

ON AN INTEGRAL INVOLVING \bar{I} -FUNCTION

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ABSTRACT. In this paper, an interesting integral involving the \bar{I} -function of one variable introduced by Rathie has been derived. Since \bar{I} -function is a very generalized function of one variable and includes as special cases many of the known functions appearing in the literature, a number of integrals can be obtained by reducing the \bar{I} function of one variable to simpler special functions by suitably specializing the parameters. A few special cases of our main results are also discussed.

1. Introduction

The well known H-function of one variable is defined by Fox [3] and he proved the H-function as a symmetric Fourier kernel to Meijer G-function [2]. The H-function is often called Fox H-function. Later on many researchers studied and developed H-function. In 1997, Rathie [4] introduced a new function in the literature namely the I-function which is useful in Mathematics, Physics and other branches of applied mathematics. The I-function introduced by Rathie [4] is defined and represented by the following Mellin Barnes type contour integral:

$$(1) \quad \begin{aligned} I_{p,q}^{m,n}(z) &\equiv I_{p,q}^{m,n} \left[z \left| \begin{array}{c} (a_1, e_1, A_1), \dots, (a_p, e_p, A_p) \\ (b_1, f_1, B_1), \dots, (b_q, f_q, B_q) \end{array} \right. \right] \\ &= \frac{1}{2\pi i} \int_{\mathcal{L}} \theta(s) z^s ds, \end{aligned}$$

where

$$(2) \quad \theta(s) = \frac{\prod_{j=1}^m \Gamma^{B_j}(b_j - f_j s) \prod_{j=1}^n \Gamma^{A_j}(1 - a_j + e_j s)}{\prod_{j=m+1}^q \Gamma^{B_j}(1 - b_j + f_j s) \prod_{j=n+1}^p \Gamma^{A_j}(a_j - e_j s)}.$$

Also

- (i) $i = \sqrt{-1}$;
- (ii) $z \neq 0$;
- (iii) m, n, p, q are integers satisfying $0 \leq m \leq q, 0 \leq n \leq p$;
- (iv) \mathcal{L} is a suitable contour in the complex plane;

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- (v) an empty product is to be interpreted as unity;
- (vi) $e_j, j = 1, \dots, p; f_j, j = 1, \dots, q; A_j, j = 1, \dots, p;$ and $B_j, j = 1, \dots, q$ are positive numbers;
- (vii) $a_j, j = 1, \dots, p$ and $b_j, j = 1, \dots, q$ are complex numbers such that no singularity of $\Gamma^{B_j}(b_j - f_j s), j = 1, \dots, m$, coincides with any singularity of $\Gamma^{A_j}(1 - a_j + e_j s), j = 1, \dots, n$. In general these singularities are not poles.
- (viii) The contour \mathcal{L} goes from $\sigma - i\infty$ to $\sigma + i\infty$ (σ real) so that all the singularities of $\Gamma^{B_j}(b_j - f_j s), j = 1, \dots, m$, lie to the right of \mathcal{L} , and all the singularities of $\Gamma^{A_j}(1 - a_j + e_j s), j = 1, \dots, n$, lie to the left of \mathcal{L} .

In short, (1) will be denoted by

$$I_{p,q}^{m,n} \left[z \left| \begin{array}{l} 1(a_j, e_j, A_j)_p \\ 1(b_j, f_j, B_j)_q \end{array} \right. \right].$$

The function defined by (1) is convergent if

$$(3) \quad \Delta > 0, \quad |\arg(z)| < \frac{1}{2}\Delta\pi,$$

where

$$(4) \quad \Delta = \sum_{j=1}^m B_j f_j - \sum_{j=m+1}^q B_j f_j + \sum_{j=1}^n A_j e_j - \sum_{j=n+1}^p A_j e_j.$$

When $A_1 = A_2 = \dots = A_p = 1 = B_1 = B_2 = \dots = B_q$, (1) reduces to the H-function introduced by Fox [3] and studied by Braaksma [1].

