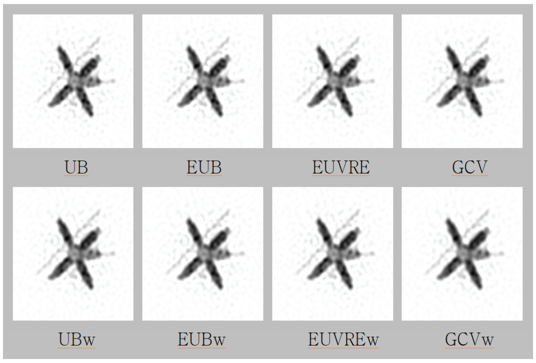

Fig. 2 Restored images with isotropic TV smoother.

Table 2. Performance Results using isotropic TV smoother

Measure \ Method	RE ↓	SNR ↑	PSNR	ISNR	Remark λ
UB	0.3434	9.284	22.91	6.243	5.61e-4
UBw	0.3411	9.342	22.97	6.302	1.83e-8
EUB	0.3433	9.286	22.91	6.245	5.77e-4
EUBw	0.3409	9.348	22.98	6.307	1.73e-8
EUBRE	0.3431	9.293	22.92	6.252	6.94e-4
EUBREw	0.3408	9.351	22.98	6.310	1.58e-8
GCV	0.3433	9.285	22.91	6.245	8.45e-4
GCVw	0.3425	9.308	22.94	6.267	2.23e-8
Wiener	0.3243	9.781	23.41	6.740	N/A

Table 3. Performance Results using anisotropic TV smoother

Measure \ Method	RE ↓	SNR ↑	PSNR	ISNR	Remark λ
UB	0.3454	9.233	22.86	6.193	3.30e-4
UBw	0.3423	9.312	22.94	6.272	1.36e-8
EUB	0.3454	9.235	22.86	6.194	3.34e-4
EUBw	0.3421	9.318	22.95	6.277	1.26e-8
EUBRE	0.3448	9.248	22.88	6.207	4.31e-4
EUBREw	0.3420	9.320	22.95	6.280	1.19e-8
GCV	0.3450	9.244	22.87	6.204	4.90e-4
GCVw	0.3435	9.282	22.91	6.241	1.62e-8
Wiener	0.3243	9.781	23.41	6.740	N/A

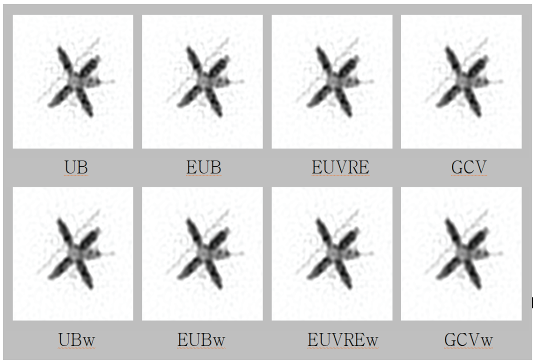

Fig. 3 Restored images with anisotropic TV smoother.

Table 4. Performance Results with identity smoothing operator

Measure \ Method	RE ↓	SNR ↑	PSNR	ISNR	Remark λ
UB	0.5694	4.891	18.52	1.850	1.72e-5
UBw	0.3521	9.066	22.69	6.025	3.89e-9
EUB	0.3496	9.129	22.76	6.088	1.59e-4
EUBw	0.3455	9.230	22.86	6.189	1.20e-8
EUBRE	0.3492	9.139	22.77	6.098	1.84e-4
EUBREw	0.3408	9.351	22.98	6.310	7.22e-9
GCV	0.3575	8.935	22.56	5.895	9.80e-5
GCVw	0.3408	9.349	22.98	6.309	7.91e-9
Wiener	0.3243	9.781	23.41	6.740	N/A

Table 5. Performance Results using Laplacian smoother

Measure \ Method	RE ↓	SNR ↑	PSNR	ISNR	Remark λ
UB	0.3489	9.146	22.77	6.106	7.91e-3
UBw	0.3499	9.121	22.75	6.080	9.92e-8
EUB	0.3433	9.287	22.92	6.247	2.54e-3
EUBw	0.3422	9.314	22.94	6.273	4.39e-8
EUBRE	0.3433	9.287	22.92	6.247	2.47e-3
EUBREw	0.3417	9.326	22.95	6.285	3.64e-8
GCV	0.3449	9.247	22.87	6.206	4.45e-3
GCVw	0.3447	9.250	22.88	6.210	6.31e-8
Wiener	0.3243	9.781	23.41	6.740	N/A

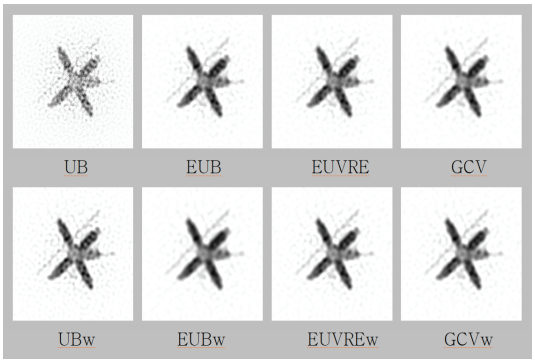

Fig. 4 Restored images with identity smoother.

