

HOMOTOPY RESULTS FOR THE BETTER ADMISSIBLE CHANDRABHAN TYPE MULTIMAPS

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ABSTRACT. First, we generalize homotopy results of O'Regan [6] for Mönch type multimaps to Chandrabhan type multimaps. Second, we show that the better admissible Chandrabhan type multimaps have fixed point properties whenever their ranges are Klee approximable. Finally, we give examples of essential maps for various class of multimaps including Φ -condensing multimaps.

1. Introduction and Preliminaries

O'Regan [6] presented homotopy results for the better admissible Mönch type multimaps on Hausdorff topological vector spaces. From this, generalized Leray–Schauder alternatives for some general classes of maps were obtained, which contain the well known Leray–Schauder principles in [2, 5, 7]. Park introduced the better admissible class in [8, 9] and Dhage [3] introduced Chandrabhan multimaps which generalizes Mönch type multimaps.

In Section 2, we modify the definition of Chandrabhan type multimaps and generalize homotopy results of [2, 6] to Chandrabhan type multimaps. In Section 3, we show that the better admissible Chandrabhan type multimaps have fixed point properties whenever their ranges are Klee approximable. Finally, we give examples of essential multimaps for various class of multimaps including Φ -condensing multimaps. Our new results improve and extend theorems in [2, 6, 10].

A *multimap* (or simply, a *map*) $F : X \multimap Y$ is a function from a set X into the power set of Y ; that is, a function with the *values* $F(x) \subset Y$ for all $x \in X$. For $A \subset X$, let $F(A) := \bigcup\{F(x) \mid x \in A\}$. Throughout

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this paper, we assume that multimaps have nonempty values otherwise explicitly stated or obvious from the context. The graph of F is denoted by $\text{Gr}F$.

A t.v.s. means a Hausdorff topological vector space.

Let E be a t.v.s., X be a nonempty subset of E , and Y be a topological space. A *polytope* P in a subset X of a t.v.s. E is a nonempty compact convex subset of X contained in a finite dimensional subspace of E .

The *better admissible class* \mathfrak{B} of maps from X into Y is defined as follows:

$F \in \mathfrak{B}(X, Y) \iff F : X \multimap Y$ is a map such that, for each polytope P in X and for any continuous function $\phi : F(P) \rightarrow P$, the composition $\phi \circ F|_P : P \multimap P$ has a fixed point.

$F \in \mathfrak{B}^\kappa(X, Y) \iff F : X \multimap Y$ is a map such that, for any compact, convex subset K of X , there exists a closed map $G \in \mathfrak{B}(K, Y)$ with $G(x) \subset F(x)$ for each $x \in K$.

Clearly $\mathfrak{B}(X, Y) \subset \mathfrak{B}^\kappa(X, Y)$.

$F \in \mathcal{D}(X, Y) \iff F \in \mathfrak{B}^\kappa(X, Y)$ is a closed map (i.e., $\text{Gr}F$ is closed), takes compact sets into relatively compact sets, and satisfies one of the following conditions for a compact subset B of X :

- (C) if $A \subset X \cap \overline{\text{co}}(B \cup F(A))$, then \overline{A} is compact, or
- (CC) if $A \subset X \cap \text{co}(B \cup F(A))$, then \overline{A} is compact.

In a Banach space, Dhage [3] called a closed valued map as a *Chandrabhan type* map if it satisfies the condition (C) with a countable set B and specially called it a *Mönch type* map when B is a point in X .

Let C be a closed, convex subset of E , B be a compact subset of C and U be an open subset of C containing B .

$F \in \mathcal{D}_{\partial U}(\overline{U}, C) \iff F \in \mathcal{D}(\overline{U}, C)$ with $x \notin F(x)$ for all $x \in \partial U$ where ∂U denotes the boundary of U in C .