In the present paper we establish an integral formulae of \bar{I} -function of one variable which is defined and represented by the following Mellin Barnes type contour integral:

$$(5) \quad \begin{aligned} \bar{I}(z) &= \bar{I}_{p,q}^{m,n} \left[z \left| \begin{array}{l} 1(a_j, e_j, A_j)_n, \quad n+1(a_j, e_j, A_j)_p \\ 1(b_j, f_j, 1)_m, \quad m+1(b_j, f_j, B_j)_q \end{array} \right. \right] \\ &= \frac{1}{2\pi i} \int_{\mathcal{L}} \phi(s) z^s ds, \end{aligned}$$

where

$$(6) \quad \phi(s) = \frac{\prod_{j=1}^m \Gamma(b_j - f_j s) \prod_{j=1}^n \Gamma^{A_j}(1 - a_j + e_j s)}{\prod_{j=m+1}^q \Gamma^{B_j}(1 - b_j + f_j s) \prod_{j=n+1}^p \Gamma^{A_j}(a_j - e_j s)}.$$

The function defined by (2) is convergent if

$$(7) \quad \Delta > 0, \quad |\arg(z)| < \frac{1}{2}\Delta\pi,$$

where

$$(8) \quad \Delta = \sum_{j=1}^m f_j - \sum_{j=m+1}^q f_j + \sum_{j=1}^n A_j e_j - \sum_{j=n+1}^p A_j e_j.$$

Also, in the same paper, Rathie [4] showed that

$$(9) \quad \bar{I}(z) \sim z^c, \text{ where } c = \min_{1 \leq j \leq m} \left(\operatorname{Re} \left[\frac{b_j}{f_j} \right] \right).$$

In our present investigation, we shall require the following result [2].

$$(10) \quad \int_0^\pi (\sin \phi)^{c-1} e^{i\delta\phi} d\phi = \frac{\pi \Gamma(c) e^{\frac{1}{2}i\delta\pi}}{2^{c-1} \Gamma(\frac{c+\delta+1}{2}) \Gamma(\frac{c-\delta+1}{2})}$$

provided $\operatorname{Re}(c) > 0$.

2. Main result

The integral involving \bar{I} -function to be established in this note is the following.

Consider the integral

$$(11) \quad S = \int_0^\pi (\sin \phi)^{c-1} e^{-i\delta\phi} \bar{I}_{p,q}^{m,n} \left[z (\sin \phi)^\sigma e^{i\sigma\phi} \left| \begin{matrix} 1(a_j, e_j, A_j)_n, n+1(a_j, e_j, A_j)_p \\ 1(b_j, f_j, 1)_m, m+1(b_j, f_j, B_j)_q \end{matrix} \right. \right].$$

Expressing the \bar{I} -function with the help of its definition, we get

$$(12) \quad S = \int_0^\pi (\sin \phi)^{c-1} e^{-i\delta\phi} \frac{1}{2\pi i} \int_L \theta(s) [z (\sin \phi)^\sigma e^{i\alpha\phi}]^s ds d\phi.$$

On changing the order of integration, we get

$$(13) \quad S = \frac{1}{2\pi i} \int_L \theta(s) z^s ds \int_0^\pi (\sin \phi)^{\sigma s + c - 1} e^{-i\delta\phi + i\alpha\phi s} d\phi.$$

Using (10) we obtain,

$$(14) \quad S = \frac{1}{2\pi i} \int_L z^s \theta(s) \frac{\pi \Gamma(\sigma s + c) e^{\frac{1}{2}i(\alpha s - \delta)\pi}}{2^{\sigma s + c - 1} \Gamma(\frac{\sigma s + c + \alpha s - \delta + 1}{2}) \Gamma(\frac{\sigma s + c - \alpha s + \delta + 1}{2})} ds$$

which can also be written as

$$(15) \quad S = \frac{1}{2\pi i} \int_L z^s \theta(s) \frac{\Gamma(c + \sigma s) 2^{-\sigma s} e^{\frac{1}{2}i\pi\alpha s}}{\Gamma(\frac{c+\delta+1}{2} + \frac{(\sigma-\alpha)s}{2}) \Gamma(\frac{c-\delta+1}{2} + \frac{(\sigma+\alpha)s}{2})} ds \frac{\pi 2^{1-c}}{e^{\frac{1}{2}i\pi\delta}}.$$

This represents the constant $\frac{\pi 2^{1-c}}{e^{\frac{1}{2}i\pi\delta}}$ multiplied by an \bar{I} -function with the argument $z 2^{-\sigma} e^{\frac{1}{2}i\pi\alpha}$.

