

Higher Spectral Efficiency of 3-User Cross CSC NOMA in 5G Systems

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Abstract

As a new paradigm in non-orthogonal multiple access (NOMA), correlated superposition coding (CSC) has gained an attention in the literature of NOMA, in contrast to standard independent superposition coding (ISC). In the conventional 3-user CSC scheme, it has been reported that the average allocated power can be reduced, owing to an introduced correlation between transmitted signals. Thus, this paper proposes a 3-user cross CSC scheme with purely-imaginary correlation coefficients.

First, we derive the achievable data rates of the proposed 3-user cross CSC scheme, for each of the three users. Then, simulations demonstrate that for the proposed 3-user cross CSC scheme, the achievable data rates of the first and second users increase greatly and slightly, respectively, whereas the achievable data rate of the third user decreases little, compared to those of the conventional 3-user CSC scheme. In addition, we also show that the sum rate of the three users of the proposed 3-user cross CSC scheme is much larger than that of the three users of the conventional 3-user CSC scheme.

As a result, the proposed 3-user cross CSC scheme could be a solution to the problem of the reduction of the average allocated total power in the conventional 3-user CSC scheme toward the fifth-generation (5G) NOMA mobile networks.

Keywords: NOMA, 5G, Correlation Coefficient, Superposition coding, Power allocation.

1. Introduction

The large increase of the number of mobile devices in the Internet of Thing (IoT) network has demanded the higher spectral efficiency for the future mobile communications [1], [2]. Non-orthogonal multiple access (NOMA) has a tremendous attention as a promising multiple access [3], [4]. Unlike orthogonal multiple access (OMA) in 4G mobile networks [5-7], NOMA has the larger system capacity. This superiority of NOMA over OMA was optimized in [8]. Cooperative NOMA was investigated for full-duplex relaying [9]. NOMA-based underwater visible light communication was considered in [10]. The power-outage tradeoff of NOMA was investigated [11]. The bit-error rate for NOMA were derived in [12]. Local oscillator imperfection of NOMA was studied [13].

Recently, correlated superposition coding (CSC) NOMA schemes have gained an attention in the literature of NOMA, in contrast with standard independent superposition coding (ISC) [14]. In the conventional 3-user

CSC scheme, it has been reported that the average allocated power can be reduced, owing to an introduced correlation between transmitted signals [15]. Thus, this paper proposes a 3-user cross CSC scheme with purely-imaginary correlation coefficients.

First, we propose the 3-user cross CSC scheme in order to avoid the reduction of the average allocated power. Then, simulations demonstrate that for the proposed 3-user cross CSC scheme, the achievable data rates of the first and second users increase greatly and slightly, respectively, whereas the achievable data rate of the third user decrease little, compared to those of the conventional 3-user CSC scheme. In addition, we also show that the sum rate of the three users of the proposed 3-user cross CSC scheme is much larger than that of the three users of the conventional 3-user CSC scheme.

The remainder of this paper is organized as follows. In Section 2, the system and channel model are described. The achievable sum rate of 3-user cross CSC NOMA is derived in Section 3. The numerical results are addressed and discussed in Section 4. Finally, the conclusions are presented in Section 5.

The main contributions of this paper are summarized as follows:

- We propose a 3-user cross CSC NOMA scheme with purely-imaginary correlation coefficients, in contrast to a conventional 3-user CSC NOMA scheme.
- Then, we derive the achievable data rates of the proposed 3-user cross CSC scheme, for each of the three users.
- We show that for the proposed 3-user cross CSC scheme, the achievable data rates of the first and second users increase greatly and slightly, respectively.
- It is also shown that the achievable data rate of the third user decrease little, compared to those of the conventional 3-user CSC scheme.
- In addition, we also show that the sum rate of the three users of the proposed 3-user cross CSC scheme is much larger than that of the three users of the conventional 3-user CSC scheme.