A map $F \in \mathcal{D}_{\partial U}(\overline{U}, C)$ is called *essential* in $\mathcal{D}_{\partial U}(\overline{U}, C)$ if for every $G \in \mathcal{D}_{\partial U}(\overline{U}, C)$ with $G|_{\partial U} = F|_{\partial U}$, there exists $x \in U$ with $x \in G(x)$. The essentiality is introduced in [1, 6].

Let E be a t.v.s. and Z be a lattice with a least element, which is denoted by 0. A function $\Phi : E \multimap Z$ is called a *measure of precompactness* on E provided that the following conditions hold for any $A, B \subset E$:

- (1) A is relative compact iff $\Phi(A) = 0$;
- (2) $\Phi(A \cup B) = \max\{\Phi(A), \Phi(B)\}$; and
- (3) $\Phi(\overline{\text{co}}A) = \Phi(A)$.

It follows that $A \subset B$ implies $\Phi(A) \subset \Phi(B)$.

for all $x \subset E$, a map $T : X \dashrightarrow E$ is said to be Φ -condensing provided that if $A \subset X$ and $\Phi(A) \leq \Phi(T(A))$, then A is relatively compact; that is, $\Phi(A) = 0$. For details, see [7].

Note that a Φ -condensing map satisfies the conditions (C) and (CC) for any compact subset B of X .

From now on, we assume that Y is a topological space, E is a t.v.s., X is a nonempty subset of E , C is a closed, convex subset of E , B is a compact subset of C and U is an open subset of C containing B .

2. Essential maps for the better admissible Chandrabhan type maps

THEOREM 2.1. *Suppose $F \in \mathcal{D}(\bar{U}, C)$ and assume the following conditions are satisfied:*

- (1) *the constant map $S : \bar{U} \dashrightarrow C$ defined by $S(x) = B$ for all $x \in \bar{U}$ is essential in $\mathcal{D}_{\partial U}(\bar{U}, C)$;*
- (2) *$x \notin \lambda F(x) + (1 - \lambda)B$ for all $x \in \partial U$ and $\lambda \in (0, 1]$; and*
- (3) *for any continuous function $\mu : \bar{U} \rightarrow [0, 1]$ with $\mu(\partial U) = 0$ and for any map $H \in \mathcal{D}(\bar{U}, C)$ with $H|_{\partial U} = F|_{\partial U}$, the map $R_\mu : \bar{U} \dashrightarrow C$ defined by $R_\mu(x) = \mu(x)H(x) + (1 - \mu(x))B$ for all $x \in \bar{U}$ is in $\mathfrak{B}^\kappa(\bar{U}, C)$.*

Then F is essential in $\mathcal{D}_{\partial U}(\bar{U}, C)$.

Proof. By (2), $F \in \mathcal{D}_{\partial U}(\bar{U}, C)$. Let $H \in \mathcal{D}_{\partial U}(\bar{U}, C)$ with $H|_{\partial U} = F|_{\partial U}$. It is enough to show H has a fixed point in U . Consider

$$D = \{x \in \bar{U} : x \in tH(x) + (1 - t)B \text{ for some } t \in [0, 1]\}.$$

Note $B \subset D$. Also D is closed (in C) since H is a closed map and B is compact. Furthermore D is compact, since $D \subset \text{co}(H(D) \cup B)$, and H satisfies condition (C) or (CC) for B . In addition $D \cap \partial U = \emptyset$, otherwise if $x \in D \cap \partial U$, then $F(x) = H(x)$ since $H|_{\partial U} = F|_{\partial U}$. So $x \in tF(x) + (1 - t)B$ and $x \in B$ by condition (2), but $B \cap \partial U = \emptyset$.

Because a t.v.s. is completely regular, there exists a continuous $\mu : \bar{U} \rightarrow [0, 1]$ with $\mu(\partial U) = 0$ and $\mu(D) = 1$.

