Received: 1 February 2020 | Revised: 2 July 2020 | Accepted: 12 August 2020

DOI: 10.4218/etrij.2020-0031

ORIGINAL ARTICLE

ETRI Journal WILEY

Additional degree of freedom in phased-MIMO radar signal design using space-time codes

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In this paper, an additional degree of freedom in phased multi-input multi-output (phased-MIMO) radar with any arbitrary desired covariance matrix is proposed using space-time codes. By using the proposed method, any desired transmit covariance matrix in MIMO radar (phased-MIMO radars) can be realized by employing fully correlated base waveforms such as phased-array radars and simply extending them to different time slots with predesigned phases and amplitudes. In the proposed method, the transmit covariance matrix depends on the base waveform and space-time codes. For simplicity, a base waveform can be selected arbitrarily (ie, all base waveforms can be fully correlated, similar to phased-array radars). Therefore, any desired covariance matrix can be achieved by using a very simple phased-array structure and space-time code in the transmitter. The main advantage of the proposed scheme is that it does not require diverse uncorrelated waveforms. This considerably reduces transmitter hardware and software complexity and cost. One the receiver side, multiple signals can be analyzed jointly in the time and space domains to improve the signal-to-interference-plus-noise ratio.

KEYWORDS
MIMO radar, phased-MIMO, SINR, space-time codes, waveform design

1 | INTRODUCTION

Over the past decade, multi-input multi-output (MIMO) antenna systems have attracted significant attention from researchers in the communication systems field, particularly for radar systems [1–4]. Deploying multiple antennas for both transmitters and receivers can improve diversity, which enhances channel capacity and reduces bit error rates and signal fading. This method has been considered in many advanced radar applications in recent years [5–10]. MIMO radar systems can be divided into two main types. The first type is widely separated MIMO radar, where transmit and receive antennas are located far from each other (relative to the wavelength). This type of system enhances spatial diversity [11]. The second type is co-located MIMO radar, where transmit and receive antennas are close together [5]. This type of system has advantages in terms of interference rejection, improvement of parameter identifiability, and enhanced flexibility in beam pattern design. Our focus in this paper is co-located MIMO radar. Phased-MIMO radar, which was first proposed in [12], divides an antenna array into various sub-arrays. The waveforms in each sub-array are fully correlated, similar to phased radar arrays [1,13], and different sub-arrays have different stochastic properties [12,14]. The high processing load of multi-antenna radar systems has motivated the development of different transmit signal designs and receiver structures to reduce complexity and enhance processing efficiency. One such design is space-time coding (STC). The concept of STC was introduced by Foschini et al. [15]. In radar applications, STC was deployed by De Maio et al. [16] to improve the detection probability of MIMO radar. They demonstrated that a set of orthogonal space-time codes can improve the detection probability of multi-antenna radar systems. The waveform design problem has been discussed in several studies [17–21]. In [17–19], MIMO radar waveforms with constant moduli and similarity constraints were designed. A constant modulus reduces the cost of employing multiple radio amplifiers in a transmitter. Additionally, similarity constraints can be used to make a transmitted signal similar to a reference signal to realize the desired shape of a beam pattern. In [18], a robust waveform design for MIMO radar was proposed to improve target detectability and the worst-case signal-to-interference-plus-noise ratio (SINR) in signal-dependent interference. The authors proposed algorithms to optimize the SINR for unknown target locations and interferences by defining an uncertain region for steering matrices. They divided high-dimensional problems into multiple one-dimensional problems for which optimal solutions could be found in polynomial time. Additionally, space-time transmit codes and space-time receive filter designs were discussed in [22] and [23]. In [22], transmit and receive filters were jointly designed to maximize SINRs. In [23], an iterative method was designed to improve the worst-case SINR.

One of the major concerns in traditional phased-MIMO radar is realizing predetermined beam patterns and transmit covariance matrices (the transmit beam pattern trend depends on the transmit covariance matrix, which will be discussed in the following sections). Realizing any desired transmit covariance matrix requires generating uncorrelated waveforms, which inevitably require different types of signal generators with high costs. The STC signal design proposed in this paper resolves this issue.

MIMO radars transmit beams in space uniformly and can be used in multi-target scenarios at the cost of reduced SINRs. Phased-array radars can focus transmit beams to the angle of a single target, resulting in the best SINRs among multi-antenna radar systems. The problem of waveform design for co-located MIMO radars for multi-target scenarios in the presence of interference was studied in [20]. The authors proposed a sequential quasi-convex algorithm to optimize the minimum SINR at the receiver. Additionally, the authors of [21] proposed a waveform optimization method to design transmit and receive filters jointly to improve worst-case SINRs.

To achieve focused beams at several azimuth angles in multi-target scenarios, we must use correlated MIMO radar in which the waveforms of different transmit antennas are partially correlated. The problem of finding a proper transmit covariance matrix to match the desired beam pattern (beam pattern matching design) is discussed in detail in [24]. To realize any desired partially correlated covariance matrix, we must use different types of signal generators or phased-MIMO radars, which increases cost. Additionally, phased-MIMO radar requires uneven transmit power, which requires different types of radio amplifiers in transmitters. To overcome these issues, we propose a novel scheme that uses STC and simple phased-array radar simultaneously to realize partially correlated waveforms. STC content can be selected to achieve equal transmit power for each antenna.

We propose using STC as an additional degree of freedom in MIMO radar waveform signal design. Increasing diversity makes propagation paths more reliable under undesirable fading channel conditions because there are several ways for a signal to be transmitted and received. When STC is used, diversity can be achieved by adopting predesigned transmitted weighting factors in different space/time slots, rather than sending diverse base waveforms. Our main contribution is that STC is used in a transmitter with fully correlated base waveforms to realize predetermined desired transmit covariance matrices. In other words, phased-MIMO radar with any arbitrary transmit covariance matrix can be realized by combining phased-array fully correlated base waveforms with STC. At a receiver, several data strings are received in the space and time domains. We propose a joint space-time method to analyze received data matrices in a manner that improves SINR performance, whereas traditional methods only work on received vectors in one dimension. We attempt to use the maximum degrees of freedom to design waveforms in transmitters to achieve desired beam patterns and maximize the SINR to distinguish targets from interference at receivers.

Our study on MIMO radar systems yielded two main contributions, which can be summarized as follows.