2. System and Channel Model

There are one base station and three users in a cellular downlink NOMA network. The complex channel coefficient between the m th user and base station is denoted by h_m , $m = 1, 2, 3$, and the channels are sorted as $|h_1| \geq |h_2| \geq |h_3|$. The base station will send the *correlated* superimposed signal $z = \sqrt{P_A\beta_1}c_1 + \sqrt{P_A\beta_2}c_2 + \sqrt{P_A\beta_3}c_3$, where the average *allocated* total power P_A of c_1 , c_2 , and c_3 is given by

$$P_A = \frac{P}{\sum_{i=1}^3 \sum_{j=1}^3 \rho_{i,j} \sqrt{\beta_i \beta_j}}, \quad (1)$$

where P is the average power consumed at the base station, the correlation coefficient of the messages' signals is $\rho_{i,j} = E[c_i c_j^*]$, and c_m is the signal for the m th user with the average unit power. β_m is the power allocation coefficient, with $\beta_1 + \beta_2 + \beta_3 = 1$. Note that equation (1) for P_A is derived based on $P = E[|z|^2]$,

i.e., equating the average power of z to P . The received signal y_m at the m th user is expressed as follows:

$$y_m = h_m z + n_m, \quad (2)$$

where $n_m \sim CN(0, \sigma^2)$ is complex additive white Gaussian noise (AWGN).

3. Derivation of Achievable Sum Rate of Proposed 3-User Cross CSC NOMA

To derive the spectral efficiency of the 3-user cross CSC scheme, we introduce the existing 3-user conventional CSC scheme [15]:

$$\rho_{1,2} = \text{Re}\{\rho_{1,2}\}, \quad (3)$$

$$\rho_{1,3} = \text{Re}\{\rho_{1,3}\}, \quad (4)$$

and

$$\rho_{2,3} = \text{Re}\{\rho_{2,3}\}. \quad (5)$$

Then the *conditional* achievable sum rate given $|h_1|$, $|h_2|$, and $|h_3|$ in the conventional 3-user conventional CSC NOMA scheme is expressed as [15]:

$$R_{sum}^{(3\text{-user conventional})} = R_1^{(3\text{-user conventional})} + R_2^{(3\text{-user conventional})} + R_3^{(3\text{-user conventional})}, \quad (6)$$

where

$$R_1^{(3\text{-user conventional})} = \log_2 \left(1 + \frac{|h_1|^2 P_A \beta_1 (1 - \rho_{1,2,3}^2)}{\sigma_2^2} \right), \quad (7)$$

$$R_2^{(3\text{-user conventional})} = \log_2 \left(\frac{|h_2|^2 P_A \left(\beta_1 (1 - |\rho_{1,3}|^2) + \beta_2 (1 - |\rho_{2,3}|^2) + 2\sqrt{\beta_1} \sqrt{\beta_2} \text{Re}\{\rho_{1,2} - \rho_{1,3}\rho_{2,3}\} \right) + \sigma^2}{|h_2|^2 P_A \beta_1 (1 - \rho_{1,2,3}^2) + \sigma^2} \right), \quad (8)$$

and

$$R_3^{(3\text{-user conventional})} = \log_2 \left(\frac{|h_3|^2 P + \sigma^2}{|h_3|^2 P_A \left(\beta_1 (1 - |\rho_{1,3}|^2) + \beta_2 (1 - |\rho_{2,3}|^2) + 2\sqrt{\beta_1} \sqrt{\beta_2} \text{Re}\{\rho_{1,2} - \rho_{1,3}\rho_{2,3}\} \right) + \sigma^2} \right), \quad (9)$$

with

$$\rho_{1|2,3}^2 = \frac{|\rho_{1,2} - \rho_{1,3}|^2 + 2\operatorname{Re}\{\rho_{1,2}\rho_{3,1}(1 - \rho_{2,3})\}}{1 - |\rho_{2,3}|^2}. \quad (10)$$

