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Design of an 8× Four-group Zoom System without a Moving Group by Considering the Overall Length

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We present a method to count the overall length of the zoom system in an initial design stage. In a zoom-lens design using the concept of the group, it has been very hard to precisely estimate the overall length at all zoom positions through the previous paraxial studies. To solve this difficulty, we introduce T_{eq} as a measure of the total track length in an equivalent zoom system, which can be found from the first order parameters obtained by solving the zoom equations. Among many solutions, the parameters that provide the smallest T_{eq} are selected to construct a compact initial zoom system. Also, to obtain an 8× four-group zoom system without moving groups, tunable polymer lenses (TPLs) have been introduced as a variator and a compensator. The final designed zoom lens has a short overall length of 29.99 mm, even over a wide focal-length range of 4–31 mm, and an *f*-number of F/3.5 at wide to F/4.5 at tele position, respectively.

Keywords : Aberrations, First orders, Tunable polymer lens, Zoom lens, Zoom position

OCIS codes : (080.2740) Geometric optical design; (120.4570) Optical design of instruments; (220.3620) Lens system design

I. INTRODUCTION

A zoom system consists of many lens groups, including moving groups such as a variator for zooming and a compensator for fixing the image plane. These moving groups are necessary to achieve different magnifications, but they require additional space for their motion. In addition, since this zoom system must satisfy all first-order requirements and correct aberrations at all zoom positions, it is more likely to consist of many lens elements. However, in the initial design stage of this complex optical zoom system, it is very difficult to precisely estimate the overall length of the zoom optics through a paraxial study.

Much research on initial zoom-system designs using paraxial optics has been reported [1–6]. These paraxial solutions provide the first orders of each group and zoom locus, but it is difficult to determine whether they provide a suitable solution. To have a reasonable zoom system initially, Park and Shannon proposed zoom-lens design using lens modules, in which each group was replaced with lens modules and then optimized to have a zoom system using the first orders of the modules as design parameters [7–9]. However, this design method did not provide any information on the overall length needed to achieve a compact zoom system.

To overcome this difficulty, this study proposes a method to count the overall length of the zoom system at the initial design stage. From the fact that the overall length can be estimated indirectly through the air distance and the first orders obtained by solving the zoom equations, we introduce T_{eq} as a measure of the total track length in an equivalent zoom system. When T_{eq} is small, the overall length of a real optical system is expected to be short. Therefore, if the system with the smallest T_{eq} is suggested, we can effectively obtain an optical zoom system having short overall length. This design concept is a key point of this study.

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Among many solutions for an 8× four-group zoom system employing two tunable polymer lenses (TPLs) as variator and compensator, the first orders that provide the smallest T_{eq} are selected to construct a compact initial zoom system. This design approach enables us to realize a compact 8× four-group zoom system with short overall length of 29.99 mm over a wide focal length range of 4–31 mm, although all groups are fixed for zooming and compensating.

II. OVERALL LENGTH OF THE FIRST ORDER EQUIVALENT OPTICAL SYSTEM

Each group in a zoom system can be transformed into an equivalent lens group that has the same first-order quantities, such as effective focal length (*efl*), back focal length (*bfl*), and front focal length (*ffl*). Thus, we can make an equivalent optical zoom system within paraxial optics by replacing each real lens group with its equivalent lens group, and then combining groups according to the air distances between them. Here we will define the overall length parameter T_{eq} of this equivalent zoom system as a measure of the total track length, using the first orders obtained from paraxial ray tracing. When the rays are incident parallel to the optical axis in the forward direction, as in Fig. 1(a), paraxial ray tracing yields the following equations:

$$nu \;= -k\, h, \tag{1}$$

$$h \;= h + u\, d, \tag{2}$$

$$u \;- nu \;= -k\; h\;, \tag{3}$$

$$\Delta P\, H\;\, F \quad u \;= -K h, \tag{4}$$

$$\Delta h\; A\, F \quad u \;= -\frac{h}{bfl}. \tag{5}$$

Also, another paraxial ray can be traced similarly in the reverse direction of Fig. 1(b); its parameters are marked by double primes as follows:

$$nu'' = -k\; h, \tag{6}$$

$$h'' = h + u''d'', \tag{7}$$

$$u'' - nu'' = -k\, h'', \tag{8}$$

$$\Delta P H\, F \quad u'' = -K\, h, \tag{9}$$

$$\Delta h'' A\, F \quad u'' = \frac{h''}{ffl}, \tag{10}$$

where k_1 and k_2 are the powers of each surface, and $K = 1\,/\,efl$ is the total power of an equivalent lens. Also, u_1 and u_2 are the convergence angles between paraxial ray and optical axis, and h_i is the axial ray's height. In Fig. 1(a), the signs of angles u_1, u_2 are positive for counterclockwise. On the other hand, the signs of angles u_1'', u_2'' of Fig. 1(b) are negative for counterclockwise. The distance d'' is positive when the ray travels from right to left, as opposed to d of Fig. 1(a).

Combining Eqs. (4) and (9), Eqs. (4) and (5), and Eqs. (9) and (10), yields respectively the following three equations:

$$u \;= u'' = -K\, h, \tag{11}$$

$$h \;= h K\, bfl, \tag{12}$$

$$h'' = -h K\; ffl. \tag{13}$$

Substituting Eqs. (1), (4), and (12) into Eq. (3), we obtain Eq. (14). Also, substituting Eqs. (6), (9), and (13) into Eq. (8) yields Eq. (15), and similarly combining Eqs. (14) and (15) leads to Eqs. (16) and (17), as follows:

$$k \;= K \;\; - k\;\, bfl \;, \tag{14}$$

$$k \;= K \;\; + k\;\, ffl \;, \tag{15}$$

$$k \;= \frac{K - K\;\, bfl}{+ K\;\, bfl \cdot ffl}, \tag{16}$$

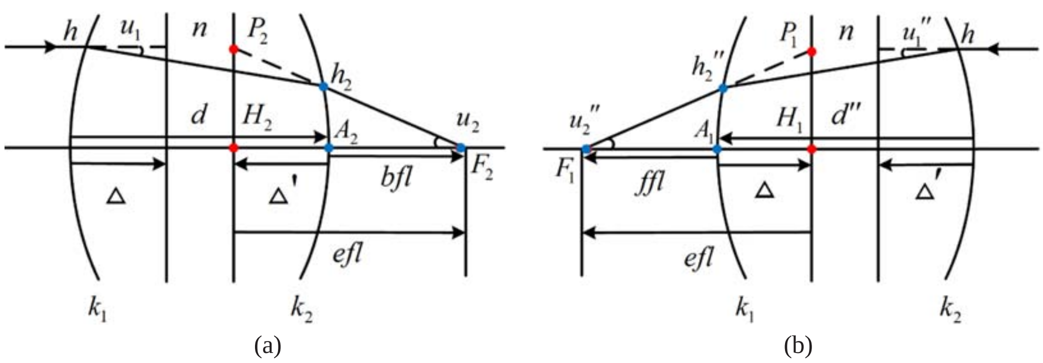


FIG. 1. Paraxial ray traces in an equivalent optical system: (a) Ray tracing in the forward direction, (b) ray tracing in the reverse direction.