- Any desired beam pattern and corresponding covariance matrix in a multi-target scenario can be realized by designing a proper STC matrix and deploying fully correlated base waveforms in transmitter-like phased-array radars. Unlike traditional correlated MIMO radar, this method does not require different types of signal generators in the transmitter, which reduces cost considerably. Additionally, the complexity of transmitters can be reduced. To achieve maximum power efficiency, the constant-envelope (CE) method can be used to deploy only a single type of power amplifier in all transmit nodes.
- On the receiver side, we receive two-dimensional data in both the space and time domains. Therefore, we must perform matrix analysis instead of vector analysis. Unlike traditional processing methods in which processing is performed only in the space domain, we propose a linear joint processing method in the space and time domains simultaneously to achieve the maximum possible SINR performance. SINR maximization is modeled as a semi-definite

programming problem. The proposed processing method and related optimization problem represent a general case of the minimum-variance distortionless response (MVDR) problem. Based on the fractional form in the proposed cost function, the Dinkelbach method is used to solve this problem.

The remainder of this paper is organized as follows. In Section 2, a model for co-located MIMO radar with multiple transmitter and receiver antenna elements with STC is described. Additionally, a covariance matrix with STC for MIMO radar and the related performance criteria are presented. Section 3 discusses how deploying STC results in extra degrees of freedom for MIMO radar signal design. In Section 4, the proposed method with CE conditions is discussed and the required conditions are satisfied. The receive filter design is discussed in Section 5. A multi-target scenario for the proposed design is discussed in Section 6. Finally, simulation results and concluding remarks are presented in Sections 7 and 8, respectively.

2 | System model

Consider a MIMO radar with M_t and M_r for transmit and receive antennas, respectively. The antennas are co-located with half-wavelength spacing on both the transmitter and receiver sides. Additionally, suppose that in addition to the target of interest, there are L interferers affecting the received signal.

As shown in Figure 1, $\mathbf{A}$ is an $M_t \times N$ STC matrix containing predesigned amplitudes and phases in different space-time slots and N is the extension in time (ie, number of pulses transmitted by an antenna). Therefore, the (i, j)-th element in the code matrix represents the amplitude and phase of the pulse transmitted by the i-th antenna ($i = 1, 2, \ldots, M_t$) in the j-th ($j = 1, 2, \ldots, N$) time extension. Suppose that the width of a single pulse in each duration is constant and equal to τ_p, and that the period between two consecutive pulses is T_p (ie, the pulse repetition). Therefore, the duration of the waveform emitted by each transmitter is $(N-1)T_p + \tau_p$. Suppose that there is point clutter in the form of L discrete interference objects. All equations presented below are baseband equations. Additionally, we consider that all targets and interfering objects are within the unambiguous range of the radar, meaning the waveform has a low pulse repetition frequency to avoid any range ambiguity. In the discrete-time signal model, the transmitted pulse in each period is modeled by K_s samples. In particular, by using STC, the discrete-time baseband signal corresponding to the n-th pulse train ($n = 1, 2, \ldots, N$) and k-th sample ($k = 1, 2, \ldots, K_s$) is an $M_t \times 1$ vector defined as

$$\mathbf{S}_{nk} = \mathbf{S}_k \odot \mathbf{a}_n, \quad (1)$$

where $\mathbf{S}_k$ has a size of $M_t \times 1$ and is the k-th sample vector of the transmitted base waveform ($k = 1, 2, \ldots, K_s$). Additionally, $\mathbf{a}_n$ is an $M_t \times 1$ column vector representing the n-th column of the space-time code matrix and $\odot$ is the Hadamard (element-wise) product operator. $\mathbf{S} = [\mathbf{S}_1, \mathbf{S}_2, \ldots, \mathbf{S}_K]$ is the transmit discrete-time base waveform and $\mathbf{A} = [\mathbf{a}_1, \mathbf{a}_2, \ldots, \mathbf{a}_N]_{M_t \times N}$ represents an $M_t \times N$ STC matrix.

As shown in Figure 1, a discrete-time baseband transmitted signal with a size of $M_t \times K_s N$ can be expressed as

$$\mathbf{X} = [\overbrace{\mathbf{a}_1, \ldots, \mathbf{a}_1}^{Ks \text{ times}}, \ldots, \overbrace{\mathbf{a}_N, \ldots, \mathbf{a}_N}^{Ks \text{ times}}] \odot [\overbrace{\mathbf{S}, \mathbf{S}, \ldots, \mathbf{S}}^{N \text{ times}}]. \quad (2)$$

The transmitted signal matrix $\mathbf{X}$ represented in (2) can be interpreted as N pulse trains each of which contains K_s samples. For the sake of simplicity and without loss of generality, we assume that each pulse is represented by a single sample($K_s = 1$).

On the receiver side, a received waveform is passed through a down-converter and matched filter. Therefore, the observed baseband signal at the receiver with a size of $M_t \times N$ can be expressed as [19]

$$\begin{aligned}\mathbf{Y} &= \beta_0 \mathbf{a}_R(\theta_0)\mathbf{a}_T^T(\theta_0)\mathbf{X} + \sum_{i=1}^{L} \beta_i \mathbf{a}_R(\theta_i)\mathbf{a}_T^T(\theta_i)\mathbf{X} + \mathbf{v} \\ &= \mathbf{Y}_0 + \mathbf{Y}_{int} + \mathbf{v},\end{aligned} \quad (3)$$

where θ_0 is the azimuth angle of the target of interest, θ_i is the angle of the i-th interference, and β_0 is the channel coefficient between the transmitter, target, and receiver (considering the effect of the radar cross-section (RCS) of the target). Additionally, β_i represents the channel and RCS effect related to the i-th interference. Suppose that a uniform linear array with half-wavelength spacing is used. Then, $\mathbf{a}_T(\theta)$ and $\mathbf{a}_R(\theta)$ are transmit and receive steering vectors that can be expressed as [1]