Define a map $R_\mu : \bar{U} \dashrightarrow C$ by $R_\mu(x) = \mu(x)H(x) + (1 - \mu(x))B$ for all $x \in \bar{U}$, then $R_\mu \in \mathfrak{B}^\kappa(\bar{U}, C)$ by condition (3). Also R_μ is a closed map. Moreover, R_μ takes compact sets into relatively compact sets. Indeed, let A be a compact subset of \bar{U} and (y_α) be a net in $R_\mu(A)$, i.e., $y_\alpha = \mu(x_\alpha)z_\alpha + (1 - \mu(x_\alpha))b_\alpha$ for some $z_\alpha \in \overline{H(x_\alpha)}$ with $x_\alpha \in A$ and $b_\alpha \in B$. Then the compactness of A , B and $\overline{H(A)}$ guarantees that

there exist $x \in A$, $b \in B$ and $z \in \overline{H(A)}$ with $x_\alpha \rightarrow x$, $b_\alpha \rightarrow b$ and $z_\alpha \rightarrow z$ without loss of generality. Since $\text{Gr}H$ and $\text{Gr}R_\mu$ are closed, we have $z \in H(x)$ and $y = \mu(x)z + (1 - \mu(x))b \in R_\mu(x)$. Thus $y_\alpha \rightarrow y$ and $y \in R_\mu(A)$. Notice $R_\mu|_{\partial U} = B$.

In addition, we verify that R_μ satisfies condition (C) for B if H satisfies condition (C) for B . To see this, let $A \subset \overline{U}$ and $A \subset \overline{\text{co}(B \cup R_\mu(A))}$. Then $R_\mu(A) \subset \overline{\text{co}(B \cup H(A))}$ yields

$$A \subset \overline{\text{co}(B \cup R_\mu(A))} \subset \overline{\text{co}(\text{co}(B \cup H(A)))} = \overline{\text{co}(B \cup H(A))}.$$

Since H satisfies condition (C) for B , \overline{A} is compact.

By the same argument, it can be shown that R_μ satisfies condition (CC) if H satisfies condition (CC) for B .

Hence $R_\mu \in \mathcal{D}_{\partial U}(\overline{U}, C)$ with $R_\mu|_{\partial U} = B$ and $D \cap \partial U = \emptyset$. Now condition (1) implies that there exists $x \in U$ with $x \in R_\mu(x)$. Thus $x \in D$ and so $\mu(x) = 1$, i.e., $x \in H(x)$. \square

REMARK 2.2. Theorem 2.1 and Theorem 2.4 in [6] are special cases in which $B = \{x_0\}$ for some $x_0 \in U$ in our Theorem 2.1.

$F \in \mathcal{M}(\overline{U}, C) \iff F : \overline{U} \dashrightarrow C$ is a nonempty, convex, compact valued upper semicontinuous map, and satisfies (C) or (CC) for a compact subset B of X .

$$F \in \mathcal{M}_{\partial U}(\overline{U}, C) \iff F \in \mathcal{M}(\overline{U}, C) \text{ with } x \notin F(x) \text{ for all } x \in \partial U.$$

A map $F \in \mathcal{M}_{\partial U}(\overline{U}, C)$ is called *essential* in $\mathcal{M}_{\partial U}(\overline{U}, C)$ if for every $G \in \mathcal{M}_{\partial U}(\overline{U}, C)$ with $G|_{\partial U} = F|_{\partial U}$, there exists $x \in U$ with $x \in G(x)$.

Note that a nonempty, convex, compact valued upper semicontinuous map $F : \overline{U} \dashrightarrow C$ is in $\mathfrak{B}(X, Y)$ [4], F is also a closed map and F takes compact sets into compact sets. Hence $\mathcal{M}(\overline{U}, C) \subset \mathcal{D}(\overline{U}, C)$ and any $F \in \mathcal{M}(\overline{U}, C)$ satisfies (3) in Theorem 2.1.

Therefore we obtain the following corollary which generalizes Theorem 2.6 in [2].

COROLLARY 2.3. Suppose $F \in \mathcal{M}(\overline{U}, C)$ and assume the following conditions are satisfied:

- (1) the constant map $S : \overline{U} \dashrightarrow C$ defined by $S(x) = B$ for all $x \in \overline{U}$ is essential in $\mathcal{M}_{\partial U}(\overline{U}, C)$; and
- (2) $x \notin \lambda F(x) + (1 - \lambda)B$ for all $x \in \partial U$ and $\lambda \in (0, 1]$.