$$k = \frac{K+K\ \ ffl}{+K\ \ bfl \cdot ffl}. \tag{17}$$

In addition, inserting Eq. (2) into Eq. (1) yields Eq. (18). Finally, if we substitute Eqs. (12) and (16) into Eq. (18), we obtain Eq. (19), as follows:

$$\frac{d}{n} = \frac{h-h}{k\,h}, \tag{18}$$

$$\frac{d}{n} = \frac{+K\ \ bfl\ ffl}{K}. \tag{19}$$

If we designate the thickness and the refractive index of the i^{th} equivalent lens group as $d_{i,eq}$ and $n_{i,eq}$ respectively, then Eq. (19) leads to Eq. (20) as

$$d_{i\ eq} = n_{i\ eq}\, efl_i \left(+\frac{bfl_i}{efl_i} \cdot \frac{ffl_i}{efl_i} \right). \tag{20}$$

Consequently, the overall length T_{eq} of an equivalent g-group zoom system within first-order optics can be given by

$$T_{eq} = \sum_{i=}^{g} \left[n_{i\ eq}\, efl_i \left(+\frac{bfl_i}{efl_i} \cdot \frac{ffl_i}{efl_i} \right) + air_i \right]. \tag{21}$$

In Eq. (21), g denotes the number of groups and air_i is the real air distance between the i^{th} group and the $(i + 1)^{th}$ group. When T_{eq} is small, the overall length of a real optical zoom system can be estimated to be short. Therefore, if the zoom system with the first orders that yield the smallest T_{eq} is suggested, we can effectively obtain an optical system having short overall length.

III. ANALYSES FOR THE PARAMETERS OF AN INITIAL FOUR-GROUP ZOOM SYSTEM EMPLOYING TWO TPLS

In this section, we determine the initial data for an 8× four-group zoom system without moving groups by employing two tunable polymer lenses (TPLs) as a variator and a compensator [10]. As seen in Fig. 2, there are 21 parameters to be determined initially at each zoom position in a four-group zoom system; they should be selected to have the smallest T_{eq}. For convenience, we deal with the zoom system at its wide and tele positions, so there are 42 parameters in total.

3.1. Optical Configuration for Higher Zoom Ratio

In this work, we present a four-group zoom system without moving groups by employing TPLs. Thus, we locate the TPLs at the second group for zooming and the fourth group for compensating, as shown in Fig. 3. To obtain a high zoom ratio of 8×, it is desirable to construct the zoom system to be of the retrofocus type at the wide position, and of the telephoto type at the tele position.

The 1st group's power is always fixed and slightly positive to collect the rays, while the TPL at the 2nd group should have a negative power at the wide position, and a positive power at the tele position. The power of the 3rd group should be positive, to relay the rays to the next group. As opposed to the 2nd group, the TPL at the 4th group should have a positive power at wide position and a negative power at tele position, as in Fig. 3. This large change in power of the TPLs can be obtained by shape-changing the polymeric membrane from plano-concave to plano-convex (or vice versa), as shown in Fig. 4 [11–14].

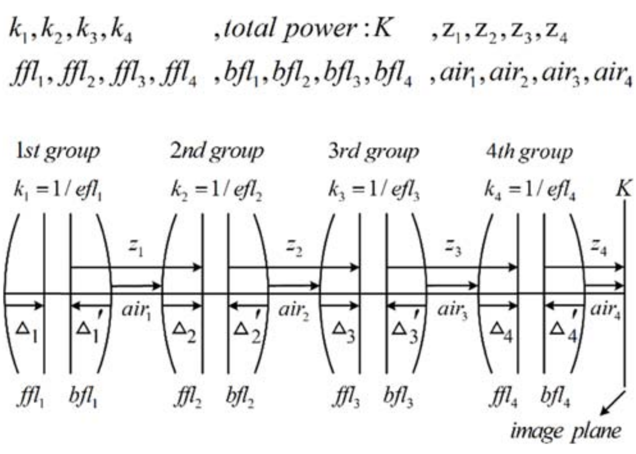


FIG. 2. Initial design parameters and distances between groups in a four-group zoom system.

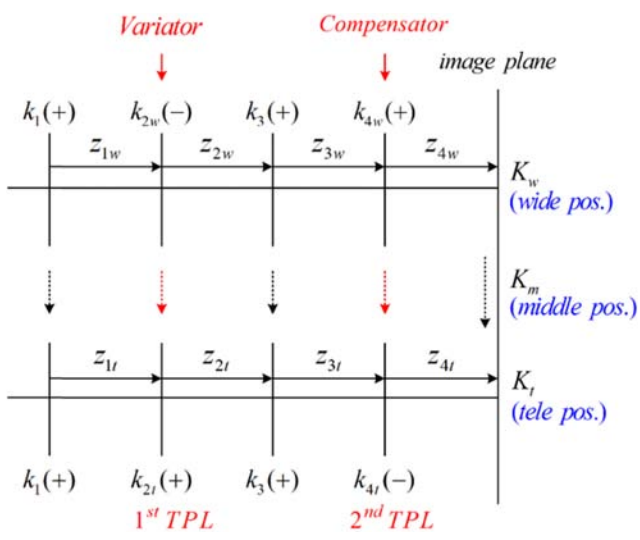


FIG. 3. Layout of the four-group zoom system without moving groups by employing tunable polymer lenses (TPLs) at the 2nd and 4th groups.