Let $P = \frac{\pi 2^{1-c}}{e^{\frac{1}{2}i\pi\delta}}$ and $Q = z 2^{-\sigma} e^{\frac{1}{2}i\pi\alpha}$.

If σ , $\sigma - \alpha$ or $\sigma + \alpha$ is zero, the corresponding gamma function inside the integral is free from the variable of integration s . Then the gamma function is a constant and can be taken outside the integral sign. For example when $\sigma = 0$, the integral is $P \Gamma(c)$ multiplied by an \bar{I} -function with argument $z e^{\frac{1}{2}i\pi\alpha}$.

If $\sigma > 0$, $\Gamma(c + \sigma s)$ will be taken with $\prod_{j=1}^n \Gamma^{A_j}(1 - a_j + \alpha_j s)$ of $\theta(s)$ and hence the value of n (and of p) will be increased by 1. The corresponding pair of parameters in the \bar{I} -function is obtained by comparing $c + \sigma s$ with $1 - a_j + \alpha_j s$.

If $\sigma < 0$, $\Gamma(c + \sigma s)$ will be taken with $\prod_{j=1}^m \Gamma^{B_j}(b_j - \beta_j s)$ of $\theta(s)$ and hence the value of m (and of q) will be increased by 1. The corresponding pair of parameters in the \bar{I} -function is obtained by comparing $c + \sigma s$ with $b_j - \beta_j s$.

If $\frac{\sigma - \alpha}{2}$ or $\frac{\sigma + \alpha}{2}$ is positive, the gamma function in the denominator is taken with $\prod_{j=m+1}^q \Gamma^{B_j}(1 - b_j + \beta_j s)$ and hence q increases by 1.

If $\frac{\sigma - \alpha}{2}$ or $\frac{\sigma + \alpha}{2}$ is negative, the gamma function in the denominator is taken with $\prod_{j=n+1}^p \Gamma^{A_j}(a_j - \alpha_j s)$ and hence p increases by 1.

The following cases can be considered.

1. $\sigma = 0, \alpha > 0$.
2. $\sigma = 0, \alpha < 0$.
3. $\alpha = \sigma, \sigma > 0$.
4. $\alpha = \sigma, \sigma < 0$.
5. $\alpha = -\sigma, \sigma > 0$.
6. $\alpha = -\sigma, \sigma < 0$.
7. $\sigma > 0, \sigma + \alpha > 0, \sigma - \alpha > 0$.
8. $\alpha < 0 < \sigma, \sigma + \alpha < 0$.
9. $\alpha < \sigma < 0$.
10. $0 < \sigma < \alpha$.
11. $\sigma < 0 < \alpha, \sigma - \alpha > 0$.
12. $\sigma < 0, \sigma + \alpha < 0, \sigma - \alpha < 0$.

Special Cases:

(i) When $\sigma = 0, \alpha > 0$, (15) takes the following form.

$$\int_0^\pi (\sin \phi)^{c-1} e^{-i\delta\phi} \bar{I}_{p,q}^{m,n} \left[z e^{i\sigma\phi} \left| \begin{matrix} 1(a_j, e_j, A_j)_n, n+1(a_j, e_j, A_j)_p \\ 1(b_j, f_j, 1)_m, m+1(b_j, f_j, B_j)_q \end{matrix} \right. \right] \\ = P \Gamma(c) \bar{I}_{p+1,q+1}^{m,n} \left[z e^{\frac{i\pi\alpha}{2}} \left| \begin{matrix} 1(a_j, e_j, A_j)_p, (\frac{c+\delta+1}{2}, \frac{\alpha}{2}, 1) \\ 1(b_j, f_j, 1)_m, m+1(b_j, f_j, B_j)_q (\frac{1-c+\delta}{2}, \frac{\alpha}{2}, 1) \end{matrix} \right. \right].$$

(ii) When $\sigma = 0, \alpha < 0$, (15) takes the following form.

$$\int_0^\pi (\sin \phi)^{c-1} e^{-i\delta\phi} \bar{I}_{p,q}^{m,n} \left[z e^{i\sigma\phi} \left| \begin{matrix} 1(a_j, e_j, A_j)_n, n+1(a_j, e_j, A_j)_p \\ 1(b_j, f_j, 1)_m, m+1(b_j, f_j, B_j)_q \end{matrix} \right. \right] \\ = P \Gamma(c) \bar{I}_{p+1,q+1}^{m,n} \left[z e^{\frac{i\pi\alpha}{2}} \left| \begin{matrix} 1(a_j, e_j, A_j)_p, (\frac{c-\delta+1}{2}, \frac{-\alpha}{2}, 1) \\ 1(b_j, f_j, 1)_m, m+1(b_j, f_j, B_j)_q (\frac{1-c-\delta}{2}, \frac{-\alpha}{2}, 1) \end{matrix} \right. \right].$$

