FRACTIONAL ORDER THERMOELASTIC PROBLEM FOR FINITE PIEZOELECTRIC ROD SUBJECTED TO DIFFERENT TYPES OF THERMAL LOADING - DIRECT APPROACH

KISHOR R. GAIKWAD[1†] AND VIDHYA G. BHANDWALKAR[1]

[1]PG DEPARTMENT OF MATHEMATICS, NES, SCIENCE COLLEGE, NANDED, M.S., INDIA-431601.
E-mail address: drkr.gaikwad@yahoo.in

ABSTRACT. The problem of generalized thermoelasticity of two-temperature for finite piezo-electric rod will be modified by applying three different types of heating applications namely, thermal shock, ramp-type heating and harmonically vary heating. The solutions will be derived with direct approach by the application of Laplace transform and the Caputo-Fabrizio fractional order derivative. The inverse Laplace transforms are numerically evaluated with the help of a method formulated on Fourier series expansion. The results obtained for the conductive temperature, the dynamical temperature, the displacement, the stress and the strain distributions have represented graphically using MATLAB.

1. INTRODUCTION

Many theories have been discovered and tried to eliminate the contradiction of the infinite speed of heat propagation in the classical theory of thermoelasticity. But two among them are very famous. That are Lord-Shulman theory [1] and Green-Lindsay theory [2]. Many of the authors gave their contribution in the theory of generalized Thermopiezoelasticity for two-temperature as seen in the work of Youssef et.al. [3], Tianhu et.al. [4], Nowacki [5], Abouelregal [6] and Chandrasekharaiah [7].

In last few decades there had been remarkable developments in the field of Fractional Calculus as shown in the many researchers work like Podlubny [8], Miller et.al. [9], Yang et.al. [10] and Losada et.al. [11]. The use of derivative of fractional order has also spread into the work of many researchers as Raslan [12], Sherief et.al. [13, 14, 15], Youssef et.al. [16, 17], and Ezzat et.al. [18]. Honig et.al. [19], discussed the numerical inversion of Laplace transform. Some contributions of this theory are in[20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34].

This paper is the modification of the work of E. Bassiouny and H. M. Youssef [35]. In this paper, a new thermoelastic model has been prepared with Caputo-Fabrizio fractional derivative

of order $0 \leq \alpha \leq 1$ and the results are derived in Laplace transform domain defined in [36] and using the direct approach. The comparative study of Thermopiezoelasticity of finite rod with three different types of thermal loading has been done and the conductive temperature, the dynamical temperature, the stress, the strain and the displacement functions are to be determined. Numerical results are illustrated graphically using MATLAB software.

Nomenclature:

λ, μ	Lame's Constants
ρ	Density
a	Two-temperature parameter
C_E	Specific heat at constant strain
T	Absolute temperature
T_0	Reference temperature
t	Time
t_0	Ramping time parameter
u_i	Components of displacement vector
Ω	The angular frequency of thermal vibration
δ_{ij}	Kronecker delta function
α_T	Coefficient of linear thermal expansion
ξ	The entropy density
ϕ	The conductive temperature
q_i	The components of the heat flux vector
e_{ij}	Strain tensor component
σ_{ij}	Stress tensor component
τ_0	One relaxation time parameter
α	Fractional order operator
γ	$(3\lambda+2\mu)\alpha_T$
ω	$ac_0^2\eta^2$, Dimensionless two-temperature parameter
c	$\dfrac{\rho C_E}{T_0}$
θ	$(T - T_0)$, The dynamical temperature increment such that, $\dfrac{|T - T_0|}{T_0} << 1$
η	$\dfrac{\rho C_E}{k}$, The thermal viscosity
β	$\dfrac{\gamma T_0}{\lambda + 2\mu}$, Dimensionless thermoelastic coupling constant
c_0	$\sqrt{\dfrac{\lambda + 2\mu}{\rho}}$, Longitudinal wave speed
ϵ	$\dfrac{\gamma}{\rho C_E}$, Dimensionless mechanical coupling constant
D	constant, the component of electric displacement
F_0	constant, the strength of shock on the boundary

2. One-dimensional formulation of the problem

Consider a piezoelectric rod of finite length h. At initial stage the rod is at rest. The one end of the rod is being heated and other is fixed at initial temperature.