3.2. Conditions to Realize an Optical System

From Fig. 3, the paraxial design of this zoom system with an infinite object can be formulated as follows [5, 6, 10]:

$$K_w = [k \quad -z_{\ w} \quad k_{\ w} \quad -z_{\ w} \quad k \quad -z_{\ w} \quad k_{\ w}], \tag{22}$$

$$K_t = [k \quad -z_{\ t} \quad k_{\ t} \quad -z_{\ t} \quad k \quad -z_{\ t} \quad k_{\ t}], \tag{23}$$

$$h_{iw} = h\ [k \quad -z_{\ w} \quad k_{\ w} \quad -z_{\ w} \quad k \quad -z_{\ w} \quad k_{\ w} \quad -z_{\ w}] = \quad , \tag{24}$$

$$h_{it} = h\ [k \quad -z_{\ t} \quad k_{\ t} \quad -z_{\ t} \quad k \quad -z_{\ t} \quad k_{\ t} \quad -z_{\ t}] = \quad , \tag{25}$$

where k_1, k_2, k_3, and k_4 are the powers of the groups, and z_1, z_2, z_3, and z_4 are the distances between adjacent principal points at a given zoom position. h_i is the axial ray height on the image plane, and K is the total power of the zoom system. The subscripts w and t denote the zoom positions at wide and narrow fields, usually designated as wide and tele positions, respectively. Finally, [ ] represent Gaussian brackets [3, 15].

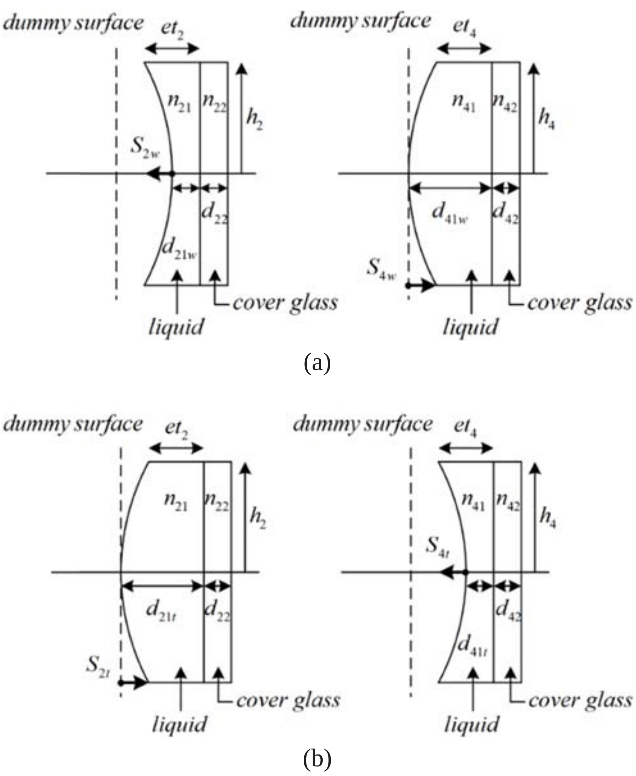


FIG. 4. Design parameters of a shape-changing tunable polymer lens (TPL) according to its curvature change: (a) Surface shapes of the 2nd (left) and 4th (right) groups at the wide position, and (b) surface shapes of the 2nd (left) and 4th (right) groups at the tele position.

Figure 4 illustrates the changes in surface sag and central thickness induced by the curvature-changing at different zoom positions. To obtain a physically meaningful 8× four-group zoom system, the TPLs should have reasonable quantities such as refractive index, thickness, aperture size, etc. These parameters should initially be specified to yield a feasible and compact TPL. In Fig. 4, S_{2w} and S_{2t} are the sags of the TPL's surface at the second group, at wide and tele positions respectively. Similarly, S_{4w} and S_{4t} are those at the fourth group at both positions. Also, et_2 and et_4 are the edge thicknesses of the TPLs at the second and fourth groups.

From the structure of a TPL, while the edge thickness is always fixed for the zooming process, its central thickness is changed by means of a shape-changing polymeric membrane, as shown in Fig. 4 [10, 12–14]. Assuming that the back surface of the TPL is a rigid plane in this study, the thicknesses at both extreme positions, aperture sizes, and refractive indices of two feasible TPLs are summarized in Table 1.

Finally, z_i is the principal plane distance between the i^{th} group and $(i + 1)^{\text{th}}$ group as in Fig. 2, given by

$$z_i = -\Delta_i' + air_i + \Delta_{i+}\ .$$

These principal-plane distances between adjacent groups at the wide position can be re-expressed in terms of first-order quantities such as the effective focal length (*efl*), the back focal length (*bfl*), the front focal length (*ffl*), and the air distance (air_i) as

$$z_{\ w} = air \ + ffl_{\ w} + efl_{\ w} - bfl \ + efl\ , \tag{26}$$

$$z_{\ w} = air \ + ffl \ + efl \ - bfl_{\ w} + efl_{\ w}, \tag{27}$$

$$z_{\ w} = air \ + ffl_{\ w} + efl_{\ w} - bfl \ + efl\ , \tag{28}$$

$$z_{\ w} = air \ - bfl_{\ w} + efl_{\ w}. \tag{29}$$

The curvature of a TPL should be changed to provide the required power at various zoom positions, which con-

TABLE 1. Design parameters of the tunable polymer lenses (TPLs) at the 2nd and 4th groups

<table>
<tr><th rowspan="2">Parameters</th><th colspan="2">The 2nd group</th><th colspan="2">The 4th group</th></tr>
<tr><th>Liquid Lens</th><th>Cover Glass</th><th>Liquid Lens</th><th>Cover Glass</th></tr>
<tr><td>Thickness (wide)</td><td>d_{21w} = 0.4 mm</td><td rowspan="2">d_{22} = 0.6 mm</td><td>$d_{41w} = d_{41t} + (S_{4w} - S_{4t})$</td><td rowspan="2">$d_{42}$ = 0.6 mm</td></tr>
<tr><td>Thickness (tele)</td><td>$d_{21t} = d_{21w} + (S_{2t} - S_{2w})$</td><td>$d_{41t}$ = 0.4 mm</td></tr>
<tr><td>Semi-aperture</td><td colspan="2">h_2 = 4 mm</td><td colspan="2">h_4 = 3 mm</td></tr>
<tr><td>Refractive Index</td><td>n_{21} = 1.38</td><td>n_{22} = 1.5168</td><td>n_{41} = 1.42</td><td>n_{42} = 1.5168</td></tr>
</table>

sequently causes the principal-point positions to shift. Since the distances between adjacent principal points at the tele position, z_{it} (i = 1, 2, 3, 4), can be expressed using the amount of movement Δz_i (i = 1, 2, 3, 4) of the principal points from the wide position, the parameters related to those at the tele position should be excluded from the total of 42 degrees of freedom of Fig. 2.