(iii) When $\alpha = \sigma > 0$, (15) takes the following form.

$$\int_0^\pi (\sin \phi)^{c-1} e^{-i\delta\phi} \bar{I}_{p,q}^{m,n} \left[z (\sin \phi)^\sigma e^{i\sigma\phi} \left| \begin{matrix} 1(a_j, e_j, A_j)_n, n+1(a_j, e_j, A_j)_p \\ 1(b_j, f_j, 1)_m, m+1(b_j, f_j, B_j)_q \end{matrix} \right. \right] \\ = \frac{P}{\Gamma(\frac{c+\delta+1}{2})} \bar{I}_{p+1,q+1}^{m,n+1} \left[z 2^{-\sigma} e^{\frac{i\pi\sigma}{2}} \left| \begin{matrix} (1-c, \sigma, 1), 1(a_j, e_j, A_j)_p \\ 1(b_j, f_j, 1)_m, m+1(b_j, f_j, B_j)_q, (\frac{1-c+\delta}{2}, \sigma, 1) \end{matrix} \right. \right].$$

(iv) When $\alpha = \sigma < 0$, (15) takes the following form.

$$\int_0^\pi (\sin \phi)^{c-1} e^{-i\delta\phi} \bar{I}_{p,q}^{m,n} \left[z (\sin \phi)^\sigma e^{i\sigma\phi} \left| \begin{matrix} 1(a_j, e_j, A_j)_n, n+1(a_j, e_j, A_j)_p \\ 1(b_j, f_j, 1)_m, m+1(b_j, f_j, B_j)_q \end{matrix} \right. \right]$$

$$= \frac{P}{\Gamma(\frac{c+\delta+1}{2})} \bar{I}_{p+1,q+1}^{m+1,n} \left[z 2^{-\sigma} e^{\frac{i\pi\sigma}{2}} \left| \begin{matrix} 1(a_j, e_j, A_j)_p, (\frac{1+c-\delta}{2}, -\sigma, 1) \\ (c, -\sigma, 1), {}_1(b_j, f_j, 1)_m, {}_{m+1}(b_j, f_j, B_j)_q \end{matrix} \right. \right].$$

(v) When $\alpha = -\sigma$, $\sigma > 0$, (15) takes the following form.

$$\int_0^\pi (\sin \phi)^{c-1} e^{-i\delta\phi} \bar{I}_{p,q}^{m,n} \left[z (\sin \phi)^\sigma e^{i\sigma\phi} \left| \begin{matrix} 1(a_j, e_j, A_j)_n, {}_{n+1}(a_j, e_j, A_j)_p \\ {}_1(b_j, f_j, 1)_m, {}_{m+1}(b_j, f_j, B_j)_q \end{matrix} \right. \right] \\ = \frac{P}{\Gamma(\frac{c-\delta+1}{2})} \bar{I}_{p+1,q+1}^{m,n+1} \left[z 2^{-\sigma} e^{\frac{-i\pi\sigma}{2}} \left| \begin{matrix} (1-c, \sigma, 1), {}_1(a_j, e_j, A_j)_p \\ {}_1(b_j, f_j, 1)_m, {}_{m+1}(b_j, f_j, B_j)_q, (\frac{1-c-\delta}{2}, \sigma, 1) \end{matrix} \right. \right].$$

(vi) When $\alpha = -\sigma$, $\sigma < 0$, (15) takes the following form.

$$\int_0^\pi (\sin \phi)^{c-1} e^{-i\delta\phi} \bar{I}_{p,q}^{m,n} \left[z (\sin \phi)^\sigma e^{i\sigma\phi} \left| \begin{matrix} 1(a_j, e_j, A_j)_n, {}_{n+1}(a_j, e_j, A_j)_p \\ {}_1(b_j, f_j, 1)_m, {}_{m+1}(b_j, f_j, B_j)_q \end{matrix} \right. \right] \\ = \frac{P}{\Gamma(\frac{c-\delta+1}{2})} \bar{I}_{p+1,q+1}^{m+1,n} \left[z 2^{-\sigma} e^{\frac{-i\pi\sigma}{2}} \left| \begin{matrix} 1(a_j, e_j, A_j)_p, (\frac{1+c+\delta}{2}, -\sigma, 1) \\ (c, -\sigma, 1), {}_1(b_j, f_j, 1)_m, {}_{m+1}(b_j, f_j, B_j)_q \end{matrix} \right. \right].$$

