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The Effect of the Credit Period on Inventory Policy under Trade Credit with Ordering Cost inclusive of a Freight Cost

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Abstract

In this paper we analyze the effect of the credit period on inventory policy under trade credit with ordering cost including a fixed cost and freight cost, where the freight cost has a quantity discount. For marketing purposes, some supplier offers credit period to his buyer to stimulate the demand for the product he produces. The delay in payments during the credit period has the effect of reducing the buyer's capital opportunity cost. It is also assumed that the buyer pays the freight cost for the order and hence, the ordering cost consists of a fixed ordering cost and a variable freight cost which depends on the order quantity. As a result, the possibilities of trade credit and discounts on freight costs are expected to play an important role in the buyer's inventory policy. Based on the economic order quantity inventory model, we analyze how the buyer can determine the optimal inventory policy and we examine the effect of the length of credit period on the buyer's inventory policy.

Keywords: *Inventory, EOQ, Credit period, Freight Cost, Sensitivity analysis*

1. INTRODUCTION

One of the practical extension models of the classical Economic Order Quantity (EOQ) model is the inventory model under trade credit. Such credit transactions would play an important role in normal business for many reasons. From the point view of a supplier offering a credit transaction, it is provided as a means of price differentiation which circumvents antitrust measures and is also provided as a means of stimulating the demand of the product. On the other hand, from the buyer's point of view, trade credit is an efficient method of bonding a supplier from the risk of receiving inferior quality products and is also an effective means of reducing the capital opportunity cost of holding inventory. In this sense, as stated by Fewings, the advantage of trade credit from the point of view of a supplier offering a credit period is substantial in terms of influence on the buyer's purchasing decisions [1]. Also, Mehta stated that one of the main reasons for allowing credit transactions from a supplier's point of view is to stimulate the demand for the product he produces [2]. As a result, the possibility of trade credit plays an important role in the decision of the buyer's inventory policy. In this regard, there were numerous research papers considering trade credit transactions. Chung and Teng et al. analyzed the retailer's inventory model assuming that the supplier allows a fixed credit period. And they investigated the properties of the inventory model under trade credit and proposed the inventory policy [3, 4]. Mahata and Goswami examined the economic ordering policy of deteriorating items under trade credit [5]. Also, considering the main reasons why suppliers allow buyers to trade on credit is that they can expect to stimulate demand for products through credit transactions, several research assuming that the length of credit

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period is a function of the amount purchased by the buyer have been published. In this regard, Chang et al. and Ouyang et al. presented the joint pricing and ordering problem with order-size dependent trade credit [6, 7].

All the research papers mentioned above implicitly assumed that the buyer's ordering cost is fixed without the consideration of any transportation cost for the quantity ordered. But, in practical situations, the buyer would pay the freight cost for the order quantity. In this regard, Aucamp and Lee examined the classical inventory model with the buyer's ordering cost consisting of a fixed cost and freight cost charged by the order quantity [8, 9]. Also, Shinn et al. introduced the trade credit model assuming that the buyer's ordering cost consist of a fixed cost and a variable freight cost [10]. However, in many common business transactions, the quantity ordered may be transported in unit loads, i.e., pallets, boxes, containers, and others. In this case, the transportation cost is charged as follows. There is a basic fee for the first unit load and there is an incremental fee for each more transportation unit considered as special model suggested by Lee [9]. Recently, Shinn considered the problem of determining EOQ under trade credit [11]. He assumed that the buyer's ordering cost consists of a fixed cost and a variable freight cost to be charged depending on each additional unit load. And he provided more efficient solution procedure than the solution algorithm presented by Shinn et al. [10].

The purpose of this paper is to analyze the effect of the credit transaction period on inventory policy under trade credit with ordering cost including a fixed cost and a freight cost. For the analysis, the freight cost adopted in this model is assumed as a special model stated by Lee [9], i.e., there is a basic fee for the first unit load and there is an incremental fee for each more freight unit loads. (Note that the freight unit can be in terms of weight or volume.) As mentioned above, the possibility of credit transactions permitted by the supplier is expected to play an important role in the buyer's inventory policy. Also, the ordering cost consists of a fixed ordering cost and a freight cost that depends on the buyer's order quantity is expected to affect the buyer's inventory policy. From this point of view, we analyze the effect of the length of the credit period on the buyer's inventory policy based on the results of the model presented by Shinn [11].

2. MODEL DEVELOPMENT AND DETERMINATION OF EOQ

Based on the trade credit inventory model with ordering cost inclusive of a freight cost considered by Shinn [11], this study aims to analyze the effect of the length of credit period on the buyer's inventory policy. Therefore, the assumptions and notations used in this paper are essentially same as those of Shinn [11].