3.3. Specific Conditions for a Targeted Zoom System

(1) Since the 1st and 3rd groups have fixed focal lengths, the first orders of both groups are always fixed at all zoom positions, as follows:

$$\begin{array}{lll} k_{w} = k_{t} = k & ffl_{w} = ffl_{t} = ffl & bfl_{w} = bfl_{t} = bfl \\ k_{w} = k_{t} = k & ffl_{w} = ffl_{t} = ffl & bfl_{w} = bfl_{t} = bfl \end{array} \tag{30}$$

(2) We initially set the focal length of 4 mm at the wide position. To have a zoom ratio of 8×, that at the tele position should be 32 mm. Thus their powers are given by

$$K_w = - \quad ^{-}, \quad K_t = \text{—} \quad ^{-}. \tag{31}$$

(3) To avoid changing the air distances between adjacent groups owing to shape change of the polymeric membrane, dummy surfaces are placed at the TPLs of the 2nd and 4th groups, as shown in Fig. 4. Since air_i is given by the air distance between the final real lens surface and the dummy surface of the TPL, four air distances are fixed at all zoom positions as

$$\begin{array}{ll} air_{w} = air_{t} = air & air_{w} = air_{t} = air \\ air_{w} = air_{t} = air & air_{w} = air_{t} = air \end{array} \tag{32}$$

(4) Referring to Figs. 2, 4, and 5, when the zoom position is switched from wide to tele, the amount of movement of the principal points of each group, Δz_i (i = 1, 2, 3, 4), is calculated as follows [10]:

$$\begin{aligned} \Delta z &= z_{t} - z_{w} = \Delta_{t} - \Delta_{w} = -\left(S_{t} - S_{w}\right), \\ \Delta z &= z_{t} - z_{w} = -\left(\Delta_{t}' - \Delta_{w}'\right) = \left(\frac{d_{t}}{n} - \frac{d_{w}}{n}\right), \\ \Delta z &= z_{t} - z_{w} = \Delta_{t} - \Delta_{w} = -\left(S_{t} - S_{w}\right), \\ \Delta z &= z_{t} - z_{w} = -\left(\Delta_{t}' - \Delta_{w}'\right) = \left(\frac{d_{t}}{n} - \frac{d_{w}}{n}\right). \end{aligned} \tag{33}$$

Here, note that the amount of movement Δz_i of the principal points of each group is only affected by the parameters of the TPL, such as the sag, central thickness, and refractive index.

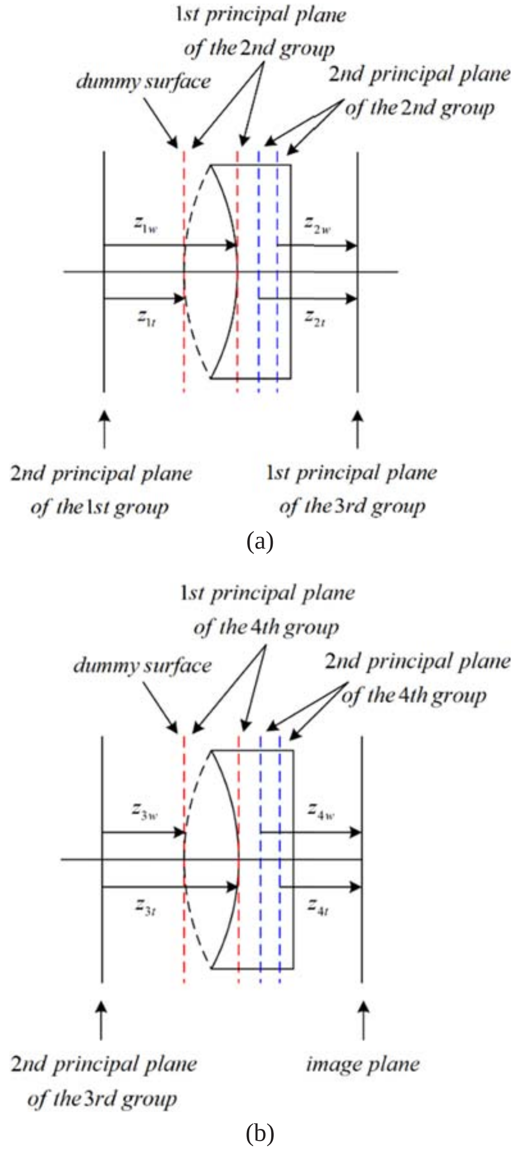


FIG. 5. Changes of the principal planes of the 2nd and 4th groups with zoom position: (a) Tunable polymer lens at the 2nd group, (b) tunable polymer lens at the 4th group.

3.4. Actual Independent Parameters

Among 42 parameters in total, since the distances between the adjacent principal points at the tele position can be expressed using the amount of movement Δz_i of the principal points from the wide position, the 19 parameters at the tele position should be excluded, except for the TPL powers of the 2nd and 4th groups. If the conditions of Eqs. (30)–(33) in Section 3.3 are applied, we have 18 actual independent parameters at both extreme positions, as follows:

$$\left(\begin{array}{cccccccc} f & f_{w} & f_{t} & f & f_{w} & f_{t} & z_{w} & z_{w} \; z_{w} \; z_{w} \\ ffl & bfl & ffl & bfl & air & air & air & air \end{array}\right).$$

Here, if the zoom system satisfies Eqs. (22)–(29), only 10 free parameters remain. It is desirable to select free parameters that can be estimated effectively, such as the focal length of each group and the air distances between groups, rather than the front focal lengths, back focal lengths, and distances between adjacent principal planes.

First, for the given six free parameters f_1, f_{2w}, f_{2t}, f_3, f_{4w}, and f_{4t}, solving Eqs. (22)–(25) yields four distances (z_{iw}) between adjacent principal planes at the wide position.