Now, we propose the novel 3-user cross CSC scheme, as follows:

$$\rho_{1,2} = \operatorname{Im}\{\rho_{1,2}\}, \quad (11)$$

$$\rho_{1,3} = \operatorname{Im}\{\rho_{1,3}\}, \quad (12)$$

and

$$\rho_{2,3} = \operatorname{Im}\{\rho_{2,3}\}. \quad (13)$$

Based on the proposed 3-user cross CSC scheme, the average *allocated* total power P_A of c_1 , c_2 , and c_3 in equation (1) is simplified as

$$\begin{aligned} P_A &= \frac{P}{\sum_{i=1}^3 \sum_{j=1}^3 \rho_{i,j} \sqrt{\beta_i \beta_j}} \\ &= \frac{P}{\beta_1 + \beta_2 + \beta_3 + 2\operatorname{Re}\{\rho_{1,2}\}\sqrt{\beta_1 \beta_2} + 2\operatorname{Re}\{\rho_{1,3}\}\sqrt{\beta_1 \beta_3} + 2\operatorname{Re}\{\rho_{2,3}\}\sqrt{\beta_2 \beta_3}} \\ &= \frac{P}{1 + 2\operatorname{Re}\{\rho_{1,2}\}\sqrt{\beta_1 \beta_2} + 2\operatorname{Re}\{\rho_{1,3}\}\sqrt{\beta_1 \beta_3} + 2\operatorname{Re}\{\rho_{2,3}\}\sqrt{\beta_2 \beta_3}} \\ &= P, \end{aligned} \quad (14)$$

where the third equality holds with $\beta_1 + \beta_2 + \beta_3 = 1$, and the last equality holds by equations (11), (12), and (13). The *conditional* achievable sum rate given $|h_1|$, $|h_2|$, and $|h_3|$ in the proposed 3-user cross CSC NOMA scheme is expressed as:

$$R_{sum}^{(3\text{-user cross})} = R_1^{(3\text{-user cross})} + R_2^{(3\text{-user cross})} + R_3^{(3\text{-user cross})}, \quad (15)$$

where

$$R_1^{(3\text{-user cross})} = \log_2 \left(1 + \frac{|h_1|^2 P \beta_1 (1 - \rho_{1|2,3}^2)}{\sigma_2} \right), \quad (16)$$

$$R_2^{(3\text{-user cross})} = \log_2 \left(\frac{|h_2|^2 P \left(\beta_1 (1 - |\rho_{1,3}|^2) + \beta_2 (1 - |\rho_{2,3}|^2) + 2\sqrt{\beta_1}\sqrt{\beta_2} \operatorname{Re}\{\rho_{1,2} - \rho_{1,3}\rho_{2,3}\} \right) + \sigma^2}{|h_2|^2 P \beta_1 (1 - \rho_{1|2,3}^2) + \sigma^2} \right), \quad (17)$$

and

$$R_3^{(3\text{-user cross})} = \log_2 \left(\frac{|h_3|^2 P + \sigma^2}{|h_3|^2 P \left(\beta_1 (1 - |\rho_{1,3}|^2) + \beta_2 (1 - |\rho_{2,3}|^2) + 2\sqrt{\beta_1}\sqrt{\beta_2} \operatorname{Re}\{\rho_{1,2} - \rho_{1,3}\rho_{2,3}\} \right) + \sigma^2} \right). \quad (18)$$

4. Numerical Results and Discussions

We investigate the impact of the proposed 3-user cross CSC scheme on the sum rate of existing 3-user conventional CSC scheme. For this end, we assume that the conditionals are $|h_1| = \sqrt{2}$, $|h_2| = 1$, and $|h_3| = 0.1$, and the average total transmitted signal-to-noise power ratio (SNR) is $P/\sigma^2 = 50$. Then, for the existing 3-user conventional CSC scheme, $\rho_{1,2} = \rho_{1,3} = \rho_{2,3} = \sqrt{0.50}$, whereas for the proposed cross CSC scheme, $\rho_{1,2} = \rho_{1,3} = \rho_{2,3} = j\sqrt{0.50}$, with $j = \sqrt{-1}$. Remarkably, this study investigates only the *conditional* achievable sum rates given the channel gain realizations, so that the simulation results are applicable to any fading channel models, for example, Rayleigh fading channel.

