International Journal of Internet, Broadcasting and Communication Vol.13 No.4 39-47 (2021)
http://dx.doi.org/10.7236/IJIBC.2021.13.4.39

IJIBC 21-4-5

Impacts of Non-Uniform Source on BER for SSC NOMA (Part I): Optimal MAP Receiver's Perspective

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Abstract

Lempel-Ziv coding is one of the most famous source coding schemes. The output of this source coding is usually a non-uniform code, which requires additional source coding, such as arithmetic coding, to reduce a redundancy. However, this additional source code increases complexity and decoding latency. Thus, this paper proposes the optimal maximum a-posteriori (MAP) receiver for non-uniform source non-orthogonal multiple access (NOMA) with symmetric superposition coding (SSC).

First, we derive an analytical expression of the bit-error rate (BER) for non-uniform source NOMA with SSC. Then, Monte Carlo simulations demonstrate that the BER of the optimal MAP receiver for the non-uniform source improves slightly, compared to that of the conventional receiver for the uniform source. Moreover, we also show that the BER of an approximate analytical expression is in a good agreement with the BER of Monte Carlo simulation.

As a result, the proposed optimal MAP receiver for non-uniform source could be a promising scheme for NOMA with SSC, to reduce complexity and decoding latency due to additional source coding.

Keywords: *NOMA, 5G, Lempel-Ziv coding, Superposition coding, Power allocation.*

1. Introduction

In the optical communications, symmetric superposition coding (SSC) [1] has been proposed to achieve the performance of the near-perfect successive interference cancellation (SIC) for non-orthogonal multiple access (NOMA) [2-4]. This performance for NOMA with can be obtained via smart bit-to-symbol mapping. In NOMA, unipodal binary pulse amplitude modulation (2PAM) for NOMA was studied in [5]. Asymmetric 2PAM non-SIC NOMA was proposed in [6]. Quadrature correlated superposition modulation was investigated in NOMA [7]. Low-correlated superposition coding NOMA was studied in [8]. Meanwhile, Lempel-Ziv coding [9] is one of the most famous source coding schemes. The output produced by this source coding is usually a non-uniform code, which requires additional source coding, such as arithmetic coding, to reduce a redundancy. However, this additional source code increase complexity and decoding latency. Thus, this paper proposes the optimal maximum a-posteriori (MAP) receiver for non-uniform source NOMA with SSC. First, we derive an analytical expression of the bit-error rate (BER) for non-uniform source NOMA with SSC. Then,

Manuscript Received: August. 25, 2021 / Revised: August. 29, 2021 / Accepted: September. 7, 2021
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Monte Carlo simulations demonstrate that the BER of the optimal MAP receiver for the non-uniform source improves, compared to that of the conventional receiver for the uniform source. Moreover, we also show that the BERs of approximate analytical expressions are in a good agreement with the BERs of Monte Carlo simulations.

The remainder of this paper is organized as follows. In Section 2, the system and channel model are described. The BER of non-uniform source NOMA with SSC is derived in Section 3. Monte Carlo simulations are addressed and discussed in Section 4. Finally, the conclusions are presented in Section 5.

The main contributions of this paper are summarized as follows:

- We propose the optimal MAP receiver for non-uniform source NOMA with SSC.
- Then, we derive an analytical expression of the BER for non-uniform source NOMA with SSC.
- Monte Carlo simulations demonstrate that the BER of the optimal MAP receiver for the non-uniform source improves slightly, compared to that of the conventional receiver for the uniform source.
- Moreover, we also show that the BERs of approximate analytical expressions are in a good agreement with the BERs of Monte Carlo simulations.