Here, the displacement component is depends only on x co-ordinate. Thus, we have,

$$u_x = u(x,t), \qquad u_y = u_z = 0$$

The initial conditions are assumed as follows:

$$u(x,0) = 0, \qquad 0 \leq x \leq h$$

$$\frac{\partial u}{\partial t}(x,0) = 0, \qquad 0 \leq x \leq h$$

$$\sigma(x,0) = 0, \qquad 0 \leq x \leq h$$

$$\phi(x,0) = \dot{\phi}(x,0) = 0, \qquad 0 \leq x \leq h$$

Suppose, $F(t)$ is a heating function which is being applied to one end of the rod and as the rod is traction free, the boundary conditions are as follows:

$$\phi_0(0,t) = F(t) \tag{2.1}$$

$$e(0,t) = e(h,t) = 0, \qquad \sigma(0,t) = 0 \tag{2.2}$$

For this problem we will consider the following basic equations:

$$(\lambda + 2\mu)\frac{\partial^2 u}{\partial x^2} - \gamma \frac{\partial \theta}{\partial x} = \rho \frac{\partial^2 u}{\partial t^2} \tag{2.3}$$

$$\sigma = (\lambda + 2\mu)\frac{\partial u}{\partial x} - \gamma \theta - hD \tag{2.4}$$

$$k\frac{\partial^2 \phi}{\partial x^2} = \left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^{\alpha+1}}{\partial t^{\alpha+1}}\right)[\rho C_E \theta + \gamma T_0 e] \tag{2.5}$$

$$\phi - T = a\frac{\partial^2 \phi}{\partial x^2} \tag{2.6}$$

$$e = \frac{\partial u}{\partial x} \tag{2.7}$$

$$E = -\frac{\partial v}{\partial x} \tag{2.8}$$

$$\frac{\partial D}{\partial x} = 0 \tag{2.9}$$

For our convenience, we consider the following non-dimensional variables,

$$u' = c_0 \eta u, \quad \phi' = \frac{\phi - T_0}{T_0}, \quad \tau' = c_0^2 \eta \tau, \quad \sigma' = \frac{\sigma}{\lambda + 2\mu}, \quad t' = c_0^2 \eta t, \quad \theta' = \frac{T - T_0}{T_0},$$

$$D' = \frac{hD}{\lambda + 2\mu}, \quad \eta = \frac{\rho C_E}{k}, \quad c_0^2 = \frac{\lambda + 2\mu}{\rho}, \quad t_0' = c_0^2 \eta t_0, \quad x' = c_0 \eta x$$

Thus, the equations from Eq.(2.3)–Eq.(2.9)(dropping the primes) takes the form,

$$\frac{\partial^2 e}{\partial x^2} - \beta \frac{\partial^2 \theta}{\partial x^2} = \frac{\partial^2 e}{\partial t^2} \tag{2.10}$$

$$\sigma = e - \beta\theta - D \tag{2.11}$$

$$\frac{\partial^2 \phi}{\partial x^2} = \left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^{\alpha+1}}{\partial t^{\alpha+1}} \right)(\theta + \epsilon e) \tag{2.12}$$

$$\theta = \phi - \omega \frac{\partial^2 \phi}{\partial x^2} \tag{2.13}$$

3. SOLUTION IN THE LAPLACE TRANSFORM DOMAIN

The Laplace transform defined as follows:

$$L[f(t)] = \bar{f}(s) = \int_0^\infty e^{-st} f(t) dt \tag{3.1}$$

The Caputo-Fabrizio fractional order derivative is defined in [36] as follows:

$$_aD_t^{(\alpha)} f(t) = \frac{M(\alpha)}{1-\alpha} \int_a^t f'(\tau) \exp\left[-\frac{\alpha(t-\tau)}{1-\alpha} \right] d\tau \tag{3.2}$$

where, $M(\alpha)$ is the normalization function such that,

$$M(0) = M(1) = 0, \quad 0 \le \alpha \le 1, \quad -\infty < a < t, \quad f \in H^1(a,b), \quad a < b$$