Next, out of the remaining parameters ffl_1, bfl_1, ffl_3, bfl_3, air_1, air_2, air_3, and air_4, if we additionally select the free parameters ffl_1, air_1, air_2, and air_3 and substitute the solutions z_{iw} of Eqs. (22)–(25) into Eqs. (26)–(29), we obtain bfl_1, ffl_3, bfl_3, and air_4. From Section 3.2, note that if the focal lengths of the TPLs at the 2nd and 4th groups are given, we can immediately calculate their front focal lengths and back focal lengths. Thus, solving Eq. (29) yields the value of parameter air_4. In summary, there are 10 independent parameters as follows:

$$(f \;\; f_w \;\; f_t \;\; f \;\; f_w \;\; f_t \;\; ffl \;\; air \;\; air \;\; air).$$

To effectively enlarge the magnitude of the power variation at a TPL, the focal lengths of the TPLs at the 2nd and 4th groups are set to be symmetric with opposite signs at wide and tele positions. This configuration reduces two degrees of freedom, as follows:

$$f_w = f \qquad f_t = -f$$
$$f_w = f \qquad f_t = -f$$

According to design experience, the closer the 1st group and 2nd group are to each other, the more advantageous they are for a short overall length. When the same rule is applied to the 3rd and 4th groups, their air distances are constrained to be 0.3 mm for a compact zoom system. This eliminates an additional two degrees of freedom:

$$air = \qquad\qquad air = \qquad\qquad .$$

Finally, only six independent parameters remain; they are

$$(f \;\; f \;\; f \;\; f \;\; ffl \;\; air).$$

Whenever these six independent parameters are properly given, the remaining 36 design variables are determined by solving the zoom equations (22)–(29) and satisfying the four conditions of Section 3.3, with the TPL specifications of Table 1.

IV. DATA ACQUISITION FOR A COMPACT FOUR-GROUP ZOOM SYSTEM EMPLOYING TWO TPLS

For the given six parameters f_1, f_2, f_3, f_4, ffl_1, and air_2 within reasonable limits, the solutions with the smallest T_{eq} can be selected by examining the paraxial solutions obtained from the process above. These solutions are expected to provide a zoom system with short overall length.

4.1. Analysis for the Distances between Adjacent Principal Planes

Whenever the focal lengths (f_1, f_2, f_3, f_4) of each group are properly given, solving Eqs. (22)–(25) yields four combinations for solutions of distances (z_{1w}, z_{2w}, z_{3w}, z_{4w}) between adjacent principal planes at the wide position. To obtain a physically meaningful four-group zoom system, all combinations for each solution are examined using the Mathematica software.

The solutions for these four principal-plane distances at the wide position are initially investigated for f_1 = 60 mm, f_2 = −16.5 mm, and f_4 = 10.5 mm, for which combinations of solutions are designated as $Z11$–$Z44$ (Zij, i: combination number of solution, j: lens group number). For the focal lengths of the 3rd group, ranging from f_3 = 5 mm to f_3 = 25 mm, the solutions for Eqs. (22)–(25) yield four combinations for each distance, as illustrated in Fig. 6. Figure 6(a) shows four combinations for the principal-plane distances between the 1st and 2nd groups at the wide position. Similarly, four combinations for the principal-plane distances between other adjacent groups at the wide position are listed in Figs. 6(b)–(d).

In Fig. 6, note that as f_3 gets longer, there is the specific focal length of f_3 = 10.8 mm, which dramatically change the principal plane distances between groups. Here it is confirmed that these results appear the same, even if we check the solutions by changing the focal lengths of another group instead of f_3.

The solutions for the 1st and 4th combinations of $Z11$, $Z41$, $Z14$, and $Z44$ change very rapidly and have unrealistic values, such that these two combinations should be excluded. Therefore, zoom systems suitable for the purpose of this study appear in the 2nd and 3rd combinations of solutions in Fig. 6. Depending on the value of f_3, we can choose either solution combination. In other words, we choose the 2nd combination for $f_3 < 10.8$ mm, or the 3rd one for $f_3 > 10.8$ mm, the selection of which is advantageous for obtaining a short T_{eq}. From Fig. 6, note that all solutions investigated from f_3 = 5 mm to f_3 = 25 mm have a negative distance for z_{3w}, unlike the positive distances for z_{1w}, z_{2w}, and z_{4w}.

When the focal lengths of all groups, not only f_3, change within reasonable limits, we investigate the solutions for distances between adjacent principal planes at the wide position with a power distribution (+ − + +) for each group, as discussed in Section 3.1. That is, to obtain the solutions that provide a retro focus-type zoom system at the wide position, all groups should have these reasonable powers.

By inputting the focal lengths ranging from f_1 = 50 mm to f_1 = 100 mm, from f_2 = −22 mm to f_2 = −16 mm, from f_3 = 8 mm to f_3 = 24 mm, and from f_4 = 10 mm to f_4 = 13 mm, into the zoom equations of (22)–(25), we numerically obtain the physically meaningful solutions for our four distances (z_{1w}, z_{2w}, z_{3w}, z_{4w}). It is confirmed that all solutions