2. System and Channel Model

There are a base station and two users in a cellular downlink NOMA network. The complex channel coefficient between the *m*th user and base station is denoted by h_m, $m=1,2$, and the channels are sorted as $|h_1| \geq |h_2|$. The base station transmits the superimposed signal $x=\sqrt{P_A \alpha} s_1 + \sqrt{P_A(1-\alpha)} s_2$, where given the average total transmitted power P of x, at the base station, by solving $P=\mathbb{E}\left[|x|^2\right]=\mathbb{E}\left[\left|\sqrt{P_A \alpha} s_1 + \sqrt{P_A(1-\alpha)} s_2\right|^2\right]=P_A\left(1+\sqrt{2}\sqrt{\alpha}\sqrt{(1-\alpha)}\right)$, P_A is expressed by

$$P_A = \frac{P}{1+\sqrt{2}\sqrt{\alpha}\sqrt{(1-\alpha)}}, \tag{1}$$

s_m is the signal for the *m*th user with the average unit power, and α is the power allocation coefficient. The received signal r_m at the *m*th user is expressed as follows:

$$r_m = |h_m| x + n_m, \tag{2}$$

where $n_m \sim N(0, N_0/2)$ is additive white Gaussian noise (AWGN). Let the information bits for the user-1 and user-2 be $b_1, b_2 \in \{0,1\}$. A joint probability mass function (PMF) $P(b_1, b_2)$ is given by

$$\begin{array}{c|c|c|c} & & \multicolumn{2}{c}{P(b_2)} \\ & P(b_1,b_2) & P(b_2=0)=\frac{1}{2} & P(b_2=1)=\frac{1}{2} \\ \hline P(b_1) & P(b_1=0)=2\delta_{0,0} & P(b_1=0,b_2=0)=\delta_{0,0} & P(b_1=0,b_2=1)=\delta_{0,1}=\delta_{0,0} \\ \hline & P(b_1=1)=1-2\delta_{0,0} & P(b_1=1,b_2=0)=\delta_{1,0}=\frac{1}{2}-\delta_{0,0} & P(b_1=1,b_2=1)=\delta_{1,1}=\frac{1}{2}-\delta_{0,0} \end{array} \tag{3}$$

Note that the marginal PMF of the user-1 is $P(b_1=0)=2\delta_{0,0}$ and $P(b_1=1)=1-2\delta_{0,0}$; hence, if $\delta_{0,0}\neq\frac{1}{4}$, $P(b_1)$ is not uniformly distributed. Specifically, if $P(b_1=0)=P(b_1=1)=\frac{1}{2}$, then $P(b_1)$ is uniformly distributed, which corresponds $\delta_{0,0}=\frac{1}{4}$. We assume the binary phase shift keying (BPSK) modulation $s_1,s_2\in\{+1,-1\}$. The SSC scheme in this paper is represented as [1]

$$\begin{gathered}\begin{cases} s_1(b_1=0,b_2=0)=+1 \\ s_1(b_1=1,b_2=0)=-1 \end{cases} \quad \begin{cases} s_1(b_1=0,b_2=1)=-1 \\ s_1(b_1=1,b_2=1)=+1 \end{cases} \\ \begin{cases} s_2(b_2=0)=+1 \\ s_2(b_2=1)=-1 \end{cases}. \end{gathered} \tag{4}$$

3. Derivation of BER of Optimal MAP Receiver for Non-Uniform Source NOMA with SSC

In this section, the derivation is based on the similar procedure in [1]. We consider the optimal MAP receiver, which is formally expressed as

$$\hat{b}_1=\underset{b_1\in\{0,1\}}{\arg\max}\left\{P_{R_1|B_1}(r_1\,|\,b_1)P(b_1)\right\}, \tag{5}$$

where the likelihood $P_{R_1|B_1}(r_1\,|\,b_1)$ for the first user is given by

$$P_{R_1|B_1}(r_1\,|\,b_1)=\sum_{b_2=0}^{1}\frac{\delta_{b_1,b_2}}{P(b_1)}\frac{1}{\sqrt{2\pi N_0/2}}e^{-\frac{\left(r_1-|h_1|\sqrt{P_A}\left((-1)^{b_2}\sqrt{(1-\alpha)}+(-1)^{b_1}\sqrt{\alpha}\right)\right)^2}{2N_0/2}}. \tag{6}$$