First, in order to investigate the impact of the proposed 3-user cross CSC scheme on the sum rate of the existing 3-user conventional CSC scheme, we depict the achievable data rates of the first user both for the proposed 3-user cross CSC scheme and the existing 3-user conventional CSC in Fig. 1.

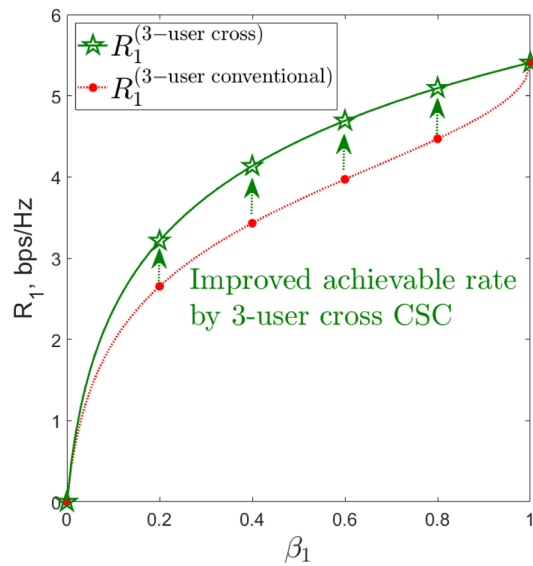


Figure 1. Comparison of achievable data rates of existing 3-user conventional CSC scheme and proposed 3-user cross CSC scheme, for first user.

As shown in Fig. 1, for the first user, the achievable data rate of the proposed 3-user cross CSC scheme increase greatly, compared to that of the conventional 3-user CSC scheme, over the entire power allocation range, i.e., $0 \leq \beta_1 \leq 1$. Notably, this improvement of the achievable data rate of the proposed 3-user cross CSC scheme can be achieved by avoiding the reduction of the average allocated power in the conventional 3-user CSC scheme.

Second, in order to investigate the effect of the proposed 3-user cross CSC scheme on the sum rate of the existing 3-user conventional CSC scheme, we depict the achievable data rates of the second user both for the proposed 3-user cross CSC scheme and the existing 3-user conventional CSC in Fig. 2.

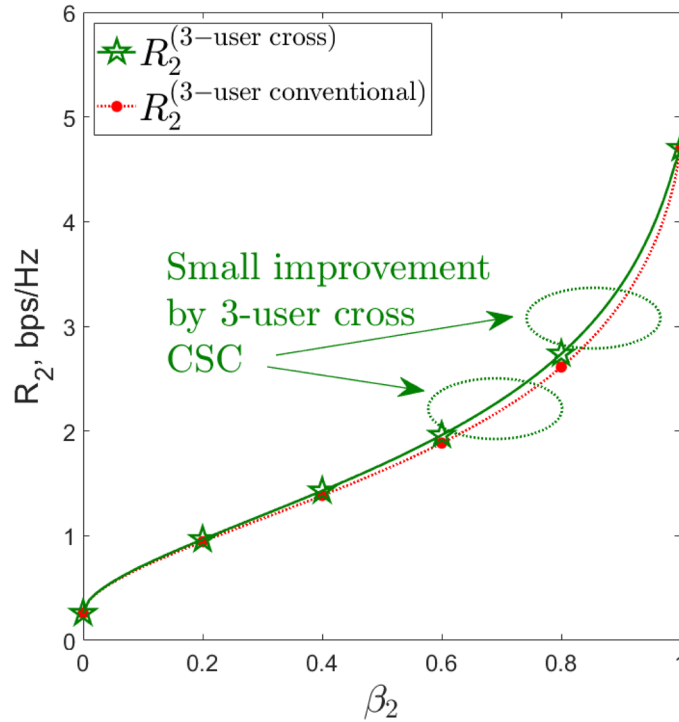


Figure 2. Comparison of achievable data rates of existing 3-user conventional CSC scheme and proposed 3-user cross CSC scheme, for second user.