Then F is essential in $\mathcal{M}_{\partial U}(\overline{U}, C)$.

3. Examples of essential maps

THEOREM 3.1. *Suppose that for any closed, convex subset K of C and for any map $G \in \mathcal{D}(K, K)$, G has a fixed point in K . For any map $\theta \in \mathcal{D}_{\partial U}(\bar{U}, C)$ with $\theta|_{\partial U} = B$ and $Q = \overline{\text{co}}(\theta(U) \cup B)$, assume that the following conditions are satisfied:*

- (4) *for θ satisfying condition (C) [or (CC)] for B and for any set $A \subset Q$ with $A \subset \overline{\text{co}}(B \cup \theta(A \cap U))$ [or $A \subset \text{co}(B \cup \theta(A \cap U))$], respectively], if $\theta(\overline{A \cap U})$ is relatively compact, then \overline{A} is compact; and*
- (5) *the map $J : Q \rightarrow Q$ defined by*

$$J(x) = \begin{cases} \theta(x), & x \in \overline{U} \cap Q \\ B, & \text{otherwise} \end{cases}$$

is in $\mathfrak{B}^k(Q, Q)$.

Then the constant map $S : \bar{U} \rightarrow C$ defined by $S(x) = B$ for all $x \in \bar{U}$ is essential in $\mathcal{D}_{\partial U}(\bar{U}, C)$.

Proof. Let $\theta \in \mathcal{D}_{\partial U}(\bar{U}, C)$ with $\theta|_{\partial U} = B$. We must show that there exists an $x \in U$ with $x \in \theta(x)$. To do this, we claim $J \in \mathcal{D}(Q, Q)$.

By condition (5), $J \in \mathfrak{B}^k(Q, Q)$. Note J is a closed map and J takes compact sets into relatively compact sets.

Next we assert that J satisfies condition (C) if θ satisfies condition (C) for B . To see this, notice if $A \subset Q$ with $A \subset \overline{\text{co}}(B \cup J(A))$, then

$$(*) \quad A \subset \overline{\text{co}}(B \cup \theta(A \cap U)),$$

since $\theta|_{\partial U} = B$. Thus $A \cap U \subset \overline{\text{co}}(B \cup \theta(A \cap U))$, and since θ satisfies condition (C) for B , we have that $\overline{A \cap U}$ is compact, and so $\theta(\overline{A \cap U})$ relatively compact. This, together with conditions (4) and (*), yields that \overline{A} is compact. Therefore, J satisfies condition (C) for B .

By the same argument, we can show that J satisfies condition (CC) if θ satisfies condition (CC) for B .

Therefore $J \in \mathcal{D}(Q, Q)$. Since Q is a closed convex subset of C , there exists $x \in Q$ with $x \in J(x)$ by the assumption. If $x \notin U$, we have $x \in J(x) = B$, which is a contradiction since $B \subset U$. Thus $x \in U$ and $x \in J(x) = \theta(x)$. □

REMARKS 3.2. 1. Theorem 3.1 generalizes Theorem 2.2 and Theorem 2.5 in [6].

2. As mentioned in [6], condition (4) holds if one of the following conditions is satisfied:

- 1) θ is Φ -condensing, since $\Phi(A) \leq \Phi(\theta(A \cap U)) \leq \Phi(\overline{\theta(A \cap U)}) = 0$;
- 2) $\overline{\text{co}}(K)$ is compact for any compact subset K of Q .

Let X be a subset of a t.v.s. E and \mathcal{V} denotes a fundamental system of open neighborhoods of the origin 0 of E . A compact subset K of X is said to be *Klee approximable* in X if for any $V \in \mathcal{V}$, there exists a continuous function $h : K \rightarrow X$ such that $x - h(x) \in V$ for all $x \in K$ and $h(K)$ is contained in a polytope in X .