We suppose that the function $M(\alpha) = 1$ and substituting $a = 0$ in the definition defined in Eq.(3.2), we obtain the Laplace transform of Caputo-Fabrizio fractional derivative in s variable as follows:

$$L[_0D_t^{(\alpha)} f(t)] = \frac{1}{1-\alpha} \int_0^\infty e^{-st} \int_0^t f'(\tau) \exp\left[-\frac{\alpha(t-\tau)}{1-\alpha} \right] d\tau dt$$

this implies,

$$L[_0D_t^{(\alpha)} f(t)](s) = \frac{sL[f(t)] - f(0)}{s + \alpha(1-s)}, \qquad 0 \le \alpha \le 1$$

Similarly,

$$L[_0D_t^{(\alpha+1)} f(t)](s) = \frac{s^2 L[f(t)] - sf(0) - f'(0)}{s + \alpha(1-s)}, \qquad 0 \le \alpha \le 1 \tag{3.3}$$

Taking Laplace transform of Eqs.(2.10)–(2.13) using the Eqs.(3.1) and (3.3), we get,

$$\frac{d^2\bar{e}}{dx^2} - \beta \frac{d^2\bar{\theta}}{dx^2} = s^2 \bar{e} \tag{3.4}$$

$$\bar{\sigma} = \bar{e} - \beta\bar{\theta} - \frac{D}{s} \tag{3.5}$$

$$\frac{d^2\bar{\phi}}{dx^2} = \left[s + \frac{\tau_0 s^2}{s + \alpha(1-s)}\right]\bar{\theta} + \epsilon\left[s + \frac{\tau_0 s^2}{s + \alpha(1-s)}\right]\bar{e} \tag{3.6}$$

$$\bar{\theta} = \bar{\phi} - \omega\frac{d^2\phi}{dx^2} \tag{3.7}$$

The boundary conditions (2.1)–(2.2) takes the form,

$$\bar{\phi}(0,s) = \bar{F}(s), \qquad \bar{\phi}(h,s) = 0 \tag{3.8}$$

$$\bar{e}(0,s) = \bar{e}(h,s) = 0, \qquad \bar{\sigma}(0,s) = 0 \tag{3.9}$$

Now by combining Eqs.(3.6)–(3.7), we obtain,

$$\frac{d^2\bar{\phi}}{dx^2} = L\bar{\phi} + L\epsilon\bar{e} \tag{3.10}$$

where,

$$L = \frac{(1+\tau_0)s^2 + \alpha s(1-s)}{(1+\omega s)(s+\alpha(1-s)) + \omega\tau_0 s^2}$$

Putting the value from Eq.(3.10) into Eq.(3.7), we obtain,

$$\bar{\theta} = (1 - \omega L)\bar{\phi} - \omega L\epsilon\bar{e} \tag{3.11}$$

Differentiating Eq.(3.11) twice w.r.t. x, we obtain,

$$\frac{d^2\bar{\theta}}{dx^2} = (1 - \omega L)\frac{d^2\bar{\phi}}{dx^2} - \omega L\epsilon\frac{d^2\bar{e}}{dx^2} \tag{3.12}$$

Substituting this value from Eq.(3.12) into Eq.(3.4) and using Eq.(3.10), we obtain,

$$\frac{d^2\bar{e}}{dx^2} = M\bar{\phi} + N\bar{e} \tag{3.13}$$

where,

$$M = \frac{\beta L(1-\omega L)}{1+\beta\omega L\epsilon}, \qquad N = \frac{s^2 + \beta L\epsilon(1-\omega L)}{1+\beta\omega L\epsilon}$$

4. DIRECT APPROACH

Eliminating $\bar{e}$ between Eqs.(3.10) and (3.13), we get,

$$\left[\frac{d^4}{dx^4} - (L+N)\frac{d^2}{dx^2} + (LN - \epsilon LM) \right] \bar{\phi} = 0 \tag{4.1}$$

The solution of Eq.(4.1) can be written as follows:

$$\bar{\phi}(x,s) = \sum_{k=1}^{2} (A_k e^{-xP_k} + B_k e^{xP_k}) \tag{4.2}$$

Similarly, eliminating $\bar{\phi}$ between Eqs.(3.10) and (3.13), we get,

$$\left[\frac{d^4}{dx^4} - (L+N)\frac{d^2}{dx^2} + (LN - \epsilon LM) \right] \bar{e} = 0 \tag{4.3}$$