(vii) When $\alpha > 0$, $\alpha + \sigma > 0$, $\sigma - \alpha > 0$, (15) takes the following form.

$$\int_0^\pi (\sin \phi)^{c-1} e^{-i\delta\phi} \bar{I}_{p,q}^{m,n} \left[z (\sin \phi)^\sigma e^{i\sigma\phi} \left| \begin{matrix} 1(a_j, e_j, A_j)_n, {}_{n+1}(a_j, e_j, A_j)_p \\ {}_1(b_j, f_j, 1)_m, {}_{m+1}(b_j, f_j, B_j)_q \end{matrix} \right. \right] \\ = P \bar{I}_{p+1,q+2}^{m,n+1} \left[z 2^{-\sigma} e^{\frac{i\pi\sigma}{2}} \left| \begin{matrix} (1-c, \sigma, 1), {}_1(a_j, e_j, A_j)_p \\ {}_1(b_j, f_j, 1)_m, {}_{m+1}(b_j, f_j, B_j)_q, (\frac{1-c+\delta}{2}, \frac{\sigma+\alpha}{2}, 1), (\frac{1-c-\delta}{2}, \frac{\sigma-\alpha}{2}, 1) \end{matrix} \right. \right].$$

(viii) When $\alpha < 0 < \sigma$, $\alpha + \sigma < 0$, (15) takes the following form.

$$\int_0^\pi (\sin \phi)^{c-1} e^{-i\delta\phi} \bar{I}_{p,q}^{m,n} \left[z (\sin \phi)^\sigma e^{i\sigma\phi} \left| \begin{matrix} 1(a_j, e_j, A_j)_n, {}_{n+1}(a_j, e_j, A_j)_p \\ {}_1(b_j, f_j, 1)_m, {}_{m+1}(b_j, f_j, B_j)_q \end{matrix} \right. \right] \\ = P \bar{I}_{p+2,q+1}^{m,n+1} \left[z 2^{-\sigma} e^{\frac{i\pi\sigma}{2}} \left| \begin{matrix} (1-c, \sigma, 1), {}_1(a_j, e_j, A_j)_p, (\frac{1+c-\delta}{2}, \frac{-(\sigma+\alpha)}{2}, 1) \\ {}_1(b_j, f_j, 1)_m, {}_{m+1}(b_j, f_j, B_j)_q, (\frac{1-c-\delta}{2}, \frac{\sigma-\alpha}{2}, 1) \end{matrix} \right. \right].$$

(ix) When $\alpha < \sigma < 0$, (15) takes the following form.

$$\int_0^\pi (\sin \phi)^{c-1} e^{-i\delta\phi} \bar{I}_{p,q}^{m,n} \left[z (\sin \phi)^\sigma e^{i\sigma\phi} \left| \begin{matrix} 1(a_j, e_j, A_j)_n, {}_{n+1}(a_j, e_j, A_j)_p \\ {}_1(b_j, f_j, 1)_m, {}_{m+1}(b_j, f_j, B_j)_q \end{matrix} \right. \right] \\ = P \bar{I}_{p+1,q+2}^{m+1,n} \left[z 2^{-\sigma} e^{\frac{i\pi\sigma}{2}} \left| \begin{matrix} 1(a_j, e_j, A_j)_p, (\frac{1+c-\delta}{2}, \frac{-(\sigma+\alpha)}{2}, 1) \\ (c, -\sigma, 1), {}_1(b_j, f_j, 1)_m, {}_{m+1}(b_j, f_j, B_j)_q, (\frac{1-c-\delta}{2}, \frac{\sigma-\alpha}{2}, 1) \end{matrix} \right. \right].$$

(x) When $0 < \sigma < \alpha$, (15) takes the following form.

