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THE SOC ESTIMATION OF THE LEAD-ACID BATTERY USING KALMAN FILTER†

YONGHO JEON

Abstract. In general, secondary batteries are widely used as an electric energy source. Among them, the state of energy storage of mobile devices is very important information. As a method of estimating a state, there is a method of estimating the state by integrating the current according to an energy storage state of a battery, and a method of designing a state estimator by measuring a voltage and estimating a charge amount based on a battery model. In this study, we designed the state estimator using an extended Kalman filter to increase the precision of the state estimation of the charge amount by including the error of the system model and having the robustness to the noise.

AMS Mathematics Subject Classification : 62L12, 93E10.
Key words and phrases : Lead-acid battery, Kalman filter, battery charge rate, state of charge.

1. Introduction

The battery charges or discharges an electric charge by an electrochemical reaction between an electrolyte and an active material[1-13]. Lead-Acid Battery, which has been widely used as a secondary battery for a long time, is cheaper and has higher stability than other secondary batteries[3-4]. In ordinary vehicles, the electrical load plays an important role in providing additional energy beyond the capacity of the supply, including starting, lighting and ignition functions. In addition, this battery is the key energy source that is cheaper than lithium-based batteries in industrial electric vehicles. A battery model that can predict the charging and discharging characteristics of lead-acid batteries used in actual vehicles needs[3-11]. Based on this, it requires skill to maximize the use efficiency of lead-acid batteries and to change the specifications of the lead-acid batteries that are already used. In the voltage measurement method, because the

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SOC(state of charge) measurement accuracy is determined by the precise simulation of the battery terminal voltage and various related parameters, proper battery modeling is essential.

2. SOC ESTIMATION

In the process of charging and discharging, the charge and discharge characteristics are determined by the electrochemical reaction occurring inside the lead-acid battery, the flow of the electrolyte, the transfer of ions, and the change of the pore of the electrode, etc. Thus, battery models include electrical equivalent circuits, electrochemical and mathematical battery models. The battery equivalent model consists of a series connected resistor and an RC (resistor capacity) network in series, as shown in Fig. 1.

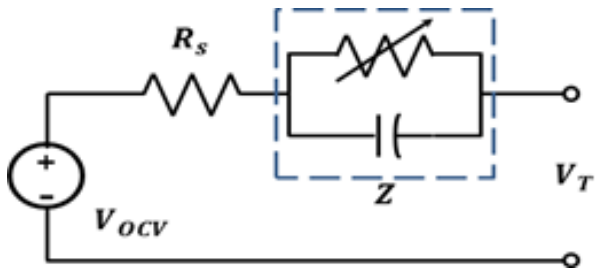

FIGURE 1. Lead-acid Battery Equivalent Circuit Model.

Among the battery models for estimating the charge rate based on the model, the equivalent model uses Fig. 1 consisting of a series connected resistor and a series of RC networks. Based on Thevenin's equivalent circuit model, OCV (open circuit voltage) and internal impedance Z are expressed in parallel circuits of R1//C1. The configuration of the circuit equation is as shown in from Eq. (1) to Eq. (3).

$$V_T(t) = V_{OCV}(t) - V_1(t) - R_s i(t) \tag{1}$$

$$\frac{d}{dt}V_1(t) = -\frac{1}{R_1C_1}V_1(t) + \frac{1}{C_1}i(t) \tag{2}$$

$$\frac{d}{dt}SOC(t) = \frac{1}{Q_{full}}i(t) \tag{3}$$

The state variable at time t is $x(t) = \begin{bmatrix} SOC(t) & V_1(t) \end{bmatrix}^T$, the input is $u(t) = i(t)$, and the output is $y(t) = V_T(t)$. $V_1(t)$ is the voltage drop in the capacitor, $i(t)$ is the current flowing through the circuit, and $V_T(t)$ is the terminal voltage of the entire circuit. In order to design the state estimator, the state-space representation of the circuit equation model from Eq. (1) to Eq. (3) is as shown in from Eq. (4) to Eq. (6).