and the solution of Eq.(4.3) takes the form as,

$$\bar{e}(x,s) = \sum_{k=1}^{2} (C_k e^{-xP_k} + D_k e^{xP_k}) \tag{4.4}$$

where, A_k, B_k, C_k and D_k, for $k = 1, 2$ are all parameters depending on s.
By applying boundary conditions defined in Eq.(3.8) and Eq.(3.9) on Eqs.(4.2) and (4.4), we can determined the parameters as follows:

$$A_1 = \frac{\phi_0(L - P_2^2)e^{2hP_1}}{2(P_1^2 - P_2^2)\sin \mathrm{h}(hP_1)}, \qquad B_1 = \frac{\phi_0(P_2^2 - L)}{2(P_1^2 - P_2^2)\sin \mathrm{h}(hP_1)} \tag{4.5}$$

$$A_2 = \frac{\phi_0(P_1^2 - L)e^{2hP_2}}{2(P_1^2 - P_2^2)\sin \mathrm{h}(hP_2)}, \qquad B_2 = \frac{\phi_0(L - P_1^2)}{2(P_1^2 - P_2^2)\sin \mathrm{h}(hP_2)} \tag{4.6}$$

from these relations we can write,

$$C_k = \frac{(P_k^2 - L)A_k}{\epsilon L}, \qquad D_k = \frac{(P_k^2 - L)B_k}{\epsilon L}, \qquad k = 1, 2 \tag{4.7}$$

Here $\pm P_1, \pm P_2$ are zeros of characteristic equation,

$$P^4 - (L+N)P^2 + LN - \epsilon LM = 0$$

and which satisfy the relations,

$$P_1 + P_2 = L + N$$
$$P_1 \cdot P_2 = LN - \epsilon LM$$

Substituting the values from Eqs.(4.5)–(4.7) in Eq.(4.2) and Eq.(4.4), we obtain the conductive heat function and strain function as follows:

$$\bar{\phi}(x,s) = \phi_1 \sin \mathrm{h}[(h - x)P_1] + \phi_2 \sin \mathrm{h}[(h - x)P_2] \tag{4.8}$$

where

$$\phi_1 = 2A_1 e^{-2hP_1} \quad \text{and} \quad \phi_2 = 2A_2 e^{-2hP_2} \tag{4.9}$$

and

$$\bar{e}(x,s) = e_1 \sin h[(h-x)P_1] + e_2 \sin h[(h-x)P_2] \qquad (4.10)$$

where

$$e_1 = 2C_1 e^{-2hP_1} \quad \text{and} \quad e_2 = 2C_2 e^{-2hP_2} \qquad (4.11)$$

Substituting Eqs.(4.8) and (4.10) in Eq.(3.11), we obtain the thermodynamical heat function as follows:

$$\bar{\theta}(x,s) = \theta_1 \sin h[(h-x)P_1] + \theta_2 \sin h[(h-x)P_2] \qquad (4.12)$$

where

$$\theta_1 = (1 - \omega L)\phi_1 - \omega \epsilon L e_1 \quad \text{and} \quad \theta_2 = (1 - \omega L)\phi_2 - \omega \epsilon L e_2 \qquad (4.13)$$

Substituting Eqs.(4.10) and (4.12) in Eq.(3.5), we obtain the stress function as follows:

$$\bar{\sigma}(x,s) = \sigma_1 \sin h[(h-x)P_1] + \sigma_2 \sin h[(h-x)P_2] - \frac{D}{s} \qquad (4.14)$$

where

$$\sigma_1 = e_1 - \beta\theta_1 \quad \text{and} \quad \sigma_2 = e_2 - \beta\theta_2 \qquad (4.15)$$

Now from Eq.(2.8) and Eq.(4.12) we can write the expression in the Laplace transform domain as,

$$\frac{\partial \bar{u}}{\partial x} = e_1 \sin h[(h-x)P_1] + e_2 \sin h[(h-x)P_2] \qquad (4.16)$$

Thus we have,

$$\bar{u} = u_1 \cosh[(h-x)P_1] + u_2 \cosh[(h-x)P_2] \qquad (4.17)$$

where

$$u_1 = -\frac{e_1}{P_1} \quad \text{and} \quad u_2 = -\frac{e_2}{P_2} \qquad (4.18)$$

which completes the solution.

5. APPLICATION OF BOUNDARY CONDITIONS

Consider a finite rod $0 \leq x \leq h$ at a uniform temperature T_0.