First, we solve the equal MAP equation:

$$P_{R_1|B_1}(r_1\,|\,b_1=0)P(b_1=0)=P_{R_1|B_1}(r_1\,|\,b_1=1)P(b_1=1), \tag{7}$$

which is expressed by

$$
\begin{aligned}
&P(b_1=0)\sum_{b_2=0}^{1}\frac{\delta_{b_1=0,b_2}}{P(b_1=0)}\frac{1}{\sqrt{2\pi N_0/2}}e^{-\frac{\left(r_1-|h_1|\sqrt{P_A}\left((-1)^{b_2}\sqrt{(1-\alpha)}+\sqrt{\alpha}\right)\right)^2}{2N_0/2}}\\
&=\\
&P(b_1=1)\sum_{b_2=0}^{1}\frac{\delta_{b_1=1,b_2}}{P(b_1=1)}\frac{1}{\sqrt{2\pi N_0/2}}e^{-\frac{\left(r_1-|h_1|\sqrt{P_A}\left((-1)^{b_2}\sqrt{(1-\alpha)}-\sqrt{\alpha}\right)\right)^2}{2N_0/2}}.
\end{aligned}
\tag{8}
$$

Then, the one exact decision boundary and the two approximate decision boundaries are given by

$$
\begin{aligned}
&r_1=0,\\
&r_1=+|h_1|\sqrt{P_A}\left(\sqrt{1-\alpha}-\frac{N_0\log_e\frac{\delta_{0,0}}{\delta_{1,0}}}{4|h_1|^2P_A\sqrt{\alpha}}\right),
\end{aligned}
\tag{9}
$$

and

$$
r_1=-|h_1|\sqrt{P_A}\left(\sqrt{1-\alpha}-\frac{N_0\log_e\frac{\delta_{0,0}}{\delta_{1,0}}}{4|h_1|^2P_A\sqrt{\alpha}}\right).
\tag{10}
$$

Then, the decision regions are given by

$$
\begin{cases}
b_1=0: \begin{cases} r_1>+|h_1|\sqrt{P_A}\left(\sqrt{1-\alpha}-\dfrac{N_0\log_e\frac{\delta_{0,0}}{\delta_{1,0}}}{4|h_1|^2P_A\sqrt{\alpha}}\right)\\ r_1<-|h_1|\sqrt{P_A}\left(\sqrt{1-\alpha}-\dfrac{N_0\log_e\frac{\delta_{0,0}}{\delta_{1,0}}}{4|h_1|^2P_A\sqrt{\alpha}}\right)\end{cases}\\
b_1=1:\ -|h_1|\sqrt{P_A}\left(\sqrt{1-\alpha}-\dfrac{N_0\log_e\frac{\delta_{0,0}}{\delta_{1,0}}}{4|h_1|^2P_A\sqrt{\alpha}}\right)<r_1<+|h_1|\sqrt{P_A}\left(\sqrt{1-\alpha}-\dfrac{N_0\log_e\frac{\delta_{0,0}}{\delta_{1,0}}}{4|h_1|^2P_A\sqrt{\alpha}}\right).
\end{cases}
\tag{11}
$$

Hence, by using the average value Σ_1 of $|h_1|^2$, the approximate average BER can be expressed as