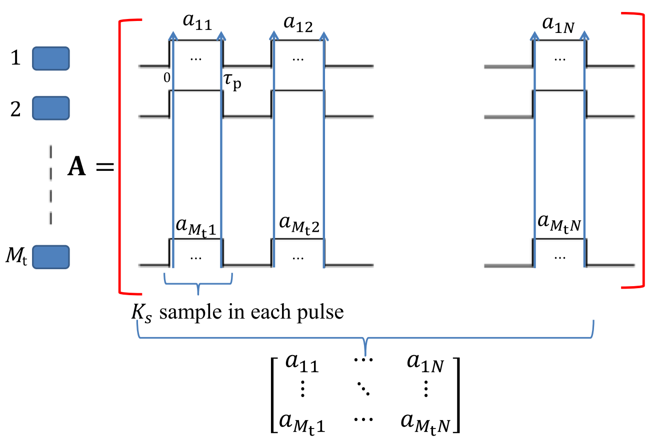

FIGURE 1 STC design for phased-MIMO radar

$$\begin{aligned} a_T(\theta) &= [1,\ e^{j\pi\sin(\theta)},\ e^{j2\pi\sin(\theta)},\ \ldots,\ e^{j(M_t-1)\pi\sin(\theta)}]^T, \\ a_R(\theta) &= [1,\ e^{j\pi\sin(\theta)},\ e^{j2\pi\sin(\theta)},\ \ldots,\ e^{j(M_r-1)\pi\sin(\theta)}]^T. \end{aligned} \quad (4)$$

Additionally, $\mathbf{Y}_0$ and $\mathbf{Y}_{\text{int}}$ are the received signals corresponding to the target of interest and interference term, respectively. $\mathbf{v}$ is a matrix of white Gaussian noise with zero mean and a covariance matrix $\sigma_v^2\mathbf{I}$. According to (3), we have N separate columns of received vectors that can be analyzed to improve the performance of MIMO radar. Joint processing in the space and time domains at the receiver will be detailed in the following sections. For each transmit/receive sensor pair, there is a forward transmit channel to the target and reverse receive channel from the target. These channels are modeled as lossless time delay and phase shift channels. According to (3), the channel between a transmitter and target is modeled as $\mathbf{a}_T(\theta)$, which has different phase shifts for each antenna element. Furthermore, the channel between a target and receiver is modeled as $\mathbf{a}_R(\theta)$. In this model, the phase shifts of different sub-channels are reflected by steering vectors. Therefore, channel state information is known for both transmitters and receivers.

Suppose that the desired transmit beam pattern is given. The corresponding covariance matrix can be achieved using the algorithms proposed in [24]. In next section, the transmit covariance matrix and its relationship with the STC matrix will be described.

3 | Proposed STC matrix design

Suppose that the covariance matrix of the transmit discrete-time base waveform is denoted as $\mathbf{R}$ (ie, $\mathbf{R} = \mathrm{E}[\mathbf{S}\mathbf{S}^H]$). Then, the transmit signal covariance matrix is denoted as $\mathbf{R}_{\text{new}}$ and can be expressed as

$$\begin{aligned} \mathbf{R}_{\text{new}} &= \mathrm{E}[\mathbf{X}\mathbf{X}^H] = \mathrm{E}[\mathbf{S}\mathbf{S}^H] \odot (\mathbf{A}\mathbf{A}^H) \\ &= \mathbf{R} \odot (\mathbf{A}\mathbf{A}^H). \end{aligned} \quad (5)$$

Therefore, the transmit covariance matrix depends on two parameters. The first parameter is the base waveform covariance matrix ($\mathbf{R}$), and the second parameter is the STC matrix ($\mathbf{A}$). In the proposed method, the STC matrix is designed achieve the desired covariance matrix and corresponding performance.

For any desired beam pattern or key performance indicator for MIMO radar, a covariance matrix can be designed using the beam pattern matching design presented in [24] or the references therein. One of the main contributions of this paper is that we can design any desired covariance matrix by using fully correlated waveforms and simply adjusting the STC elements. This means that phased-MIMO radar with any arbitrary covariance matrix can be realized by deploying a phased array and STC simultaneously. In other words, $\mathbf{R}$ can be set to all ones ($\mathbf{R} = [\mathbf{1}]_{M_t \times M_t}$), meaning the transmit covariance matrix depends solely on the transmit STC matrix. Therefore, (5) can be simplified as follows:

$$\mathbf{R}_{\text{new}} = \mathbf{R} \odot (\mathbf{A}\mathbf{A}^H) = \mathbf{A}\mathbf{A}^H. \quad (6)$$

As indicated by (6), the transmit covariance matrix depends on two parameters. The first is the covariance matrix ($\mathbf{R}$) of the base signal and the second is the space-time code ($\mathbf{A}$). It should be noted that without STC (traditional MIMO radar signal design), the transmit covariance matrix only depends on the covariance of the base waveform. By using STC, we can use the same base signals (fully correlated) in all transmitters, similar to phased-array radar, and simply adjust the space-time codes. Therefore, it is not necessary to deploy different signal generators in each antenna, which reduces cost significantly. Additionally, hardware complexity in the transmitter will be reduced and any desired covariance matrix can be achieved.

In the next section, the relationship between the STC matrix and a given specific transmit covariance matrix will be discussed.

3.1 | STC and transmit covariance matrix relationship

Consider a transmit covariance matrix derived according to (6). In this section, we find the set of STCs that can yields a specific transmit covariance matrix. Singular value decomposition (SVD) of the target transmit covariance matrix $\mathbf{R}_{\text{new}}$ is as performed follows:

$$\mathbf{R}_{\text{new}} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^H = \mathbf{U}\boldsymbol{\Sigma}\mathbf{U}^H, \quad (7)$$

where $\mathbf{U}$ is the unitary matrix, $\mathbf{U} = \mathbf{V}$ based on the symmetric property of all covariance matrices, and $\boldsymbol{\Sigma}$ is a diagonal matrix containing the singular values of $\mathbf{R}_{\text{new}}$ defined as

$$\boldsymbol{\Sigma} = \mathrm{diag}(\sigma_1,\ \sigma_2,\ \ldots,\ \sigma_r,\ 0,\ 0,\ \ldots,\ 0), \quad (8)$$

where σ_i represents the singular values of $\mathbf{R}_{\text{new}}$ in descending order and r is the rank of $\mathbf{R}_{\text{new}}$. Then, $\mathbf{A}$ in (6) can be derived by calculating the square root of $\mathbf{R}_{\text{new}}$ as follows [24]:

$$\mathbf{A} = \mathbf{U}\boldsymbol{\Sigma}^{1/2}\boldsymbol{\chi} = \mathbf{U}\begin{bmatrix} \mathbf{P}_{r\times r} & \mathbf{0}_{r\times(N-r)} \\ \mathbf{0}_{(M_t-r)\times r} & \mathbf{0}_{r\times(N-r)} \end{bmatrix}\boldsymbol{\chi}, \quad (9)$$

where $\mathbf{0}$ is a zero matrix and $\boldsymbol{\chi}$ is an $M_t \times N$ Gaussian matrix with zero mean and unit variance (ie, $E[\boldsymbol{\chi}\boldsymbol{\chi}^H] = \mathbf{I}$) [24]. Additionally $\mathbf{P}$ is an $r \times r$ diagonal matrix defined as.