5.1. **Thermal Shock.** :

Here, we have applied the thermal shock to the boundary $x = 0$, given by

$$F(0,t) = F_0 H(t) \qquad (5.1)$$

where,

$$H(t) = \{ \begin{matrix} 1 \, , \\ 0 \, , \end{matrix} \quad \begin{matrix} t \geq 0 \\ t < 0 \end{matrix} \ \ ,$$

is a Heaviside unit step function.

Taking Laplace transform of Eq.(5.1), we get,

$$\phi_0 = \bar{F}(s) = \frac{F_0}{s} \qquad (5.2)$$

After substituting value from Eq.(5.2) into Eqs.(4.8)–(4.18), we get the complete solution.

5.2. **Ramp-Type Heating.** :

Here, we have applied the heating to the boundary $x = 0$, given by

$$F(0,t) = \{ \begin{matrix} 0\,, & t \le 0 \\ \frac{F_0}{t_0}t\,, & 0 < t \le t_0 \\ F_0\,, & t > t_0 \end{matrix} \tag{5.3}$$

where, t_0 is the ramp-type parameter.

Taking Laplace transform of Eq.(5.3), we get,

$$\phi_0 = \bar{F}(s) = \frac{F_0(1 - e^{-st_0})}{t_0 s^2} \tag{5.4}$$

After substituting value from Eq.(5.4) into Eqs.(4.8)–(4.18), we get the complete solution.

5.3. **Harmonically Varying Temperature.** :

Here, we have applied the heating to the boundary $x = 0$, given by

$$F(0,t) = F_0 e^{i\,\Omega\,t} \tag{5.5}$$

where, $i = \sqrt{-1}$.

Taking the Laplace transform of Eq.(5.5), we obtain,

$$\phi_0 = \bar{F}(s) = \frac{F_0}{s - i\,\Omega} \tag{5.6}$$

After substituting value from Eq.(5.6) into Eqs.(4.8)–(4.18), we get the complete solution.

6. NUMERICAL INVERSION OF THE LAPLACE TRANSFORM

Let $\bar{f}(r,z,s)$ be the Laplace transform of $f(r,z,t)$. The inverse Laplace transform (Honig and Hirdes,1984 [19]) is defined by,

$$f(r,z,t) = \frac{e^{bt}}{2\pi} \int_{-\infty}^{\infty} e^{itv} \bar{f}(b+iv) dv$$

where, b is a positive number such that $b > \text{Re}[$ singularities of $\bar{f}(r,z,s)$ $]$

Here, the function $f(r,z,t)$ is approximated by,

$$f(r,z,t) \approx \frac{1}{2}a_0 + \sum_{k=1}^{N} a_k \approx f_N(r,z,t), \quad \text{for } 0 \le t \le 2l$$

where,

$$a_k = \frac{e^{bt}}{l} Re \left[e^{\frac{ik\pi t}{l}} \bar{f}\left(b + \frac{ik\pi}{l}\right) \right]$$

7. NUMERICAL VALUES OF CONSTANTS

For the given problem we have taken the following values of numerical constants [35]:

$$\epsilon = 0.414774, \quad \beta = 0.00254040, \quad \alpha = 0.50, \quad \tau_0 = 0.02,$$
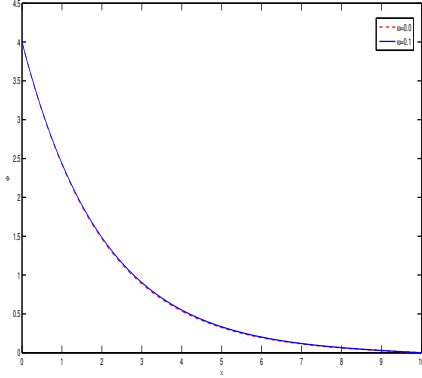$$D = 10^{-7}, \quad F_0 = 1.0, \quad \omega = 0.1, \quad h = 10, \quad \Omega = 10^{-5}.$$

Figure 1: The conductive temperature for thermal shock heating
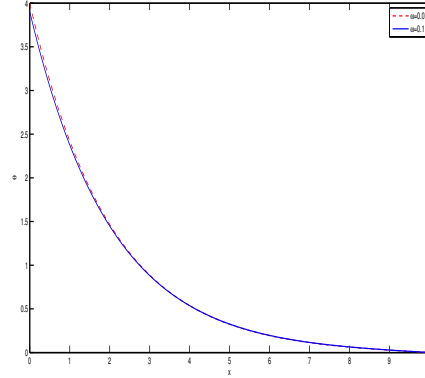

Figure 2: The thermodynamic heat distribution for thermal shock heating

8. GRAPHICAL INTERPRETATION

In this paper we have used the value $t = 0.25$ for calculating the numerical values of all field functions each for three different type of heating application and plotted the obtained results using MATLAB.