Assumptions:

(1) The buyer's demand rate is constant.

(2) Inventory depletion is not allowed.

(3) The supplier allows a certain length of credit and sales income during that period is deposited in an interest with rate I. At the end of that period, the product price is paid and the inventory investment costs with rate R ($R \geq I$) are incurred for the buyer's inventory.

(4) The buyer pays the freight cost for the transportation of the order quantity with fixed ordering cost.

Notations:

- D = annual demand rate.
- C = purchasing cost per unit.
- Q = order quantity.
- tc = credit period permitted by the supplier.
- H = inventory holding cost excluding the inventory investment cost.
- R = inventory investment cost (as a percentage).
- I = earned interest rate (as a percentage).
- U = transportation unit.
- A = fixed ordering cost.
- F_j = freight cost for Q, $(j-1)U < Q \leq jU,\ j = 1, 2, \cdots, n;\ P_0 + (j-1)P, P_0 \geq P$.
- $S(Q)$ = buyer's ordering cost for Q, $(j-1)U < Q \leq jU,\ j = 1, 2, \cdots, n;\ A + F_j$.

As Shinn [11] stated in his paper, the buyer's objective is to minimize his annual total cost, $TC(Q)$ and $TC(Q)$ consists of three elements as follows:

1) Annual ordering cost = $\frac{(A+F_j)D}{Q}$ for $(j-1)U < Q \le jU$.
2) Annual inventory holding cost = $\frac{HQ}{2}$.
3) Annual inventory investment cost
   (i) Case 1($Dtc \le Q$):
   $= \frac{C(R-I)D^2tc^2}{2Q} + \frac{CRQ}{2} - CRDtc;$
   (ii) Case 2($Dtc > Q$)
   $= \frac{CIQ}{2} - CIDtc.$

Then, $TC(Q)$ can be expressed as

$TC(Q)$ = Annual Ordering Cost + Annual Inventory Holding Cost + Annual Inventory Investment Cost.

Depending on the relative size of *Dtc* and *Q*, $TC(Q)$ can be expressed as the following two equations.

(1) Case 1($Dtc \le Q$)

$$\mathrm{TC}_{1,j}(Q) = \frac{(A+F_j)D}{Q} + \frac{HQ}{2} + \left(\frac{C(R-1)D^2tc^2}{2Q} + \frac{CRQ}{2} - CRDtc\right),\ (j-1)U < Q \le jU,\ j = 1,2,\cdots,n, \quad (1)$$

(2) Case 2($Dtc > Q$)

$$\mathrm{TC}_{2,j}(Q) = \frac{(A+F_j)D}{Q} + \frac{HQ}{2} + \left(\frac{CIQ}{2} - CIDtc\right),\ (j-1)U < Q \le jU,\ j = 1,2,\cdots,n. \quad (2)$$

The problem is to find an optimal order quantity Q^* which minimizes *TC*(*Q*). From the structure of equations (1) and (2), *TC*(*Q*) is a convex function of *Q* for every *i* and *j* and therefore, there exist an extreme value $Q_{i,j}$, which minimizes $TC_{i,j}(Q)$ as follows;

$$Q_{i,j} = \sqrt{\frac{2(A_1+F_j)D}{H_1}} \text{ where } A_1 = A + \frac{C(R-I)Dtc^2}{2} \text{ and } H_1 = H + CR, \quad (3)$$

$$Q_{2,j} = \sqrt{\frac{2(A+F_j)D}{H_2}} \text{ where } H_2 = H + CI. \quad (4)$$

Therefore, as stated by Shinn [11], we have the following useful properties and theorems which are the ones that play an important role in determining the optimal order quantity.

Property 1. For any *j*, $Q_{1,j} \ge Dtc$ if and only if $Q_{2,j} \ge Dtc$. Therefore, if $Q_{1,j} \ge Dtc$, then $TC_{2,j}(Q)$ is decreasing in *Q* over $Q_{i,j} < Q_{i,j+1}$, $j = 1,2,\cdots,n.$
Property 2. For *i* given, $Q_{1,j} \ge Dtc$ if and only if $Q_{2,j} \ge Dtc$
Property 3. For any *Q*, $TC_{i,j}(Q) < TC_{i,j+1}(Q)$, $i = 1,2$ and $j = 1,2,\cdots,n.$

From the results of properties 1 to 3, we can apply the results by Lee [9] to develop the solution algorithm for our model. Then, we have following two theorems finding the candidate value for the optimal order quantity, Q^*.(Proofs are omitted.)

Theorem 1 (for Case 1). Suppose $(k-1)U < Dtc \leq kU$ for some k. Let a be the index such that $(a-1)U < Q_{1,0} \leq aU$ where $Q_{1,0} = \sqrt{2(A_1+F_0)D/H_1}$.