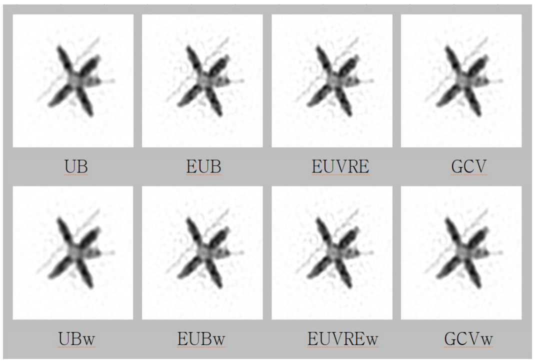

Fig. 5 Restored images with Laplacian smoother.

Suffix w denotes the result of IRL2N algorithm. Tikhonov regularization results are listed without suffix w. In IRL2N algorithm information is accumulated in the weight matrix and the regularization parameter is much smaller than the initial one. Thus constant of regularization parameter without adjustment during the iteration can prevent the process from getting the optimal state[7,8]. Tables 2-5 show that TV regularization using the IRL2N algorithm is generally more effective compared to the Tikhonov regularization.

Ⅵ. Conclusions

In this paper, the unified Bayesian Tikhonov regularization and the extended one are adopted in the IRL2N algorithm and applied to the image restoration problem. All procedures are successfully adapted to the weighted regularization method. Next plan is to develop the IRL2N method from the optimality condition providing adaption and continuation of parameters without separating shrinkage formular.

References

[1] H. Engl, M. Hanke, and A. Neubauer, *Regularization of Inverse Problems*. Dordrecht: Kluwer Academic Publishers, 1996.

[2] S. Kim, "An image denoising algorithm for the mobile phone cameras," *J. of the Korea Institute of Electronic Communication Sciences*, vol. 9, no. 5, 2014, pp. 601-608.

[3] R. C. Gonzalez and R. E. Woods, *Digital Image Processing*. MA: Addison-Wesley, 1992.

[4] J. Yoo, "A Unified Bayesian Tikhonov Regularization Method for Image Restoration," *J. of the Korea Institute of Electronic Communication Sciences*, vol. 11, no. 11, 2016, pp. 1129-1134.

[5] J. Yoo, "An Extension of Unified Bayesian Tikhonov Regularization Method and Application to Image Restoration," *J. of the Korea Institute of Electronic Communication Sciences*, vol. 15, no. 11, 2020, pp. 161-166.

[6] J. Nagy, K. Palmer, and L. Perrone, "Iterative methods for image deblurring: a Matlab object oriented approach," *Numerical Algorithms*, vol. 36, no. 1, 2004, pp. 73-93.

[7] Y. Wang, J. Yang, W. Yin, and Y. Zhang, "A New Alternating Minimization Algorithm for Total Variation Image Reconstruction," *SIAM J. IMAGING SCIENCES*, vol. 1, no. 3, 2008, pp. 248 - 272.

[8] P. Rodríguez and B. Wohlberg, "Efficient Minimization Method for a Generalized Total Variation Functional," *IEEE Transactions on Image Processing*, vol. 18, no. 2, 2009, pp. 322-332.

[9] G. Golub, M. Heath, and G. Wahba, "Generalized cross-validation as a method for choosing a good ridge parameter," *Technometrics*, vol. 21, no. 2, 1979, pp. 215-223.

[10] J. Yoo, "Self-Regularization Method for Image Restoration," *J. of the Korea Institute of Electronic Communication Sciences*, vol. 11, no.

1, 2016, pp. 45-52.
[11] H. Jeong, S. Cho and S. Kim, "Medical Image Encryption based on C-MLCA and 1D CAT," *J. of the Korea Institute of Electronic Communication Sciences,* vol. 14, no. 2, 2019, pp. 439-446.
[12] J. Choi, "Mixed Noise Cancellation by Independent Vector Analysis and Frequency Band Beamforming Algorithm in 4-channel Environments," *J. of the Korea Institute of Electronic Communication Sciences*, vol. 14, no. 5, 2019, pp. pp. 811-816.
[13] K. Chung, "On Negative Correlation Bit-to-Symbol(: B2S) Mapping for NOMA with Correlated Information Sources in 5G Systems," *J. of the Korea Institute of Electronic Communication Sciences*, vol. 15, no. 5, 2020, pp. 881-888.

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