Figures 1–5 show the graphs of field functions with thermal shock for different values of $\omega = 0.0, 0.1$.

Figure 1 shows the fractional change in conductive temperature for fixed value of $\alpha = 0.50$. The function ϕ decreases as the value of ω increases from 0.0 to 0.1 as well as for the increase in value of x. It vanishes for large value of x.

Figure 2 shows the fractional change in thermodynamic temperature for fixed value of $\alpha = 0.50$. The function θ decreases for increase in value of x and it vanishes for large value of x. Also it decreases when ω increases.

Figure 3 represents the displacement distribution for fixed fractional parameter $\alpha = 0.50$. The function u decreases gradually when x is increasing. Also as ω increases u decreases.

Figure 4 represents the stress distribution for fixed value of $\alpha = 0.50$. The absolute value of σ_{xx} decreases for $\omega = 0.0$ and $\omega = 0.1$ and vanishes for large value of x. Also as ω increases the absolute value of σ_{xx} decreases.

Figure 5 shows the strain distribution for fixed value of $\alpha = 0.50$. Here, when $\omega = 0.0$ the absolute value of function e takes maximum value 3.2×10^{-3}. It decreases as the value of ω increases. Also as x increases value of e gradually decreases and vanishes for large values of x.
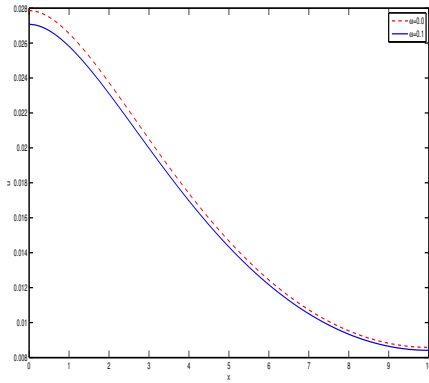
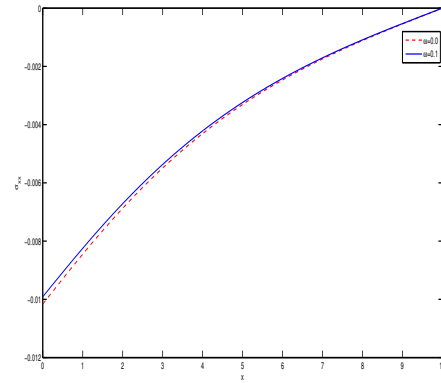Figure 3: The displacement distribution for thermal shock heating

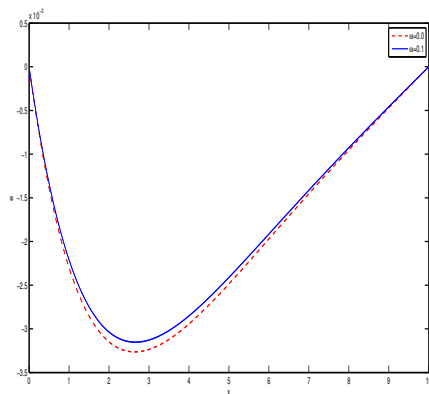Figure 4: The stress distribution for thermal shock heating

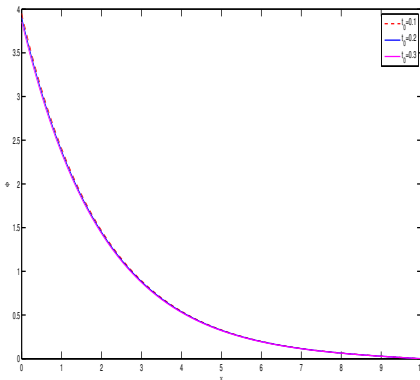Figure 5: The strain distribution for thermal shock heating

Figure 6: The conductive heat distribution for ramp-type heating

Figures 6–10 show the graphs of field functions with ramp-type heating for $t_0 = 0.1, 0.2, 0.3$