If $a > k$ and $Q_{1,a} \leq aU$, then $Q_0 = (a-1)U, Q_{1,a}$,

If $a > k$ and $Q_{1,a} > aU$, then $Q_0 = (a-1)U, aU$,

If $a \leq k$ and $Q_{1,k} \leq Dtc$, then Q^* must less than Dtc,

If $a \leq k$ and $Q_{1,k} \leq kU$, then $Q_0 = Q_{1,k}$,

If $a \leq k$ and $Q_{1,k} > kU$, then $Q_0 = kU$.

Theorem 2 (for Case 2). Suppose $(k-1)U < Dtc \leq kU$ for some k. Let b be the index such that $(b-1)U < Q_{2,0} \leq bU$ where $Q_{2,0} = \sqrt{2(A+F_0)D/H_2}$.

If $b < k$ and $Q_{2,a} \leq bU$, then $Q_0 = (b-1)U, Q_{2,b}$,

If $b < k$ and $Q_{2,a} > bU$, then $Q_0 = (b-1)U, bU$,

If $b = k$ and $Q_{2,b} \leq Dtc$, then $Q_0 = (b-1)U, Q_{2,b}$,

If $b = k$ and $Q_{2,b} \leq Dtc$, then $Q_0 = (b-1)U$,

If $b > k$, then $Q_0 = (k-1)U$.

Therefore, the same solution algorithm as proposed by Shinn [11] can be applied to determine the optimal order quantity.

Solution Algorithm

Step 1. Find index k such that $(k-1)U < Dtc \leq kU$.

Step 2. Compute $Q_{i,0} = \sqrt{\frac{2(A_1+F_0)D}{H_1}}$ and find index a such that $(a-1)U < Q_{1,0} \leq aU$.

Step 3. If $a > k$, go to step 4.
Otherwise, go to step 5.

Step 4. If $Q_{1,a} \leq aU$, then compute $TC(Q)$ with equation (1) for $Q = (a-1)U$ and $Q_{1,a}$, and go to step 6.
Otherwise, compute $TC(Q)$ with equation (1) for $Q = (a-1)U$ and aU, and go to step 6.

Step 5. If $Q_{1,k} \leq Dtc$, then go to step 6.
Otherwise, if $Q_{1,k} \leq kU$ compute $TC(Q)$ with equation (1) for $Q = Q_{1,k}$ and go to step 6.
Otherwise, compute $TC(Q)$ with equation (1) for $Q = kU$ and go to step 6.

Step 6. Compute $Q_{2,0} = \sqrt{\frac{2(A+F_0)D}{H_2}}$ and find index b such that $(b-1)U < Q_{2,0} \leq bU$.

Step 7. If $b < k$, go to step 8.
Otherwise, $b = k$, then go to step 9.
Otherwise, go to step 10.

Step 8. If $Q_{2,b} \leq bU$, then compute $TC(Q)$ with equation (2) for $Q = (b-1)U$ and $Q_{2,b}$, and go to step 11.
Otherwise, compute $TC(Q)$ with equation (2) for $Q = (b-1)U$ and bU, and go to step 11.

Step 9. If $Q_{1,b} \leq Dtc$, then go to step 6.
Otherwise, if $Q_{1,k} \leq kU$ compute $TC(Q)$ with equation (2) for $Q = (b-1)U$ and $Q_{2,b}$, and go to step 11.
Otherwise, compute $TC(Q)$ with equation (2) for $Q = (b-1)U$ and go to step 11.

Step 10. Compute $TC(Q)$ with equation (2) for $Q = (k-1)U$ and go to step 11.

Step 11. Select the one that yields the minimum cost as Q^* and stop.

3. SENSITIVITY ANALYSIS FOR CREDIT PERIOD (*tc*)

Now, we examine how much effect the length of the credit period on the buyer's optimal order quantity considering the ordering cost, which includes freight costs proportionately depending on the amount of product transportation. Unfortunately, the structure of the buyer's annual total cost in equations (1) and (2) do not allow sensitivity analysis for tc analytically. Therefore, we solve the same example problems using some different values of the credit period to answer the above question. For the analysis, the following example problem is applied.

D = 3,200 units, C = 3[$/unit], H = 0.3[$/unit·year], R = 0.15(=15%), I = 0.1(= 10%), U = 500[unit], A = 50[$/unit], P_0 = 15[$] and P = 10[$].

In order to analyze the effect of the length of the credit period on the buyer's optimal order quantity, the example problems are solved applying the credit period, tc (years) of six levels *of* 0, 0.05, 0.1, 0.2, 0.3 and 0.5. Of course, tc = 0 means a traditional inventory model that does not allow credit, as stated by Lee [9]. We also develop a computer program written "C" to solve the buyer's optimal order quantity and the results are given in Table 1.