$$\mathbf{P} = \text{diag}\ (\sigma_1,\ \sigma_2,\ \ldots,\ \sigma_r). \tag{10}$$

The STC matrix $\mathbf{A}$ derived in (9) yields the target

Algorithm 1 *Proposed STC design for realizing any desired covariance matrix*

1. **Procedure** Find the proper STC matrix ($\mathbf{A}$).
2. Desired transmit beam pattern is given as $P_t(\theta)$.
3. Find the corresponding covariance matrix $\mathbf{R}_{new}$ based on the algorithm proposed in [20].
4. Set the covariance matrix of the base signal to all ones, $\mathbf{R} = [\mathbf{1}]_{M_t \times M_t}$.
5. Generate a Gaussian $M_t \times N$ matrix $\boldsymbol{\chi} \sim N(\mathbf{0},\ \mathbf{I})$.
6. $\mathbf{U}$ and $\boldsymbol{\Sigma}$ are derived through the SVD of $\mathbf{R}_{new}$ as shown in (7).
7. $\mathbf{A} = \mathbf{U}\boldsymbol{\Sigma}^{1/2}\boldsymbol{\chi}$ is achieved using (9).

covariance matrix $\mathbf{R}_{new}$ as

$$\begin{aligned} E[\mathbf{A}\mathbf{A}^H] &= E[\mathbf{U}\boldsymbol{\Sigma}^{1/2}\boldsymbol{\chi}\boldsymbol{\chi}^H\boldsymbol{\Sigma}^{1/2}\mathbf{U}^H] \\ &= E[\mathbf{U}\boldsymbol{\Sigma}\mathbf{U}^H] = \mathbf{R}_{new}, \end{aligned} \tag{11}$$

where the expectation is given with respect to the transmit matrix $\mathbf{A}$. These procedures are summarized in Algorithm 1. The STC matrix derived in (9) has a non-CE based on Gaussian distribution of $\boldsymbol{\chi}$, which is an important issue in practice. To resolve this issue, CE methods for implementing the STC matrix and realizing any desired covariance matrix can be used as described in [24]. This topic will be reviewed later.

3.2 | Feasibility condition

In this section, the feasibility conditions for the proposed design will be discussed. In other words, the necessary conditions for the STC for implementing MIMO radar using a phased-array structure will be discussed. According to (9), the ranks of the STC matrix and covariance matrix should be equal. However, the rank of any matrix is always less than or equal to the minimum number of its rows and its columns. Therefore, we have

$$\text{rank}\ (\mathbf{R}_{new}) = \text{rank}\ (\mathbf{A}) \le \min\ (M_t, N) \le N. \tag{12}$$

Then, the feasibility condition can be summarized as follows:

$$N \ge \text{rank}\ (\mathbf{R}_{new}). \tag{13}$$

This means that the number of time extensions in STC should be greater than or equal to rank($\mathbf{R}_{new}$). The condition in (12) supports this conclusion. Based on the conditions in (9), (12), and (13), any desired covariance matrix (corresponding to the desired beam pattern or any other performance criteria) can be achieved by sending the same waveforms for all transmitters and simply adjusting the STC amplitude and phase values according to the given conditions.

4 | CE STC design

CE waveform design is an important practical constraint in signal design [17] because radio-frequency amplifiers operate at maximum efficiency when the CE property is maintained in all transmit antennas. To realize any desired beam pattern and the corresponding covariance matrix, the CE waveform design method can be adopted [17]. This method maps Gaussian random variables with varying envelopes onto binary symbols (called BPSK symbols) with a CE. These signals will yield to the desired covariance matrix [17]. The transmit beam pattern of a MIMO radar is given by [1]

$$P_t(\theta) = \mathbf{a}_T^H(\theta)\mathbf{R}_{new}\mathbf{a}_T(\theta), \tag{14}$$

where $\mathbf{R}_{new}$ is the corresponding transmit covariance matrix related to the CE STC matrix. In other words, $\mathbf{R}_{new} = E[\mathbf{A}\mathbf{A}^H]$. It should be noted that for any covariance matrix, the corresponding Gaussian signal can be found easily according to (9). Suppose that the transmit non-constant Gaussian waveform and its covariance matrix are expressed as $\mathbf{X}_g$ and $\mathbf{R}_g$ respectively. Then, we have,

$$\mathbf{R}_g = E[\mathbf{X}_g\mathbf{X}_g^H] = \mathbf{W}\Lambda\mathbf{W}^H, \tag{15}$$

where the last equality is the eigenvalue decomposition of the corresponding Gaussian covariance matrix, Λ is the $M_t \times M_t$ diagonal matrix of eigenvalues of $\mathbf{R}_g$, and $\mathbf{W}$ is the $M_t \times M_t$ matrix of Eigenvectors of $\mathbf{R}_g$ [17]. Under these conditions, the related Gaussian matrix is given by [17]

$$\mathbf{X}_g = \mathbf{W}\Lambda^{1/2}\boldsymbol{\chi}_g, \tag{16}$$

where $\boldsymbol{\chi}_g$ is an $M_t \times N$ Gaussian matrix with zero mean and a unit covariance matrix, and $\mathbf{R}_g = E[\mathbf{X}_g\mathbf{X}_g^H]$. According to [17],

we can use the sign function to map a Gaussian signal onto a CE BPSK waveform as follows [17]:

$$\mathbf{A} = \text{sign}\ (\mathbf{X}_g). \tag{17}$$

Furthermore, the covariance matrices of $\mathbf{A}$ and $\boldsymbol{\chi}_g$ are $\mathbf{R}_{new}$ and $\mathbf{R}_g$, respectively. As demonstrated in [17], after mapping the Gaussian matrix onto a CE BPSK signal, the corresponding covariance matrices can be derived as follows [17]:

$$\mathbf{R}_{\mathrm{g}} = \sin\left(\frac{\pi}{2}\mathbf{R}_{\mathrm{new}}\right). \tag{18}$$