The followings are examples of Klee approximable sets:

- (1) Any polytope in a subset of a t.v.s.;
- (2) Any compact subset K of a convex subset X in a locally convex t.v.s.;
- (3) Any compact subset K whenever $\text{co}K$ is a Klee approximable subset of X .

For details, see [10]. Note that any compact subset of a Klee approximable set is also Klee approximable.

The following proposition is Corollary 4.2 in [10]:

PROPOSITION 3.3. *Let X be a subset of a t.v.s. E and $F \in \mathfrak{B}(X, X)$ be a compact closed multimap. If $\overline{F(X)}$ is a Klee approximable subset of X , then F has a fixed point.*

The compactness of F in Proposition 3.3 is relaxed as follows:

THEOREM 3.4. *Let X be a subset of a t.v.s. E , B be a compact subset of X and $F \in \mathfrak{B}^\kappa(X, X)$ be a closed multimap satisfying condition (C) or condition (CC) for B . If $\overline{F(X)}$ is a Klee approximable subset of X , then F has a fixed point.*

Proof. Case 1. F satisfies condition (C) for B :

Put $K_0 = \overline{\text{co}}(B)$, $K_{n+1} = \overline{\text{co}}(B \cup F(K_n))$ for each $n = 0, 1, 2, \dots$ and $K = \bigcup_{n=0}^\infty K_n$. By induction, $K_0 \subseteq K_1 \subseteq \dots \subseteq K_n \subseteq K_{n+1} \dots$.

We can show that $K = \overline{\text{co}}(B \cup F(K))$. For each n , $\overline{\text{co}}(B \cup F(K_n)) \subseteq \overline{\text{co}}(B \cup F(K))$, so $K = \bigcup_{n=0}^\infty \overline{\text{co}}(B \cup F(K_n)) \subseteq \overline{\text{co}}(B \cup F(K))$. On the other hand, K is convex, since K_n is convex for each $n = 0, 1, 2, \dots$. As K contains B and $\bigcup_{n=0}^\infty F(K_n) = F(K)$, $\overline{\text{co}}(B \cup F(K)) \subseteq K$.

By condition (C) for B , K is compact. Define a map $G : K \multimap K$ by $G(x) = F(x) \cap K$. Since $F(K) \subset K$, $G(x) \neq \emptyset$ for all $x \in K$. Note that $G \in \mathfrak{B}^\kappa(K, K)$. As K is compact and convex, there exists a closed map $H \in \mathfrak{B}(K, K)$ such that $\overline{H(x)} \subset G(x)$ for each $x \in K$. Note that H is a compact multimap and $\overline{H(K)}$ is a Klee approximable

subset of X . Thus Proposition 3.3 guarantees the existence of a point $x \in H(x) \subset G(x) \subset F(x)$.

Case 2. F satisfies condition (CC) for B :

Put $K_0 = \text{co}(B)$, $K_{n+1} = \text{co}(B \cup F(K_n))$ for each $n = 0, 1, 2, \dots$ and $K = \bigcup_{n=0}^{\infty} K_n$. The same argument as above yields $K = \text{co}(B \cup F(K))$ and \overline{K} is compact.

Define a map $G : \overline{K} \rightarrow \overline{K}$ by $G(x) = F(x) \cap \overline{K}$. Then $G(x) \neq \emptyset$, since \overline{K} is compact and $F|_{\overline{K}}$ is closed. By the same argument, there exists a compact closed map $H \in \mathfrak{B}(\overline{K}, \overline{K})$ such that $H(x) \subset G(x)$ for each $x \in \overline{K}$. Note also that H satisfies all the conditions of Proposition 3.3.

□

As mentioned in Remark 3.2, Φ -condensing maps satisfy the conditions (C) and (CC) for any compact subset B of X , so the following corollary is obtained:

COROLLARY 3.5. *Let X be a subset of a t.v.s. E and $F \in \mathfrak{B}^{\kappa}(X, X)$ be a Φ -condensing closed multimap. If $\overline{F(X)}$ is a Klee approximable subset of X , then F has a fixed point.*

$F \in \mathcal{DK}(X, Y) \iff F \in \mathcal{D}(X, Y)$ and $\overline{F(X)}$ is a Klee approximable.