$$P_1^{\text{(optimal MAP receiver: non-uniform)}} \simeq$$
$$+P(b_1=0)F\left(\frac{\Sigma_1 P_A\left(\sqrt{\alpha}+\frac{N_0}{4\Sigma_1 P_A\sqrt{\alpha}}\log_e\frac{\delta_{0,0}}{\delta_{1,0}}\right)^2}{N_0}\right)$$
$$-P(b_1=0)F\left(\frac{\Sigma_1 P_A\left(2\sqrt{1-\alpha}+\sqrt{\alpha}-\frac{N_0}{4\Sigma_1 P_A\sqrt{\alpha}}\log_e\frac{\delta_{0,0}}{\delta_{1,0}}\right)^2}{N_0}\right)$$
$$+P(b_1=1)F\left(\frac{\Sigma_1 P_A\left(\sqrt{\alpha}-\frac{N_0}{4\Sigma_1 P_A\sqrt{\alpha}}\log_e\frac{\delta_{0,0}}{\delta_{1,0}}\right)^2}{N_0}\right)$$
$$+P(b_1=1)F\left(\frac{\Sigma_1 P_A\left(2\sqrt{1-\alpha}-\sqrt{\alpha}-\frac{N_0}{4\Sigma_1 P_A\sqrt{\alpha}}\log_e\frac{\delta_{0,0}}{\delta_{1,0}}\right)^2}{N_0}\right), \tag{12}$$

where

$$F(\gamma_b)=\frac{1}{2}\left(1-\sqrt{\frac{\gamma_b}{1+\gamma_b}}\right). \tag{13}$$

4. Numerical Results and Discussions

It is assumed that $\Sigma_1=\mathbb{E}\left[|h_1|^2\right]=1.8$ and $\Sigma_2=\mathbb{E}\left[|h_2|^2\right]=0.2$. We consider the average total transmitted signal power to noise power ratio (SNR) $P/N_0=40\,\text{dB}$, $\delta_{0,0}=\frac{23}{48}$, and the fixed power allocation $\alpha=0.05$. The number of the channel realizations is 10000000 for the evaluation of simulation results, from the random number generator.

First, in order to compare the BER of the proposed optimal MAP receiver for the non-uniform source with that of the conventional receiver for the uniform source, we depict the BERs of the first user, both for the proposed optimal MAP receiver and the conventional receiver, in Fig. 1.