A proof of this procedure is presented in [17]. According to (18), the CE condition holds if and only if $\sin((\pi/2)\mathbf{R}_{\mathrm{new}})$ is positive semi-definite. Therefore, we have

$$\sin\left(\frac{\pi}{2}\mathbf{R}_{\mathrm{new}}\right) = \sin\left(\frac{\pi}{2}(\mathbf{A}\mathbf{A}^{\mathrm{H}})\right) \geq 0. \tag{19}$$

In other words, the CE condition can hold with STC if and only if $\sin((\pi/2)(\mathbf{A}\mathbf{A}^{\mathrm{H}}))$ is positive semi-definite. Figure 2 presents a block diagram of the CE conditions and mapping process. This block diagram indicates that (17) will yield $\mathbf{R}_{\mathrm{new}}$, which is our target. In a realistic scenario, there is an inter-processing operation on the transmitter side before predesigned symbols are sent, and $\mathbf{A}$ and $\mathbf{R}_{\mathrm{new}}$ do not have an injective relationship (ie, for a distinct $\mathbf{R}_{\mathrm{new}}$, there is an unlimited number of solutions for $\mathbf{A}$ as a CE STC matrix), meaning the acceptable solutions are STC matrices that satisfy (13) and (19) simultaneously.

5 | Proposed receive filter design

Consider the system model and observation matrix in the space and time domains defined in (3). The observation data should be analyzed in both the space and time dimensions to improve the performance of MIMO radar. Here, we focus on the receiver to maximize the SINR. Because an observation is a matrix of size $M_{\mathrm{r}} \times N$, it is passed through two finite-impulse response (FIR) filters $\mathbf{D}$ and $\mathbf{W}$ of sizes $N \times 1$ and $M_{\mathrm{r}} \times 1$, respectively. The FIR filter $\mathbf{D}$ is related to the time domain and $\mathbf{W}$ corresponds to the space domain. The processing blocks in the receiver are presented in Figure 3. The scalar output of the bilinear filter is derived as

$$\begin{aligned} r &= \mathbf{W}^{\mathrm{H}}\mathbf{Y}\mathbf{D} = \mathbf{W}^{\mathrm{H}}(\mathbf{Y}_0 + \mathbf{Y}_{\mathrm{int}} + \mathbf{v})\mathbf{D} \\ &= \mathbf{W}^{\mathrm{H}}\mathbf{Y}_0\mathbf{D} + \mathbf{W}^{\mathrm{H}}\mathbf{Y}_{\mathrm{int}}\mathbf{D} + \mathbf{W}^{\mathrm{H}}\mathbf{v}\mathbf{D} \\ &= r_0 + r_{\mathrm{int}} + r_n. \end{aligned} \tag{20}$$

The roles of the filters $\mathbf{D}$ and $\mathbf{W}$ are to weight the observation matrix in the time and space domains, respectively. The block after the observation matrix is called a bilinear filter because it is linear with respect to $\mathbf{D}$ when $\mathbf{W}$ is constant and vice versa. The observation matrix in (3) consists of three terms. The first term is the signal reflected from the target of interest, which is called the desired signal. The second term is the sum of all reflections from interferences and the third term is white Gaussian noise. According to (20), $\mathbf{D}$ and $\mathbf{W}$ affect the output

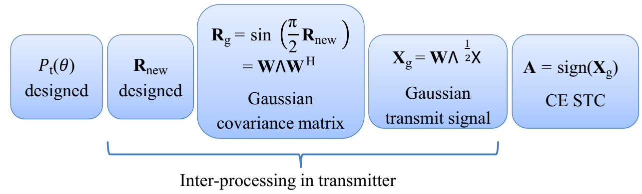

FIGURE 2 CE STC design block diagram

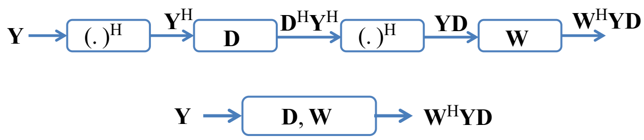

FIGURE 3 Bilinear receive filter design ($\mathbf{D}$,$\mathbf{W}$) corresponding to the space and time domains. The two blocks are equivalent

SINR. Therefore, the main problem is to find the optimal bilinear filters to maximize the SINR. According to (20), the output SINR is derived as

$$\begin{aligned} \mathrm{SINR} &= \frac{\mathrm{E}\left[\left|r_0\right|^2\right]}{\mathrm{E}\left[\left|r_{\mathrm{int}}\right|^2\right] + \mathrm{E}\left[\left|r_n\right|^2\right]} = \frac{P_0}{P_{\mathrm{int}} + P_n} \\ &= \frac{\mathrm{E}\left[\left|\beta_0\mathbf{W}^{\mathrm{H}}\mathbf{B}(\theta_0)\mathbf{X}\mathbf{D}\right|^2\right]}{\mathrm{E}\left[\left|\mathbf{W}^{\mathrm{H}}\left[\sum_{i=1}^{L}\beta_i\mathbf{B}(\theta_i)\mathbf{X}\right]\mathbf{D}\right|^2\right] + \mathrm{E}\left[\left|\mathbf{W}^{\mathrm{H}}\mathbf{v}\mathbf{D}\right|^2\right]}, \end{aligned} \tag{21}$$

where $\mathrm{E}[(.)]$ is the mathematical expectation operator and $\mathbf{B}(\theta) = \mathbf{a}_{\mathrm{R}}(\theta)\mathbf{a}_{\mathrm{T}}^{\mathrm{T}}(\theta)$. P_0, P_{int}, and P_n are the powers of the desired signal, interference, and noise, respectively, which can be derived as

$$\begin{aligned} P_0 &= \mathrm{E}\left[\left|\beta_0\mathbf{W}^{\mathrm{H}}\mathbf{B}(\theta_0)\mathbf{X}\mathbf{D}\right|^2\right], \\ P_{\mathrm{int}} &= \mathrm{E}\left[\left|\mathbf{W}^{\mathrm{H}}\left[\sum_{i=1}^{L}\beta_i\mathbf{B}(\theta_i)\mathbf{X}\right]\mathbf{D}\right|^2\right], \\ P_n &= \mathrm{E}\left[\left|\mathbf{W}^{\mathrm{H}}\mathbf{v}\mathbf{D}\right|^2\right]. \end{aligned} \tag{22}$$