$F \in \mathcal{DK}_{\partial U}(\overline{U}, C) \iff F \in \mathcal{DK}(\overline{U}, C)$ with $x \notin F(x)$ for all $x \in \partial U$.

A map $F \in \mathcal{DK}_{\partial U}(\overline{U}, C)$ is called *essential* in $\mathcal{DK}_{\partial U}(\overline{U}, C)$ if for every $G \in \mathcal{DK}_{\partial U}(\overline{U}, C)$ with $G|_{\partial U} = F|_{\partial U}$, there exists $x \in U$ with $x \in G(x)$.

From Theorem 3.1 and Theorem 3.4, we obtain the following theorem:

THEOREM 3.6. *For any map $\theta \in \mathcal{DK}_{\partial U}(\overline{U}, C)$ with $\theta|_{\partial U} = B$ and $Q = \overline{\text{co}}(\theta(U) \cup B)$, assume the following conditions are satisfied;*

- (4) *for θ satisfying condtion (C) [or (CC)] for B and for any set $A \subset Q$ with $A \subset \overline{\text{co}}(B \cup \theta(A \cap U))$ [or $A \subset \text{co}(B \cup \theta(A \cap U))$], respectively], if $\theta(\overline{A \cap U})$ is relatively compact, then \overline{A} is compact; and*
- (5) *the map $J : Q \rightarrow Q$ defined by*

$$J(x) = \begin{cases} \theta(x), & x \in \overline{U} \cap Q \\ B, & \text{otherwise} \end{cases}$$

is in $\mathfrak{B}^{\kappa}(Q, Q)$.

Then the constant map $S : \overline{U} \rightarrow C$ defined by $S(x) = B$ for all $x \in \overline{U}$ is essential in $\mathcal{DK}_{\partial U}(\overline{U}, C)$.

To find essential maps for a class of Φ -condensing multimaps, we define the followings:

$F \in \mathcal{K}(X, Y) \iff F \in \mathfrak{B}^\kappa(X, Y)$ is a closed Φ -condensing map, takes compact sets into relatively compact sets and $\overline{F(X)}$ is a Klee approximable.

$F \in \mathcal{K}_{\partial U}(\overline{U}, C) \iff F \in \mathcal{K}(\overline{U}, C)$ with $x \notin F(x)$ for all $x \in \partial U$.

A map $F \in \mathcal{K}_{\partial U}(\overline{U}, C)$ is called *essential* in $\mathcal{K}_{\partial U}(\overline{U}, C)$ if for every $G \in \mathcal{K}_{\partial U}(\overline{U}, C)$ with $G|_{\partial U} = F|_{\partial U}$, there exists $x \in U$ with $x \in G(x)$.

Since any Φ -condensing map satisfies condition (C) and condition (CC) for any compact subset B of X , $\mathcal{K}(X, Y) \subset \mathcal{DK}(X, Y) \subset \mathcal{D}(X, Y)$. Any Φ -condensing map also satisfies (4) in Theorem 3.6. Therefore the following theorem holds:

THEOREM 3.7. *For any map $\theta \in \mathcal{K}_{\partial U}(\overline{U}, C)$ with $\theta|_{\partial U} = B$ and $Q = \overline{\text{co}}(\theta(U) \cup B)$, assume the map $J : Q \dashrightarrow Q$ defined by*

$$J(x) = \begin{cases} \theta(x), & x \in \overline{U} \cap Q \\ B, & \text{otherwise} \end{cases}$$

is in $\mathfrak{B}^\kappa(Q, Q)$. Then the constant map $S : \overline{U} \dashrightarrow C$ defined by $S(x) = B$ for all $x \in \overline{U}$ is essential in $\mathcal{K}_{\partial U}(\overline{U}, C)$.

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