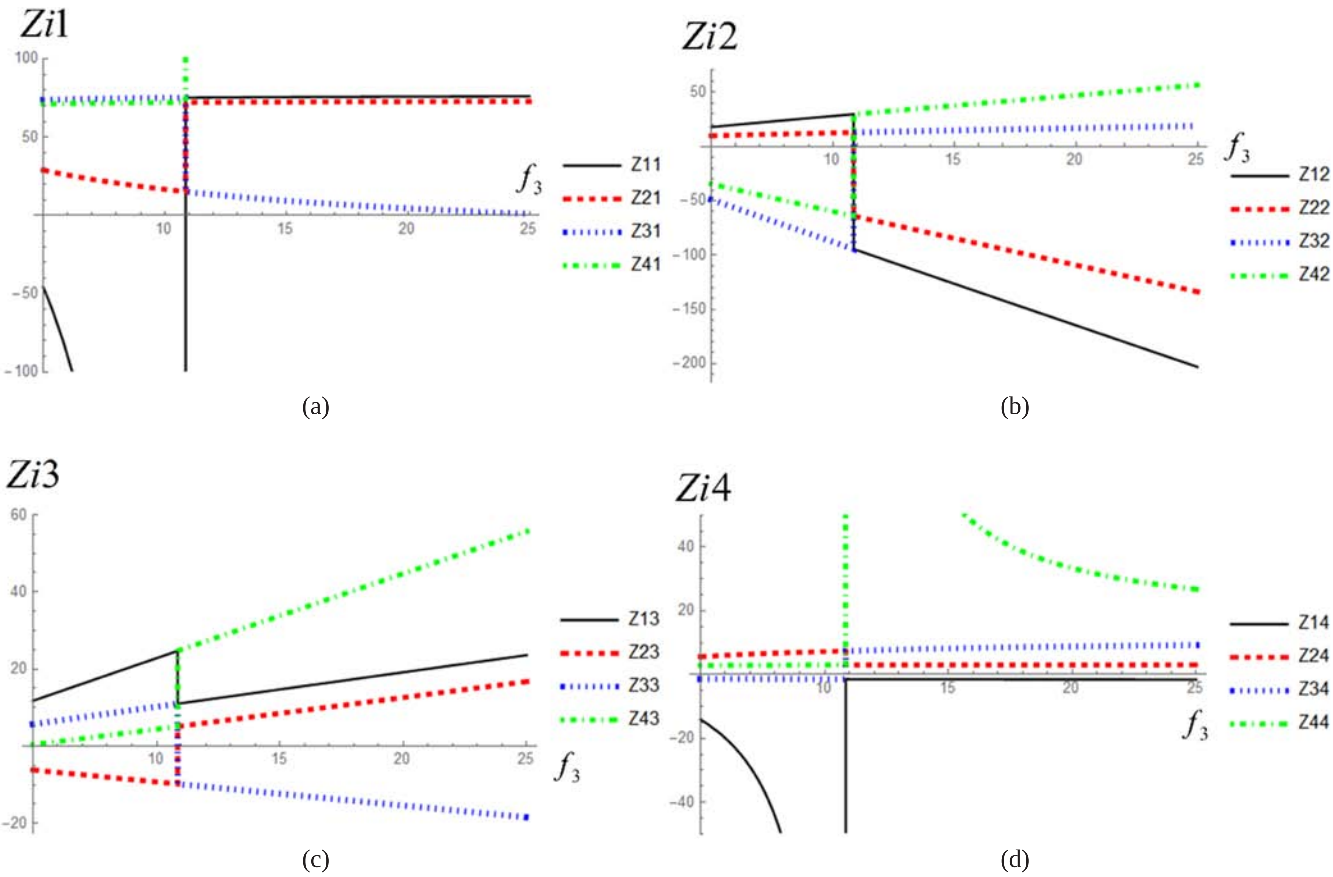


FIG. 6. Four combinations of solutions for the distances between adjacent principal planes, with the 3rd group's focal length at the wide position (Z*ij*, *i*: combination number of solution, *j*: lens group number): (a) Distances between the 1st and 2nd groups, (b) distances between the 2nd and 3rd groups, (c) distances between the 3rd and 4th groups, and (d) distances between the 4th group and image.

investigated in this study include positive distances for z_{1w}, z_{2w}, and z_{4w}, but a negative distance for z_{3w} (as in Fig. 6), the configuration of which was useful to shorten the total track length of the zoom system.

4.2. Constraints for the Principal Plane Distances

From the finding that solutions suitable for a compact zoom system have positive distances for z_{1w}, z_{2w}, and z_{4w}, and a negative distance for z_{3w}, we establish appropriate constraints for the principal-plane distances. To effectively enlarge the magnitude of the power variation at the TPL, the sags of the TPL are set to be symmetric with opposite signs at the wide and tele positions. In this case, the TPL's sag was set to −1 mm at maximum negative power and 1 mm at maximum positive power. In addition, the edge thicknesses of both liquid elements, from the TPL data in Table 1, are initially specified to be 1.4 mm, and always fixed for zooming.

z_{1w} is given by the sum of the distance $-\Delta_{1w}'$ from the second principal plane of the 1st group to the vertex of the last surface, the air distance air_1 = 0.3 mm, and twice as much (2 mm) as the TPL's sag at the 2nd group, *i.e.* $z_{1w} = (2.3\text{ mm} - \Delta_{1w}')$. The 1st lens group is preferred to have a positively powered meniscus with a convex front surface, which is useful to correct the distortion that is the dominant aberration at the wide-field position. This configuration places the second principal plane on the left side of the last surface, which leads to a positive distance for $-\Delta_{1w}'$. For this distance $-\Delta_{1w}'$ ranging from 0.2 mm to 2.2 mm, the constraints of z_{1w} are set as follows:

$$< z_{w} < \quad .$$

z_{2w} is given by the sum of the distance $-\Delta_{2w}'$ from the 2nd principal plane of the second group to the vertex of its last surface, the air distance air_2, and the distance Δ_{3w} from the front surface of the 3rd group to the first principal plane of that group, *i.e.* $z_{2w} = -\Delta_{2w}' + air_2 + \Delta_{3w}$. From Table 1, the distance $-\Delta_{2w}'$ is calculated to be $-(d_{21w} / n_{21} + d_{22} / n_{22})$ = 0.684 mm. Since the TPL at the second group plays the role of a variator, to have a high zoom ratio of 8×, the distance between the 2nd and 3rd groups should be long. Thus, if air_2 is set to a value of 4 mm or more, z_{2w} is greater than 4.6854 mm + Δ_{3w}.

The 3rd lens group should have a positive power to relay the rays to the next group and needs many elements to correct all residual aberrations. This situation probably leads the first principal plane to be placed to the right side of the front surface, which yields a positive distance for Δ_{3w}. In the appropriate range of this distance, the constraints of z_{2w} are set as follows:

$$< z_{\ w} < \quad .$$

The other distances z_{3w} and z_{4w} are set freely, without restrictions; only unrealistic solutions are excluded. To take into account the total length of the system, the sum of all principal plane distances is constrained to be positive, so that a physically meaningful system can be identified:

$$< \sum_{i=} z_{iw} < \quad .$$

4.3. Initial Data with the Smallest T_{eq}

By applying Eq. (21) to a four-group zoom system, the overall length T_{eq} of this equivalent zoom system can be re-expressed as

$$\begin{aligned} T_{eq} &= n_{\ eq}\, efl \left(+ \frac{bfl}{efl} \frac{ffl}{efl} \right) + air + d \ + air \\ &+ n_{\ eq}\, efl \left(+ \frac{bfl}{efl} \frac{ffl}{efl} \right) + air + d \ + air , \end{aligned} \quad (34)$$

where d_2 and d_4 denote the thicknesses of the TPLs at the 2nd and 4th groups including the dummy surface. Also, $n_{1,eq}$ and $n_{3,eq}$ are the indices of the equivalent 1st and 3rd lens groups; they are replaced with the value for N-BK7 (n_d = 1.516). Thus, T_{eq} can be found from the first-order quantities of the groups and the air distances between groups.