As shown in Fig. 2, for the second user, the achievable data rate of the proposed 3-user cross CSC scheme increase slightly, compared to that of the conventional 3-user CSC scheme, over the entire power allocation range. Remark that this small improvement of the achievable data rate is due to the effective increase of the inter-user interference of the first user by the proposed 3-user cross CSC scheme.

Third, in order to investigate the impact of the proposed 3-user cross CSC scheme on the sum rate of the existing 3-user conventional CSC scheme, we depict the achievable data rates of the third user both for the proposed 3-user cross CSC scheme and the existing 3-user conventional CSC in Fig. 3.

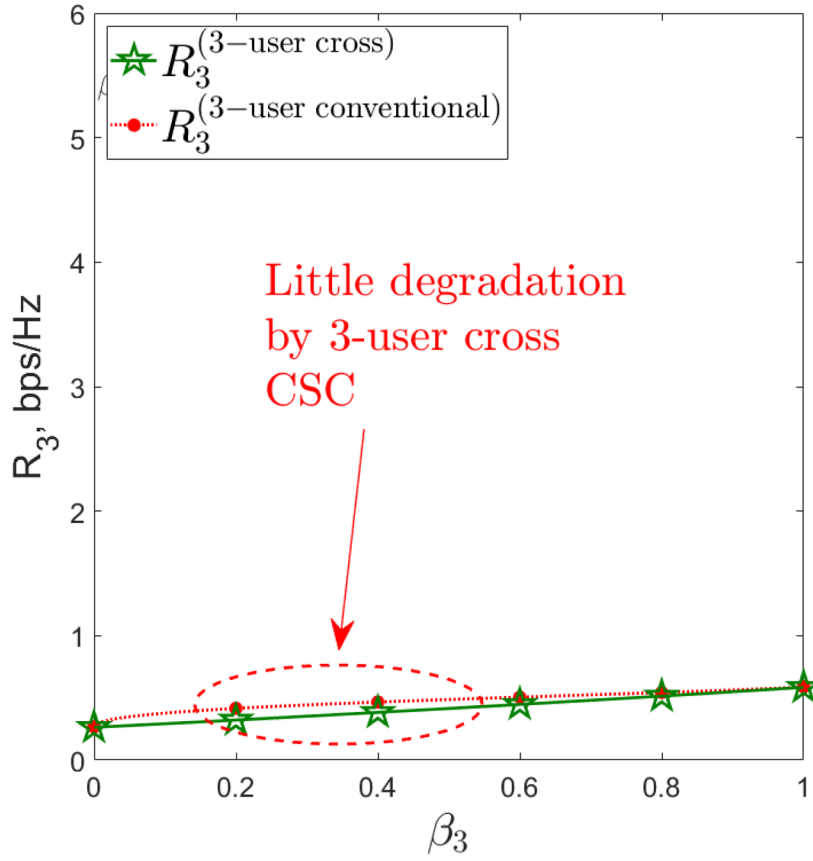


Figure 3. Comparison of achievable data rates of existing 3-user conventional CSC scheme and proposed 3-user cross CSC scheme, for third user.

As shown in Fig. 3, for the third user, the achievable data rate of the proposed 3-user cross CSC scheme degrade little, compared to that of the conventional 3-user CSC scheme, over the entire power allocation range. Note that this small degradation of the achievable data rate is due to the effective increase of the inter-user interference of the first and second users by the proposed 3-user cross CSC scheme.

Finally, in order to investigate the total impact of the proposed 3-user cross CSC scheme on the sum rate of the existing 3-user conventional CSC scheme, we depict the sum rates of the three users both for the proposed 3-user cross CSC scheme and the existing 3-user conventional CSC in Fig. 4.