$$\frac{d}{dt}x(t) = Ax(t) + Bu(t) \tag{4}$$

$$y(t) = V_{OCV}(t) - Cx(t) - Du(t) \tag{5}$$

$$A = \begin{bmatrix} 0 & 0 \\ 0 & -\frac{1}{R_1 C_1} \end{bmatrix}, B = \begin{bmatrix} \frac{1}{Q_{full}} \\ -\frac{1}{C_1} \end{bmatrix}, C = \begin{bmatrix} 0 & 1 \end{bmatrix}, D = R_s \tag{6}$$

The algorithm of the Extended Kalman Filter designed to estimate the battery charge rate is in Fig. 2. Fig. 2 shows the flow chart of current measurement and voltage processing in a two-step process. When acquiring the current value of the ADC (analog to digital convertor) conversion result of the MCU (main control unit), the conversion result corresponds to the input value of the state estimator. With the input value, this system obtains the result value of the advanced state with the mathematical model of the system for the estimated period. This is the preliminary estimate.

3. Kalman Filter Algorithm Design

The SOC of the battery and the state of overvoltage estimate using the system model of Eq. (4) to Eq. (6). When designing Kalman filter by measuring the terminal voltage of battery and designing the current state as input, the system model includes the error of the system model, so that the estimated state diverges after a certain time. This structure adds the noise of fictional system to compensate for model errors. The system model of the discrete time domain is as shown in Eq. (7) to Eq. (11).

$$x_k = F_{k-1}x_{k-1} + G_{k-1}u_{k-1} + w_{k-1} \tag{7}$$

$$y_k = H_k x_k + v_{k-1} \tag{8}$$

$$E(w_k w_j^T) = Q_k \delta_{k-j} \tag{9}$$

$$E(\nu_k \nu_j^T) = R_k \delta_{k-j} \tag{10}$$

$$E(w_k \nu_j^T) = 0 \tag{11}$$

Where x_k is the state variable in k time steps and u_k is the state input. In addition, y_k is an output state, w_{k-1} is noise included in a system model, and v_{k-1} is Gaussian normal distribution as noise included in an observation model. State estimation performs using the initial state of the filter as the best estimate. First, preliminary estimation is the process of estimating state and state covariance using a system model as an optimal estimate. The process of preliminary estimation is as Eq. (12) to Eq. (13).

$$\tilde{P}_k^- = \alpha^2 F_{k-1}\tilde{P}_{k-1}^+ F_{k-1}^T + Q_{k-1} \tag{12}$$

$$\hat{x}_k^- = F_{k-1}\hat{x}_{k-1}^+ + G_{k-1}u_{k-1} \tag{13}$$

The α of Eq. (12) is a parameter to compensate for the uncertainty of the system model, and can be set to a value greater than or equal to 1. If it is set to 1, it is a general Kalman filter, and if it is set to a value slightly larger than 1,

the uncertainty of the system model increases in preliminary estimation. Next, the post-estimation process obtains the optimal state estimate and covariance by fusing the measured values from the observer and the measured values from preliminary estimation, that is, the state $\hat{x}_k^-$ and the covariance $\tilde{P}_k^-$, and the Kalman gain K_k. This is equal to Eq. (16) in the following Eq. (14).

$$K_k = \tilde{P}_k^- H_k^T (H_k \tilde{P}_k^- H_k^T + R_k)^{-1} \tag{14}$$

$$\hat{x}_k^+ = \hat{x}_k^- + K_k(y_k - H_k \hat{x}_k^-) \tag{15}$$

$$\tilde{P}_k^+ = \tilde{P}_k^- - K_k H_k \tilde{P}_k^- \tag{16}$$

The result $\hat{x}_k^+$ obtained in Eq. (15) is the optimal estimate state, and $\tilde{P}_k^+$ in Eq. (16) is the optimal estimate covariance. This is the optimal estimate obtained in k time-steps and is the start of the pre-estimation process of $k+1$ time-steps. The discretization of Eq. (4) to Eq. (6) in the battery system is equal to Eq. (21) in Eq. (17) below.