Whenever six independent variables are given, the back (*bfl*) and front (*ffl*) focal lengths of each group along with the air distances air_j are determined by solving the zoom equations (22)–(29) and satisfying the four conditions of Section 3.3 and Table 1. By inputting these first-order data into Eq. (34), we can immediately obtain the values of T_{eq} for the corresponding independent parameters.

By inputting the focal lengths ranging from f_1 = 60 mm to f_1 = 65 mm, from f_2 = −22 mm to f_2 = −16 mm, from f_3 = 12 mm to f_3 = 21 mm, from f_4 = 10 mm to f_4 = 13 mm, from ffl_1 = −67 mm to ffl_1 = −58 mm, and from air_2 = 4 mm to air_2 = 10 mm, into the zoom Eqs. of (22)–(25), we obtain four distances (z_{1w}, z_{2w}, z_{3w}, z_{4w}) between adjacent principal planes at the wide position. Then, inserting these distances into Eqs. (26)–(29) yields the first-order quantities bfl_1, ffl_3, bfl_3, and air_4. Table 2 lists the values of T_{eq} computed by inserting these first orders into Eq. (34); here the number 100 in a cell represents a meaningless solution.

Among the solutions obtained through this design process, the zoom system with air_2 = 6 mm and ffl_1 = −63 mm is found to provide the best solution, with the smallest T_{eq}. From these free variables we can determine all zoom parameters that satisfy the requirements, as listed in Table 3. Thus, the initial zoom system with a short overall length is reasonably obtained using this paraxial analysis for zoom parameters.

V. DESIGN FOR AN 8× FOUR-GROUP ZOOM SYSTEM WITHOUT MOVING GROUPS USING TPLS

To obtain a real lens group equivalent to the zoom data of Table 3, each real lens group should be designed to have the same first-order quantities such as *efl*, *ffl*, and *bfl* given in Table 3. This design method enables us to independently design the real lens groups and combine them for an actual zoom system. In this paper, we choose an appropriate configuration for each real lens group and scale it so that its focal length is the same as that in Table 3. Using the optimization method, we match the first-order quantities of Table 3 to those of a real lens group. The real lenses of the groups are obtained after a few iterations. To realize a zoom

TABLE 2. Values of T_{eq} for various ffl_1 and air_2 at f_1 = 63 mm, f_{2w} = −17.5 mm, f_3 = 21 mm, and f_{4w} = 10 mm (in mm)

air_2 \ ffl_1	−67	⋯	−64	−63	−62	⋯	−58
4	100	⋯	100	30.267	31.751	⋯	37.690
4.5	100	⋯	100	29.481	30.966	⋯	36.904
⋯	⋯	⋯	⋯	⋯	⋯	⋯	⋯
5.5	100	⋯	100	27.909	29.394	⋯	35.333
6	100	⋯	100	**27.124**	28.608	⋯	34.547
6.5	100	⋯	100	100	100	⋯	100
⋯	⋯	⋯	⋯	⋯	⋯	⋯	⋯
9.5	⋯	⋯	100	100	100	⋯	100
10	⋯	⋯	100	100	100	⋯	100

TABLE 3. Initial zoom data with the shortest T_{eq} (in mm)

	1st group	2nd group	3rd group	4th group
efl_w	63.0	−17.5	21.0	10.0
efl_t		17.5		−10.0
z_w	4.310	17.349	−14.304	8.485
z_t	1.635	19.287	−11.783	6.71
air	0.3	6.0	0.3	6.033
ffl_w	−63.0	20.175	−10.337	−10.0
ffl_t		−17.5		12.521
bfl_w	61.664	−18.185	35.604	7.547
bfl_t		14.876		−10.677
d_{eq}	2.025	3.675	5.27	3.521
T_{eq}	27.124			

system equivalent to the zoom data of Table 3, the air distances between groups should be set according to the zoom locus at each position. This procedure yields a real zoom system equivalent to that of Table 3 within first-order optics [9, 10].

To have a compact zoom system, the 1st group is designed with two lens elements, and the 2nd and 4th groups each include only one TPL to change the power. As seen in Table 3, to have a higher zoom ratio of 8×, the principal-plane distances (z_{2w}, z_{2t}) are set to be very long. Also, other principal-plane distances (z_{3w}, z_{3t}) are negative, which denotes that the first principal plane of the 4th group is to the left of the second principal plane of the 3rd group. This configuration is useful to achieve a compact zoom system with a high zoom ratio of 8×. Also, the 3rd lens group must balance residual aberrations. These requirements lead this group to be composed of many elements, as shown in Fig. 7.

Thus, we take an initial zoom system with a half-image-size of 1 mm and f-numbers of F/5 at the wide position to F/7 at the tele position, for which reducing the aperture and field size is desirable, to avoid higher-order aberrations. Figure 7 shows an 8× initial real-lens zoom system equivalent to the zoom data of Table 3 within first-order optics. This zoom system has a fixed variator at the 2nd group and compensator at the 4th, in which the former focal lengths are changed from −17.5 mm to 17.5 mm, and the latter from 10 mm to −10 mm.

In the initial design of Fig. 7, the aperture and image size should be extended, to satisfy the specifications for a zoom camera. The aperture is increased to F/3.5 at wide and F/4.5 at tele position. The half-image-size should be 1.8 mm for a 1/6-inch CMOS image sensor with 2048 × 1536 pixels. To improve the entire performance of an extended aperture and field system, we reduce the aberrations of the starting zoom

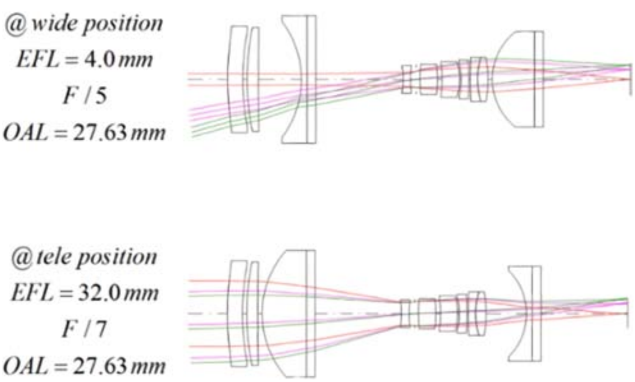


FIG. 7. An initial 8× four-group zoom system with fixed TPLs at the 2nd and 4th groups.