Figure 6 represents the fractional change in ϕ for fixed value of $\alpha = 0.50$. Here, ϕ decreases as we go on increasing the value of t_0. The function ϕ decreases in each of three cases and vanishes for large value of x.
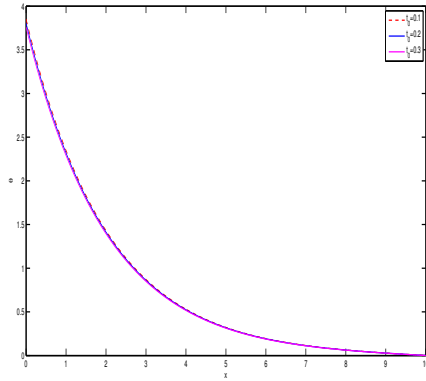
Figure 7: The thermodynamical heat distribution for ramp-type heating
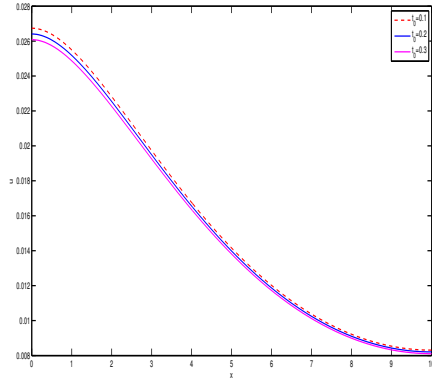

Figure 8: The displacement distribution for ramp-type heating
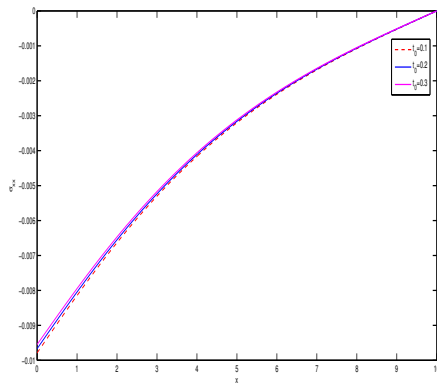

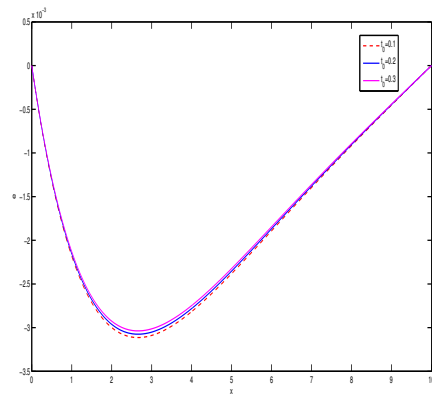Figure 9: The stress distribution for ramp-type heating

Figure 10: The strain distribution for ramp-type heating

Figure 7 represents the fractional change in θ for fixed value of $\alpha = 0.50$. Here θ decreases when t_0 is increasing. The function θ also decreases as x is goes on increasing and vanishes for large x.

Figure 8 shows the fractional change in u for fixed value of $\alpha = 0.50$. The function u decreases for both cases when t_0 is increasing and x is increasing.
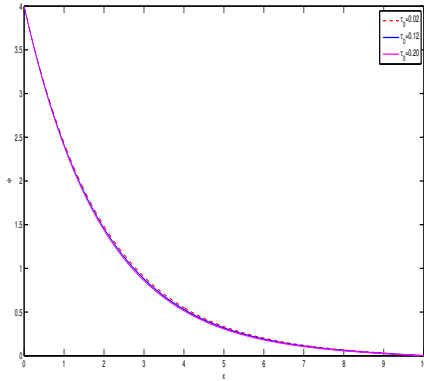
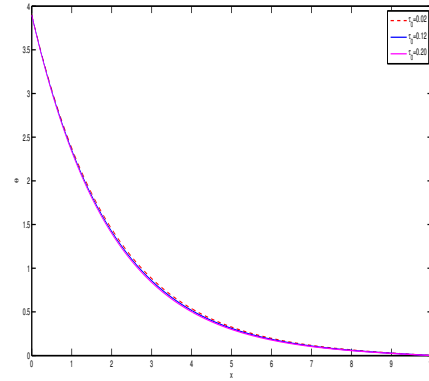Figure 11: The conductive temperature for harmonically vary heating

Figure 12: The thermodynamical distribution for harmonically vary heating

Figure 9 represents the change in stress distribution for fixed value of fractional parameter $\alpha = 0.50$. Here there is a decrease in absolute value of σ_{xx} if t_0 goes on increasing as well as when x is increasing. Moreover, it vanishes for large x.