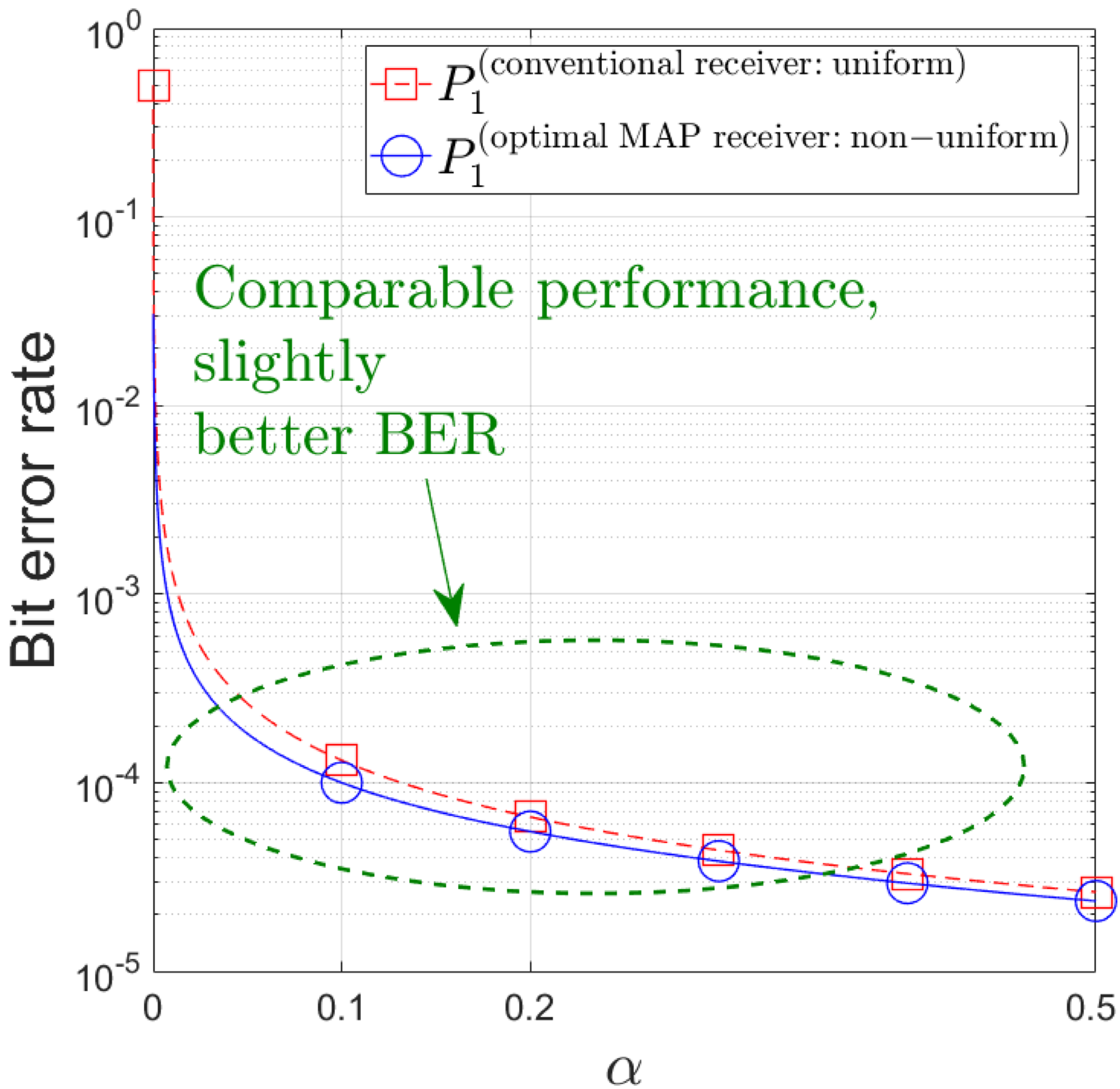

Figure 1. Comparison of BERs of optimal MAP receiver for the non-uniform source and conventional receiver for the uniform source, for first user.

As shown in Fig. 1, for the first user, the BER of the proposed optimal MAP receiver improves slightly, compared to that of the conventional receiver, over the entire power allocation range, i.e., $0 \leq \alpha \leq 1$. Notably, this improvement of the BER performance of the proposed optimal MAP receiver can be achieved, even without an additional source coding, which increases complexity and latency.

Second, to investigate the superiority of the proposed optimal MAP receiver over the conventional receiver, we depict the BERs versus the SNR, $0 \leq P / N_0 \leq 50$ (dB), with the fixed power allocation, $\alpha = 0.05$, in Fig. 2.