According to (21) and (22), the output SINR depends on the transmit STC and receive bilinear filters $\mathbf{D}$ and $\mathbf{W}$. Therefore, (21) can be simplified as

$$\mathrm{SINR} = \frac{\mathrm{SNR}\left|\mathbf{W}^{\mathrm{H}}\mathbf{Y}_0\mathbf{D}\right|^2}{\sum_{i=1}^{L}\mathrm{INR}_i\left|\mathbf{W}^{\mathrm{H}}\mathbf{Y}_{\mathrm{int},\,i}\mathbf{D}\right|^2 + \left|\mathbf{W}^{\mathrm{H}}\overline{\mathbf{v}}\mathbf{D}\right|^2}, \tag{23}$$

where $\mathrm{SNR} = \mathrm{E}[\left|\beta_0\right|^2]/\sigma_v^2$ is the signal-to-noise ratio (SNR) and $\mathrm{INR}_i = \mathrm{E}[\left|\beta_i\right|^2]/\sigma_v^2$ is the interference-to-noise ratio (INR) corresponding to the i-th interference. Additionally, $\overline{\mathbf{v}}$ is a white Gaussian noise matrix with zero mean and a unit covariance matrix. The maximization problem for finding the optimal bilinear filters **D** and **W** can be written as

$$\max_{\mathbf{D},\mathbf{W}} \frac{\mathrm{SNR}\left|\mathbf{W}^{\mathrm{H}}\mathbf{Y}_0\mathbf{D}\right|^2}{\sum_{i=1}^{L}\mathrm{INR}_i\left|\mathbf{W}^{\mathrm{H}}\mathbf{Y}_{\mathrm{int},i}\mathbf{D}\right|^2+\left|\mathbf{W}^{\mathrm{H}}\bar{\mathbf{v}}\mathbf{D}\right|^2}. \tag{24}$$

This problem is called bilinear semi-definite programming. When $N = 1$, the problem in (24) can be reduced to the well-known MVDR problem. Therefore, this optimization problem is a general case of the MVDR problem with a closed-form solution [1]. Based on the fractional form in the cost function of (24), its Dinkelbach form can be optimized instead [25]. The Dinkelbach form of (25) is derived as the numerator minus a positive factor of the denominator [25]. Therefore, (24) can be simplified as

$$\min_{\mathbf{D},\mathbf{W}}\left(\sum_{i=1}^{L}\mathrm{INR}_i\left|\mathbf{W}^{\mathrm{H}}\mathbf{Y}_{\mathrm{int},i}\mathbf{D}\right|^2+\left|\mathbf{W}^{\mathrm{H}}\bar{\mathbf{v}}\mathbf{D}\right|^2-\mathrm{SNR}\left|\mathbf{W}^{\mathrm{H}}\mathbf{Y}_0\mathbf{D}\right|^2\right). \tag{25}$$

Proposition 1 The cost function in (25) has the same form as $\sum_{p=1}^{M_r}\sum_{j=1}^{N}\sum_{k=1}^{M_r}\sum_{i=1}^{N}a_{ijkl}x_iy_ky_jy_p$, which has been discussed in [26] and [27]. Therefore, (24) and (25) are NP-hard problems. This finding was also proved in [18,26–29].

Proof. The general form of the cost function in (25) is $\left|\mathbf{W}^{\mathrm{H}}\mathbf{Y}\mathbf{D}\right|^2$, which can be simplified as

$$\begin{aligned}\left|\mathbf{W}^{\mathrm{H}}\mathbf{Y}\mathbf{D}\right|^2 &= \left|\left[w_1^*,\ w_2^*,\ \ldots,\ w_{M_r}^*\right].\mathbf{Y}.\begin{bmatrix}d_1\\ d_2\\ \vdots\\ d_N\end{bmatrix}\right|^2\\ &=\sum_{p=1}^{M_r}\sum_{j=1}^{N}\sum_{k=1}^{M_r}\sum_{i=1}^{N}d_iw_k^*d_j^*w_py_ky_{ki}y_{pj}^*.\end{aligned} \tag{26}$$

Because (26) has the same form as the cost function in [26] and [27], (25) is an NP-hard problem with no closed-form general solution. To solve this problem, we can use iterative algorithms such as particle swarm optimization. In the next section, the simulation results for the proposed method will be discussed in detail.

6 | Multi-target scenario

One of the most important issues in radar is parameter identifiability, which is the maximum number of targets that can be uniquely identified [30]. In a multi-target scenario, the received baseband signal at the receiver can be expressed as[1]

$$\begin{aligned}\mathbf{Y}&=\sum_{k=1}^{K}\beta_{k0}\mathbf{a}_{\mathrm{R}}(\theta_{k0})\mathbf{a}_{\mathrm{T}}^{\mathrm{T}}(\theta_{k0})\mathbf{X}\\&\quad+\sum_{i=1}^{L}\beta_i\mathbf{a}_{\mathrm{R}}(\theta_i)\mathbf{a}_{\mathrm{T}}^{\mathrm{T}}(\theta_i)\mathbf{X}+\mathbf{v},\end{aligned} \tag{27}$$

where K is the total number of targets and the k-th target is located at θ_{k0}. Additionally, β_{k0} is the RCS related to the k-th target. The identifiability equation can be written as follows [30]:

$$\sum_{k=1}^{K}\widehat{\beta}_{k0}\mathbf{a}_{\mathrm{R}}(\widehat{\theta}_{k0})\mathbf{a}_{\mathrm{T}}^{\mathrm{T}}(\widehat{\theta}_{k0})\mathbf{X}=\sum_{k=1}^{K}\beta_k\mathbf{a}_{\mathrm{R}}(\theta_k)\mathbf{a}_{\mathrm{T}}^{\mathrm{T}}(\theta_k)\mathbf{X}, \tag{28}$$