Figure 10 represents the change in strain distribution for fixed value of fractional parameter $\alpha = 0.50$. The absolute value of e is maximum i.e., 3.1×10^{-3} for $t_0 = 0.1$ and it decreases not only if t_0 increases but also for increase in x. Moreover, it vanishes for large x.

Figures 11–15 show the graphs of field functions with harmonically vary heating for different values of $\tau_0 = 0.02, 0.12, 0.20$

Figure 11 shows the change in function ϕ for fixed value of $\alpha = 0.50$. As we increase the value of τ_0, the function ϕ decreases. Also ϕ goes on decreasing with increase in value of x and vanishes for large x.

Figure 12 shows the change in function θ for fixed value of $\alpha = 0.50$. Here, θ decreases with increase in value of τ_0 as well as it decreases with increase in value of x and vanishes for large x.

Figure 13 represents the change in displacement function u for $\alpha = 0.50$. The function u increases when the value of τ_0 increases, but u decreases gradually with increase in x.

Figure 14 shows the change in stress function σ_{xx} for fixed value of $\alpha = 0.50$. The absolute value of σ_{xx} decreases when τ_0 increases and also when x increases the function σ_{xx} decreases and vanishes for large x.

Figure 15 shows the change in the strain distribution for fixed value of $\alpha = 0.50$. Here the absolute value of e is increases when τ_0 increases but as x increases the value of e decreases and vanishes for large x. It takes maximum absolute value 3.2×10^{-3} for $\tau_0 = 0.20$.
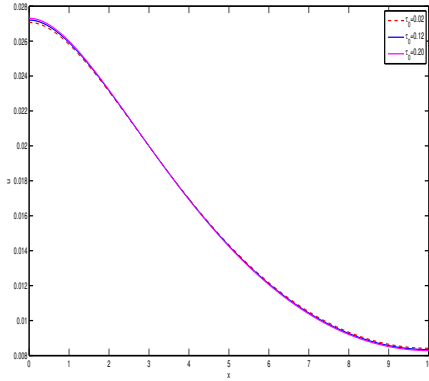
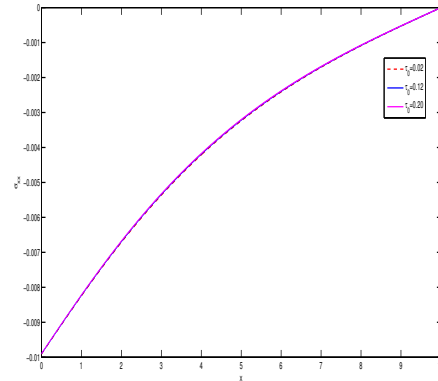Figure 13: The displacement distribution for harmonically vary heating

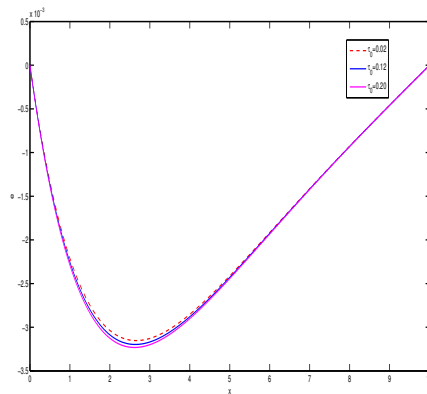Figure 14: The stress distribution for harmonically vary heating

Figure 15: The strain distribution for harmonically vary heating

9. CONCLUSION

We have solved the problem using two-type temperature method with three different heating applications and modified the work of E. Bassiouny and H. M. Youssef [35] applying the Caputo-Fabrizio fractional order derivative.

In this paper we can see that this theory has been recovered the discontinuities in the temperature, stress, strain and displacement. We observed that the fractional order parameter α,

ramp-type parameter t_0 and the relaxation time τ_0 has very significant effects on temperature, stress, strain and displacement.

ORCID ID: 0000-0003-3551-301X

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