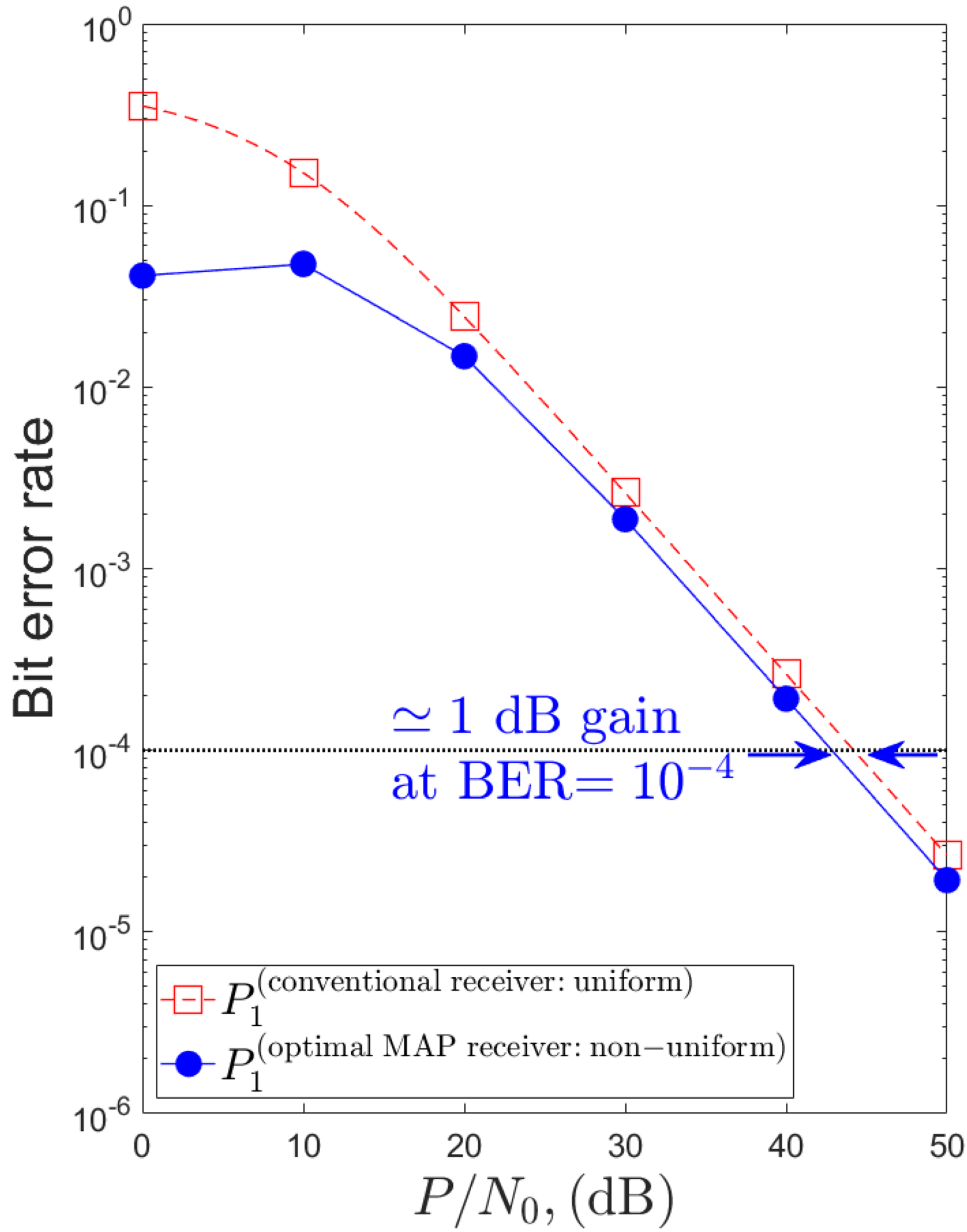

Figure 2. Comparison of BERs of optimal MAP receiver for the non-uniform source and conventional receiver for the uniform source, with varying SNR P/N_0**.**

As shown in Fig. 2, for the first user, the BER of the proposed optimal MAP receiver improves by about 1 dB, compared to that of the conventional receiver, i.e., the SNR gain of 1 dB, at the BER of 10^{-4}. It should be noted that this SNR gain can be observed, even up to at the BER of 2×10^{-5}.

Third, we validate the approximated BER expression of the proposed optimal MAP receiver, by Monte Carlo simulations, in Fig. 3.