$$\int_0^\pi (\sin \phi)^{c-1} e^{-i\delta\phi} \bar{I}_{p,q}^{m,n} \left[z (\sin \phi)^\sigma e^{i\sigma\phi} \left| \begin{matrix} 1(a_j, e_j, A_j)_n, {}_{n+1}(a_j, e_j, A_j)_p \\ {}_1(b_j, f_j, 1)_m, {}_{m+1}(b_j, f_j, B_j)_q \end{matrix} \right. \right] \\ = P \bar{I}_{p+2,q+1}^{m,n+1} \left[z 2^{-\sigma} e^{\frac{i\pi\sigma}{2}} \left| \begin{matrix} (1-c, \sigma, 1), {}_1(a_j, e_j, A_j)_p, (\frac{1+c+\delta}{2}, \frac{-(\sigma-\alpha)}{2}, 1) \\ {}_1(b_j, f_j, 1)_m, {}_{m+1}(b_j, f_j, B_j)_q, (\frac{1-c+\delta}{2}, \frac{\sigma+\alpha}{2}, 1) \end{matrix} \right. \right].$$

(xi) When $\sigma < 0 < \alpha$, $\alpha + \sigma > 0$, (15) takes the following form.

$$\int_0^\pi (\sin \phi)^{c-1} e^{-i\delta\phi} \bar{I}_{p,q}^{m,n} \left[z (\sin \phi)^\sigma e^{i\sigma\phi} \left| \begin{matrix} 1(a_j, e_j, A_j)_n, {}_{n+1}(a_j, e_j, A_j)_p \\ {}_1(b_j, f_j, 1)_m, {}_{m+1}(b_j, f_j, B_j)_q \end{matrix} \right. \right] \\ = P \bar{I}_{p+1,q+2}^{m+1,n} \left[z 2^{-\sigma} e^{\frac{i\pi\sigma}{2}} \left| \begin{matrix} 1(a_j, e_j, A_j)_p, (\frac{1+c+\delta}{2}, \frac{\sigma-\alpha}{2}, 1) \\ (c, -\sigma, 1), {}_1(b_j, f_j, 1)_m, {}_{m+1}(b_j, f_j, B_j)_q, (\frac{1-c+\delta}{2}, \frac{\sigma+\alpha}{2}, 1) \end{matrix} \right. \right].$$

(xii) When $\sigma < 0$, $\sigma + \alpha < 0$, $\sigma - \alpha < 0$, (15) takes the following form.

$$\int_0^\pi (\sin \phi)^{c-1} e^{-i\delta\phi} \bar{I}_{p,q}^{m,n} \left[z (\sin \phi)^\sigma e^{i\sigma\phi} \left| \begin{matrix} 1(a_j, e_j, A_j)_n, & n+1(a_j, e_j, A_j)_p \\ 1(b_j, f_j, 1)_m, & m+1(b_j, f_j, B_j)_q \end{matrix} \right. \right]$$

$$= P \bar{I}_{p+2,q+1}^{m+1,n} \left[z 2^{-\sigma} e^{\frac{i\pi\sigma}{2}} \left| \begin{matrix} 1(a_j, e_j, A_j)_p, & (\frac{1+c-\delta}{2}, \frac{-(\sigma+\alpha)}{2}, 1), & (\frac{1+c+\delta}{2}, \frac{-(\sigma+\alpha)}{2}, 1) \\ (c, -\sigma, 1), & 1(b_j, f_j, 1)_m, & m+1(b_j, f_j, B_j)_q \end{matrix} \right. \right].$$

3. Conclusion

Since \bar{I} -function of one variable is of very general character and includes H-function, G-function and a large number of other elementary functions as special cases, so by specializing the parameters therein, we can obtain a large number of results involving elementary functions. But we shall not include due to lack of space.

References

- [1] B. L. J. Braaksma, *Asymptotic expansions and analytic continuations for a class of Barnes-integrals*, *Compositio Math.* **15** (1964), 239–341.
- [2] A. Erdelyi, *Higher Transcendental Functions*, McGraw-Hill Book Company, New York, 1953.
- [3] C. Fox, *The G and H functions as symmetrical Fourier kernels*, *Trans. Amer. Math. Soc.* **98** (1961), 395–429. <https://doi.org/10.2307/1993339>
- [4] A. K. Rathie, *A new generalization of generalized hypergeometric functions*, *Matematiche (Catania)* **52** (1997), no. 2, 297–310.

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