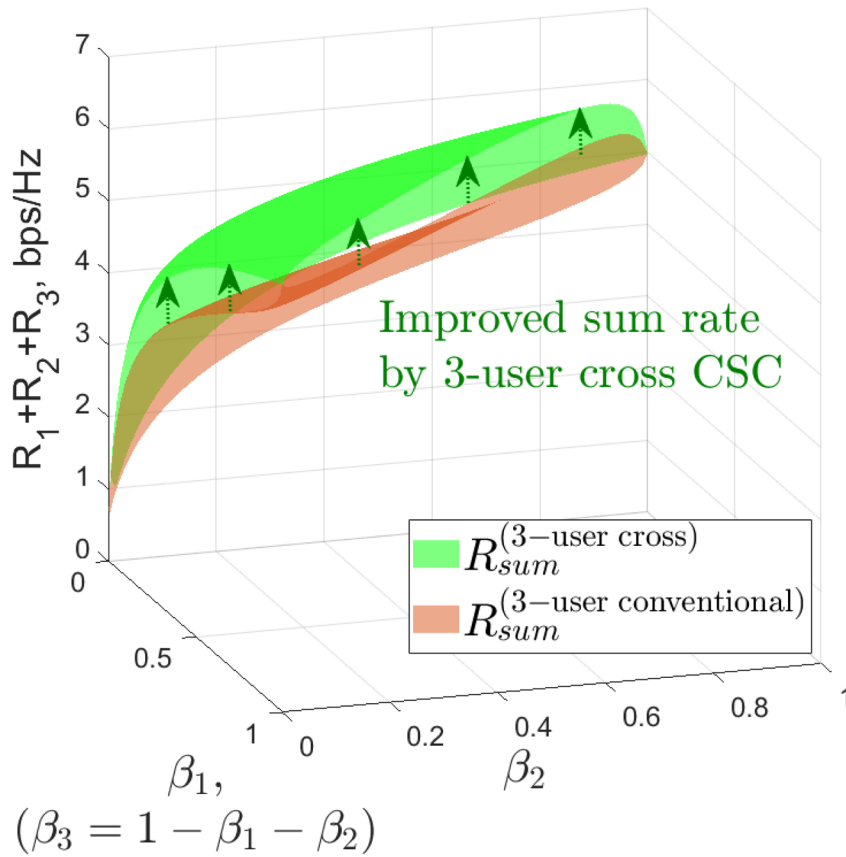


Figure 4. Comparison of achievable sum rates of three users for existing 3-user conventional CSC scheme and proposed 3-user cross CSC scheme.

As shown in Fig. 4, the achievable sum rate of the proposed 3-user cross CSC scheme increases, in comparison to that of the conventional 3-user CSC scheme. It should be noted that this improvement is obtained by the large increase of the achievable data rate of the first user, and the small improvement of the achievable data rate of the second user. Therefore, we demonstrate that the proposed 3-user cross CSC scheme improves the spectral efficiency of the conventional 3-user CSC NOMA scheme.

5. Conclusion

In this paper, we proposed the 3-user cross CSC scheme by purely-imaginary correlation coefficients. First, we derived the achievable data rates of the proposed 3-user cross CSC scheme, for each of the three users. Then, simulations demonstrated that for the proposed 3-user cross CSC scheme, the achievable data rates of the first and second users increase greatly and slightly, respectively, whereas the achievable data rate of the third user decrease little, compared to those of the conventional 3-user CSC scheme. In addition, we also showed that the sum rate of the three users of the proposed 3-user cross CSC scheme is much larger than that of the three users of the conventional 3-user CSC scheme.

As a result, the proposed 3-user cross CSC scheme could be a solution to the problem of the reduction of the average allocated total power in the conventional 3-user CSC scheme toward the 5G NOMA mobile networks.

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