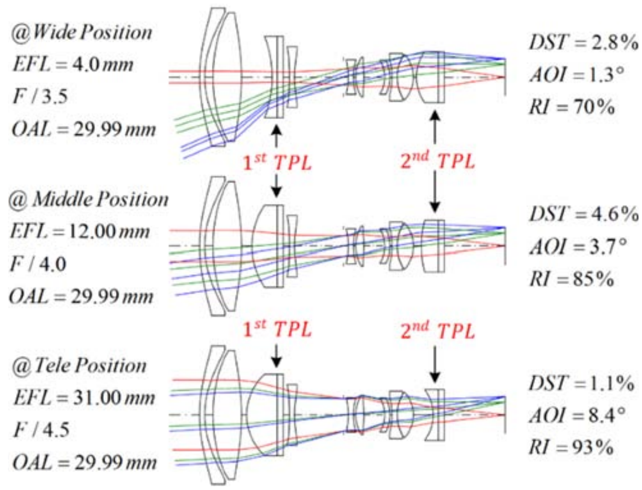
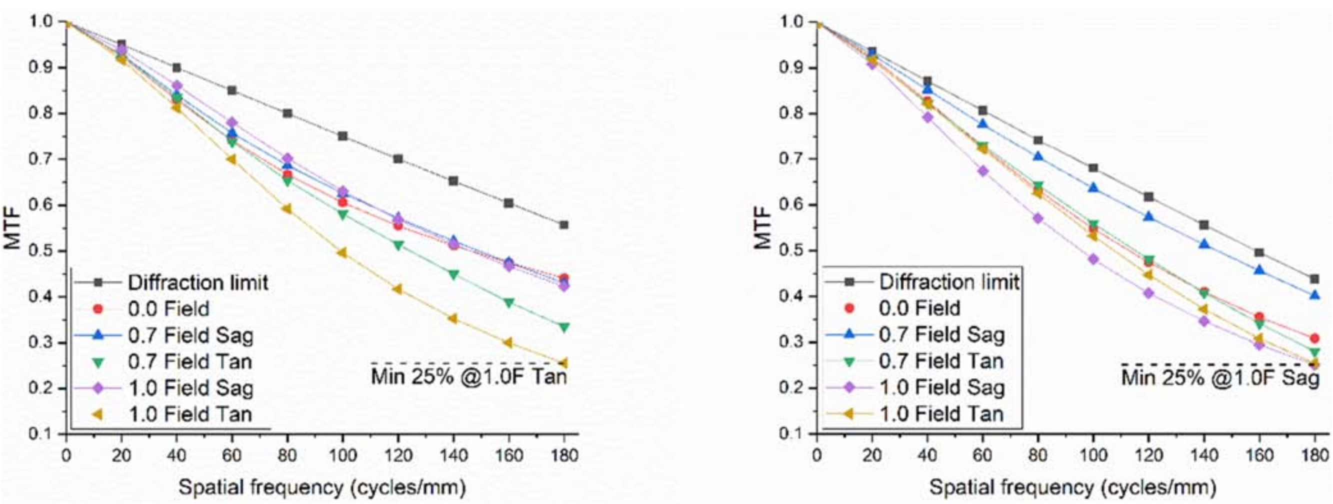


FIG. 8. Final 8× four-group zoom system with fixed variator and compensator, using TPLs at the 2nd and 4th groups (AOI, angle of incidence; DST, distortion; RI, relative illumination).



(a) (b)

FIG. 9. Modulation transfer functions (MTFs) of the final 8× four-group zoom system, at wide and tele positions: (a) MTF at wide position and (b) MTF at tele position.

lens by using aspheric surfaces.

Figure 8 illustrates the layout of the final designed zoom system, which consists of 10 elements including two TPLs and six aspheric lenses. The maximum diameter of the 1^{st} lens group is 13.59 mm. The distortions are balanced to less than 4.6%, and the ratio of relative illumination (RI) is more than 70% at the corner field when measured at all zoom positions, as in Fig. 8. The modulation transfer function (MTF) at 180 lp/mm is more than 25% over all fields at both extreme positions, from Fig. 9. Thus, all aberrations are significantly reduced.

The variation of the chief ray angle of incidence (AOI) from wide to tele position is less than 7.11 degrees. Because this variation is small, a stable image quality can be achieved for zooming. The overall length of this zoom lens, even at a large zoom ratio of 8×, is less than 29.99 mm. This is 5.5 mm shorter than the previous design, even for the same focal-length range of 4–31 mm [10]. Thus this design concept, based on the analysis for the overall length T_{eq} of an equivalent zoom system, greatly reduces the effort required to obtain a compact zoom lens system. This is an advantage of the study.

VI. CONCLUSION

To have a compact zoom system with a high zoom ratio of 8×, it is most effective to construct an optical system with a short overall length. In a zoom system consisting of several groups, however, it is very difficult to precisely estimate the overall length through a paraxial study. To solve this difficulty, this study has proposed a method to count the overall length of the zoom system in an initial design stage. From the fact that the overall length can be estimated indirectly through paraxial ray tracing, we introduce T_{eq} as a measure of the total track length in an equivalent zoom system, which can be found from the first orders obtained by solving the zoom equations.

Among many solutions, the first-order quantities that provide the smallest T_{eq} are selected to construct an initial zoom system with a high zoom ration of 8×. Also, to have a compact 8× four-group zoom system with no moving groups, TPLs have been suggested for variator and compensator.

Utilizing the design approaches proposed in this study, we have achieved a compact 8× four-group zoom system with a short overall length of 29.99 mm. This zoom lens had a wide focal-length range of 4–31 mm, and f-numbers of 3.5 at wide to 4.5 at tele position. Through this study, we illustrated the design process of a zoom system considering the overall length by introducing T_{eq}, along with a numerical, paraxial design approach.

The proposed design approaches are expected to be useful in estimating the overall length of any optical zoom system, regardless of whether there is a moving group or not. Even if a TPL were replaced by a moving group, we could estimate the overall length by inserting the first order quantities of the moving group into Eq. (34) for T_{eq}. Thus, we have been investigating an effective way to discover the solutions with the shortest overall length and applying this method to a zoom system with moving groups.

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