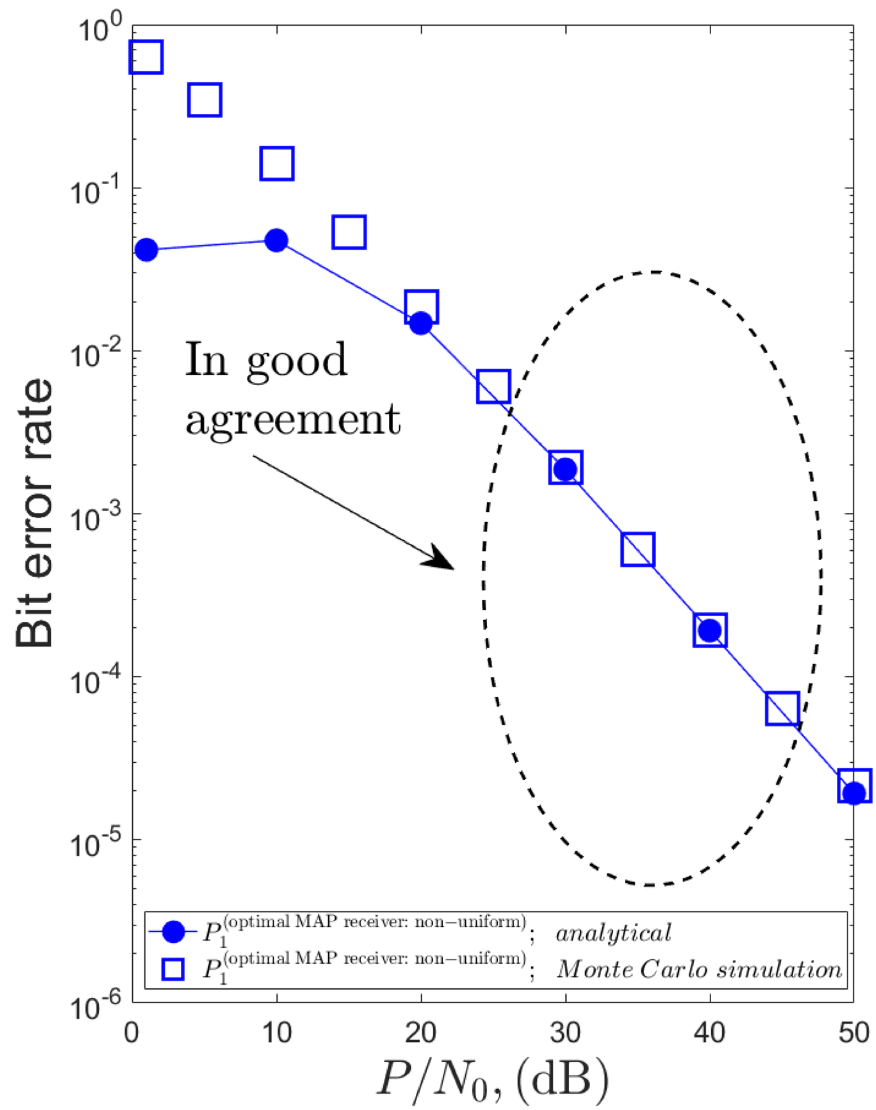

Figure 3. Comparison of BERs of optimal MAP receiver for analytical expression and Monte Carlo simulation, with varying SNR P/N_0.

As shown in Fig. 3, the Monte Carlo simulation results are well agreed with the approximate BER performance, except the low SNR, i.e., $P/N_0 \leq 20$ dB. Note that we conduct Monte Carlo simulations, each 5 dB for the SNR, $0 \leq P/N_0 \leq 50$ (dB), which is a sufficient increment for the validation of the analytical expression.

5. Conclusion

In this paper, we proposed the optimal MAP receiver for non-uniform source NOMA with SSC. This MAP receiver was based on the statistical structure in the output of Lempel-Ziv coding, i.e., a non-uniformly distributed source.

First, we derived an analytical expression of the BER for non-uniform source NOMA with SSC. Then, Monte Carlo simulations demonstrated that the BER of the optimal MAP receiver for the non-uniform source improves slightly, compared to that of the conventional receiver for the uniform sources. Moreover, we also showed that the BERs of approximate analytical expressions are in a good agreement with the BERs of Monte Carlo simulations. As a direction for future researches, it would be interesting to consider different modulation schemes for the proposed receiver.

As a result, the proposed optimal MAP receiver for non-uniform source NOMA with SSC could be a promising scheme for a non-uniform source in NOMA with SSC, to reduce complexity and decoding latency due to additional source coding.

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