Table 1. Results with various values of *tc*.

tc	$TC(Q^*)$	Q^*
0.00	10,225	600
0.05	10,156	600
0.10	10,094	600
0.20	9,986	898
0.30	9,884	900
0.50	9,692	900

From the results in Table 1, the following facts can be confirmed. As mentioned above, a major reason for the supplier to offer a credit period to the buyer is to stimulate the demand for the product he produces. And therefore, the availability of trade credit plays an important role in the decision of the buyer's inventory policy. As a result, the length of the credit period affects the buyer's order quantity. Also, the freight cost depending on the order quantity tend to make the buyer's order quantity large enough to qualify for a certain freight cost break. Moreover, in the case of $P_0 > P$, the inequality $P_0 > P$ implies that there is some quantity discount in the freight cost for changing the order size from jU to $(j+1)U$ and it is also a factor in increasing the buyer's order quantity. According to Table 1, as the length of the credit period increases, the buyer's order quantity increases while the buyer's annual total cost decreases as expected. Also, note that for the case with tc = 0, the solution algorithm gives the same results as the previous results by Lee [9].

4. CONCLUSION

In this paper, we analyzed the effects of the length of the credit period on the buyer's order quantity. For the analysis, it was assumed that the buyer's ordering cost consists of the fixed ordering cost and the variable freight cost charged by the transport unit. For sensitivity analysis on the length of the credit period to the buyer's inventory policy, we formulated the buyer's annual total cost function. Unfortunately, the structure of mathematical expressions does not allow sensitivity analysis for the credit period analytically and therefore, the same example problems are solved using some different values of the credit period, tc. According to the results of the sensitivity analysis, when credit transactions are permitted from the supplier, the delayed in payments during the credit transaction period has been shown to have the effect of reducing the stock investment cost of the buyer. Therefore, the allowance of credit transactions plays an important role in determining the buyer's inventory policy. Also, the freight cost depending on the order quantity tends to make the buyer's order quantity large enough to qualify for a certain freight cost break. With an example problem, sensitivity analysis is examined. The results show that as the credit transaction period increases, the buyer's order quantity increases while the buyer's annual total cost decreases as expected. Also, the freight cost depending on the order quantity tend to make the buyer's order quantity large enough to qualify for a certain freight cost break.

REFERENCES

[1] D. R. Fewings, "A credit limit decision model for inventory floor planning other extended trade credit arrangement," Decision Science, Vol. 23, pp. 200-220, 1992.

[2] D. Mehta, "The formulation of credit policy models," *Management Science*, Vol. 15, pp. B30-B50, 1968.

[3] K. J. Chung, "A theorem on the determination of economic order quantity under conditions of permissible delay in payments," *Computers & Operations Research*. Vol. 25, No. 1, pp. 49-52, 1998.

[4] J. T. Teng, C. T. Chang, M. S. Chern and Y. L. Chan, "Retailer's optimal ordering policies with trade credit financing," *International Journal of Systems Science*, Vol. 38, No. 3, pp. 269-278, 2007.

[5] G.C. Mahata and A. Goswami, "An EOQ model for deteriorating items under trade credit financing in the fuzzy sense.," Production Planning & Control: The management of Operations, Vol. 18, pp. 681-692, 2007.

[6] H. C. Chang, C. H. Ho, L. Y. Ouyang and C. H. Su, "The optimal pricing and ordering policy for an integrated inventory model when trade credit linked to order quantity," Applied Mathematical Modeling, Vol. 33, pp. 2978-2991, 2009.

[7] L. Y. Ouyang, C. H. Ho, and C. H. Su, "An optimization approach for joint pricing and ordering problem in an integrated inventory system with order-size dependent trade credit," *Computers & Industrial Engineering*, Vol. 57, pp. 920-930, 2009.

[8] D. C. Aucamp, "Nonlinear freight costs in the EOQ problem," European Journal of Operational Research. Vol. 9, pp. 61-63, 1982.

[9] C. Y. Lee, "The economic order quantity for freight discount costs," IIE Transactions, Vol. 18, pp. 318-320, 1986.

[10] S. W. Shinn, H. Hwang and S. S. Park, "Joint price and lot size determination under conditions of permissible delay in payments and quantity discounts for freight cost," European Journal of Operational Research, Vol. 91, pp. 528-542, 1996.

[11] S. W. Shinn, "Distributor's Lot-sizing and Pricing Policy with Ordering Cost inclusive of a Freight Cost under Trade Credit in a Two-stage Supply Chain," International Journal of Advanced Culture Technology, Vol. 8, No. 1, pp. 62-70, 2020. https://doi.org/10.17703//IJACT2020.8.1.62.