where $\widehat{\beta}_{k0}$ and $\widehat{\theta}_{k0}$ are the estimates of the RCS and the azimuth angle of the k-th target, respectively. Parameter identifiability is a consistency concept that is used to establish the uniqueness of the parameter estimation problem as the SNIR or the number of snapshots N goes to infinity [30]. In the proposed design, the transmit covariance matrix is partially correlated and the rank of the transmit signal matrix is represented by r, where $1 \le r \le M_{\mathrm{t}}$. In the MIMO radar case, all transmit waveforms are completely uncorrelated, meaning $r_{\mathrm{MIMO}} = \mathrm{rank}(\mathbf{A}) = M_{\mathrm{t}}$. Additionally, in phased-array radar, all transmit waveforms are fully correlated, meaning $r_{\mathrm{Ph}} = \mathrm{rank}(\mathbf{A}) = 1$. To discuss the uniqueness of the solutions to (28), it can be expressed as $\widehat{\mathbf{B}}\widehat{\boldsymbol{\beta}} = \mathbf{B}\boldsymbol{\beta}$, where $\widehat{\mathbf{B}}$ is the estimation of $\mathbf{B}$ and the k-th column of $\mathbf{B}$ is equal to $\mathbf{a}_{\mathrm{R}}(\theta_{k0})\otimes\mathbf{a}_{\mathrm{T}}(\theta_{k0})$ and $\boldsymbol{\beta}=\left[\beta_{10},\beta_{20},\ldots,\beta_{K0}\right]^{\mathrm{T}}$. Some of the equations in $\widehat{\mathbf{B}}\widehat{\boldsymbol{\beta}} = \mathbf{B}\boldsymbol{\beta}$ may be identical. Therefore, we let $\widehat{\mathbf{C}}\widehat{\boldsymbol{\beta}} = \mathbf{C}\boldsymbol{\beta}$ denote the system of equations in which identical equations are eliminated. $\mathbf{C}$ is an $L_{\mathrm{c}} \times K$ matrix (K is the number of targets), and $\boldsymbol{\beta}$ is a $K \times 1$ vector containing the RCSs of the targets.

Depending on the geometry of the transmit and receive arrays, and how many antennas they share, some equations in $\widehat{\mathbf{B}}\widehat{\boldsymbol{\beta}} = \mathbf{B}\boldsymbol{\beta}$ may be identical [31]. In other words, it is important to determine how many distinct elements $\mathbf{a}_{\mathrm{R}}(\theta_{k0})\otimes\mathbf{a}_{\mathrm{T}}(\theta_{k0})$ exist. For example, when the transmit and receive arrays share no antennas in MIMO radar, $\mathbf{a}_{\mathrm{R}}(\theta_{k0})\otimes\mathbf{a}_{\mathrm{T}}(\theta_{k0})$ will have $M_{\mathrm{t}}M_{\mathrm{r}}$ possible distinct values. This is the case in which the most distinct targets can be distinguished. Additionally, suppose that the transmit antennas form a uniform linear array (ULA) that is a subset of the receive antenna ULA. In this case, the transmit and receive arrays share the maximum number of antennas, meaning $\mathbf{a}_{\mathrm{R}}(\theta_{k0})\otimes\mathbf{a}_{\mathrm{T}}(\theta_{k0})$ may only contain $M_{\mathrm{t}}+M_{\mathrm{r}}-1$ distinct elements. This is the worst scenario for the identification of distinct targets. Therefore, in MIMO radar, we have

$$L_{\mathrm{c}} \in \left[M_{\mathrm{t}}+M_{\mathrm{r}}-1,\ M_{\mathrm{t}}M_{\mathrm{r}}\right]. \tag{29}$$

Based on the results presented in [30] and [31], a sufficient and necessary condition for parameter identifiability is

[1]Here, another target located in the same range bin is considered as interference.

$$L_c + 1 > 2K, \text{ ie, } K_{max} = \left\lceil \frac{L_c - 1}{2} \right\rceil, \quad (30)$$

where $\lceil . \rceil$ denotes the smallest integer greater than or equal to a given number. Substituting (29) into (30) yields

$$K_{max,\,MIMO} \in \left[\frac{M_t + M_r - 2}{2}, \frac{M_t M_r + 1}{2} \right). \quad (31)$$

(31) indicates that the maximum number of targets that a MIMO radar can identify depends on the geometry of the transmit/receive arrays.

In phased-array radar, this number is reduced because there is only one type of signal to be transmitted, meaning M_t should be replaced with one, which yields

$$K_{max,\,Ph} \in \left[\frac{M_r - 1}{2}, \frac{M_r + 2}{2} \right). \quad (32)$$

In our proposed design, the number of independent transmit waveforms is equal to the rank of the STC matrix (ie, $r_{Ph\text{-}MIMO} = \text{rank}(\mathbf{A})$). Therefore, the maximum number of targets that can be uniquely identified can be calculated as follows:

$$K_{max,\,Ph\text{-}MIMO} \in \left[\frac{r_{Ph\text{-}MIMO} + M_r - 2}{2}, \frac{r_{Ph\text{-}MIMO} M_r + 1}{2} \right]. \quad (33)$$

7 | Simulation results

In this section, simulation results are presented in two parts. The first part focuses on realizing the desired covariance matrix by using the appropriate STC in the transmitter. The second part focuses on the optimization of the bilinear FIR filters to achieve better output SINR values.

Example 1: Suppose there are $M_t = 4$ and $M_r = 4$ co-located antennas with half-wavelength spacing on the transmit and receive sides, respectively. Additionally, suppose there are three targets located at $\theta = 0^\circ$, $\theta = -45^\circ$, and $\theta = -135^\circ$ with the desired transmit beam pattern shown in Figure 4. This desired beam pattern is compared with phased-array radar, MIMO radar, and phased-MIMO radar with $k = 2$ fully overlapped sub-arrays and equal weights. Based on the results in [24] and the results of beam pattern matching design, the corresponding transmit covariance matrix for the desired beam pattern is defined as follows:

$$\mathbf{R}_{new} = \begin{bmatrix} 1 & 0.28-0.90i & 0.11+0.05i & 0.72+0.12i \\ 0.28-0.90i & 1 & 0.08+0.20i & 0.29+0.63i \\ 0.11-0.05i & 0.08-0.02i & 1 & 0.07+0.29i \\ 0.72-0.12i & 0.29+0.63i & 0.07-0.28i & 1 \end{bmatrix}. \quad (34)$$

Additionally, the number of time extensions required to implement STC is $N = 4$, which is equivalent to the rank of the transmit covariance matrix in this example. This covariance matrix indicates that each waveform in an individual transmitter is not fully correlated like phased-array radar or fully uncorrelated like MIMO radar. Traditionally, to implement this transmit covariance matrix, phased-MIMO radar with a predesigned weight or correlated MIMO radar scheme was used, but these methods required different types of signal generators. Our proposed method using STC has much simpler hardware and software requirements. The proposed method can be implemented using simple phased-array radar and STC with $N = 4$ time extensions. Simulation is performed over 100 000 iterations to achieve the desired covariance matrix. Based on (9) and the SVD decomposition of the desired transmit covariance matrix in (34), the corresponding STC matrix can be derived as $\mathbf{A} = \mathbf{U}\Sigma^{1/2}\boldsymbol{\chi}$, where $\mathbf{U}$ and Σ are derived in this example as

$$\mathbf{U} = \begin{bmatrix} -0.60+0.00i & -0.12+0.00i & 0.45-0.00i & -0.64-0.00i \\ -0.17+0.56i & -0.25-0.00i & -0.12-0.25i & 0.29+0.63i \\ -0.02-0.02i & -0.41+0.81i & 0.20+0.25i & 0.07+0.28i \\ -0.53+0.08i & 0.23+0.19i & -0.71+0.31i & 1.00+0.00i \end{bmatrix} \quad (35)$$

and

$$\Sigma = \begin{bmatrix} 2.5928 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 1.1704 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.2351 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0018 \end{bmatrix}, \quad (36)$$

respectively, where $\boldsymbol{\chi}$ is a Gaussian matrix of size $M_t \times N$ with zero mean and an identity covariance matrix. The corresponding transmit beam pattern is illustrated in Figure 4 and compared with the patterns generated by MIMO radar and phased-array systems.

One of the main advantages of the proposed scheme is its compatibility with multi-target scenarios. Any arbitrary

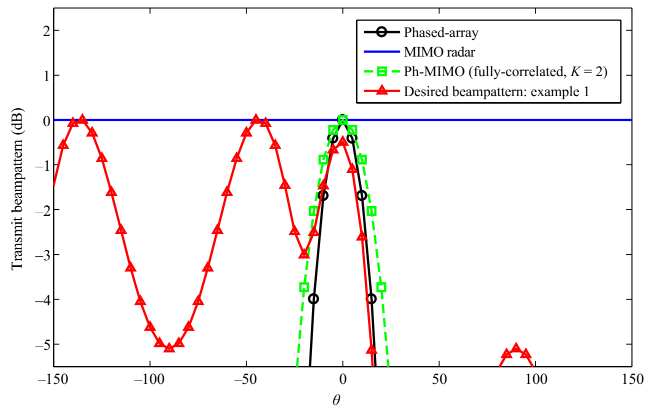

FIGURE 4 Normalized transmit beam pattern for Example 1

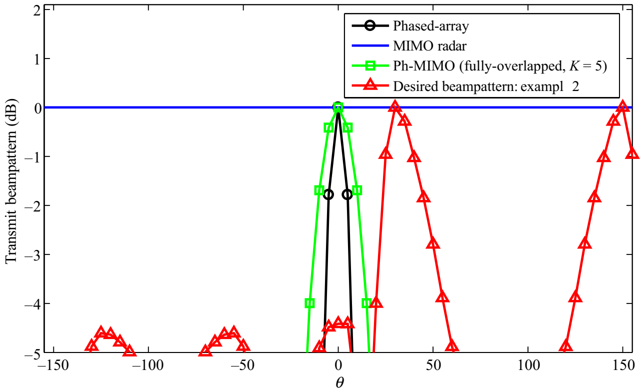

FIGURE 5 Normalized transmit beam pattern for Example 2

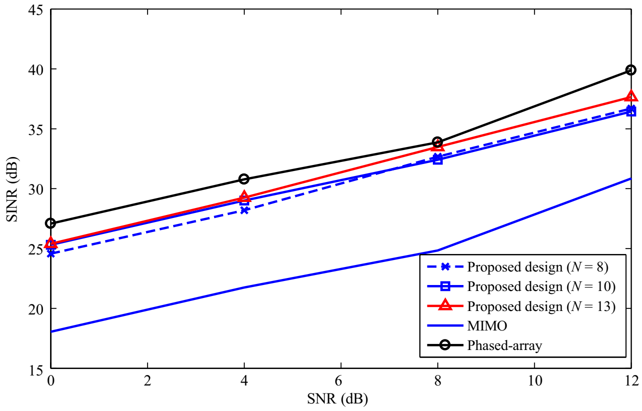

FIGURE 6 SINR comparisons for Example 2 with phased-array, phased MIMO, and MIMO radar systems

transmit covariance matrix and corresponding STC matrix can be derived using the proposed method. The cost of the proposed method is also much lower than that of traditional phased-MIMO and correlated MIMO radar because only one type of signal generator must be deployed.

Example 2: Suppose there are $M_t = 8$ and $M_r = 8$ co-located antennas with half-wavelength spacing on the transmitter and receiver sides, respectively. Additionally, suppose that $N = 8$ time extensions are used to implement STC and that there are two targets located at $\theta = 30^{\circ}$ and $\theta = 150^{\circ}$ with the desired transmit beam pattern shown in Figure 5. Then, there are $K = 5$ fully overlapped sub-arrays with equal weights in the phased-MIMO radar scheme. The corresponding STC matrix can be derived easily, similar to the previous example (which is not detailed here based on space limitations).

Suppose that there are two interferences located at $\theta_1 = -60^{\circ}$ and $\theta_2 = 45^{\circ}$ that affect the received signal strength and that the INR is constant at 15 dB. SINR versus SNR values were simulated based on the proposed receive FIR filter design. The results are presented in Figure 6. One can see that as the number of time extensions increases, a better SINR is achieved. For $N = 13$ and SNR = 12 dB, the SINR gap between the proposed design and phased array, which can only track one target, is approximately 2.24 dB.

8 | CONCLUSION

The proposed scheme provides additional degrees of freedom in transmit signal design. We demonstrated that with the proposed design, diversity can be improved and the desired covariance matrix can be derived easily by using a phased-array signal structure (fully correlated base waveforms) and by simply adjusting the STC values. Diversity is achieved by deploying and designing the STC as discussed in Section III. Therefore, STC is a key factor for providing additional degrees of freedom. Furthermore, the complexity of the transmitter can be reduced and there is no need to use multiple different signal generators in a transmitter, which reduces cost significantly. At a receiver, received vectors are analyzed to improve radar performance in terms of the SINR. By using the proposed design, we achieved phased-MIMO radar with the desired covariance matrix based on STC and a phased-array radar signal structure. Additionally, it was demonstrated that CE BPSK waveforms can be used with the STC structure and the required STC matrix conditions can be derived. Furthermore, any arbitrary covariance matrix can be derived by using an STC matrix with a BPSK signal design, as shown in Figure 2. On the receiver side, a novel STC analysis method was proposed to optimize SINR values.

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