$$x_k = \begin{bmatrix} SOC & V_1 & V_{OCV} \end{bmatrix}_k^T \tag{17}$$

$$u_k = i_k \tag{18}$$

$$F_k = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 - \frac{T_s}{R_1 C_1} & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{19}$$

$$G_k = \begin{bmatrix} -\frac{T_s}{Q_{full}} & \frac{T_s}{C_1} & 0 \end{bmatrix}_k^T \tag{20}$$

$$H_k = \begin{bmatrix} 0 & -1 & 1 \end{bmatrix}_k \tag{21}$$

Eq. (17) adds to the state variable as a discrete state variable of the battery V_{OCV} and estimates it. V_{OCV} is estimated because the noise is updated by using a constant feature in a certain time interval. Next, using the voltage value obtained from AD7284 as the observed value, this system obtains the optimal estimation result through matching process with a pre-estimated state value. We call that a post estimation or estimated state update process.

4. PERFORMANCE EVALUATION

To verify the performance of the proposed SOC estimator, we simulated the battery model using the measured parameters. As a result, we confirmed that the performance of the estimator can be compared and reviewed. The PSIM program was used to simulate the charge/discharge state of the lead-acid battery. The lead acid battery model used in the simulation is the same as Eq. (22) to Eq. (23).[13]

$$V_T = E_0 - K \frac{Q}{Q - \int idt} (\int idt + i^*) + V_{exp}(t) \tag{22}$$

$$\dot{V}_{exp}(t) = B|i(t)|(-V_{exp}(t) + Au(t)) \tag{23}$$

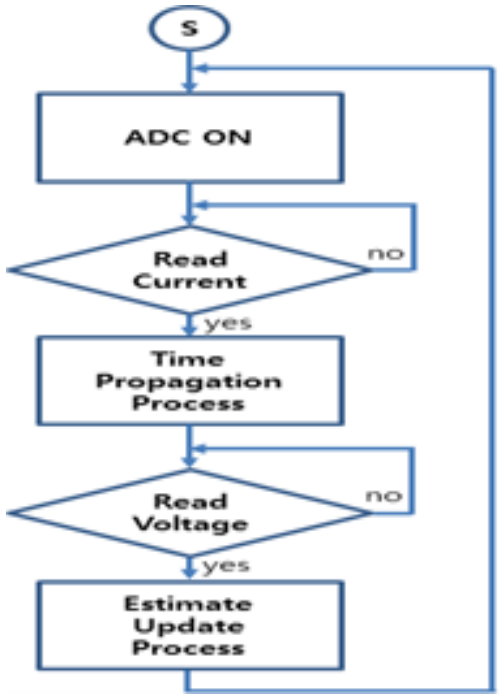

FIGURE 2. SOC Estimation Algorithm.

Here, $V_{exp}(t)$ represents the voltage in the exponential zone from the start of the discharge in the full charge state to the point where the exponential decrease ends, and has the characteristics of Eq. (23). E_0 is the battery constant voltage at full charge. K is the polarization constant. Q is the battery capacity and i^* is the measured current through a low pass filter. A is the size of the exponential zone in which $V_{exp}(t)$ acts, and B is the reciprocal of the time constant. And $u(t)$ is applied as a value of 0 in discharge. Eq. (23) was used to find a solution on the assumption that the current acts as a nonlinear constant. The applied parameters are shown in Table 1.

TABLE 1. battery parameter

Parameter	value
E_0	12.4659 V
R	0.04 Ω
K	0.047 V/Ah
A	0.83 V
B	125 $(Ah)^{-1}$

The battery operation was implemented using the lead-acid battery model of Eq. (22) to Eq. (23). The observed state of the observer measured the output current and used it as the input of the observer, and the measured output terminal voltage was used as the output state to estimate the internal state of the battery and the amount of charge. The electrical model of the battery used for state estimation is shown in Fig. 1. The noise added to the SOC state of the system model of the estimator was 0.01, and the noise added to the state V_1 was 0.01 $[V^2]$, the noise added to the state V_{OCV} was set to 0.1 $[V^2]$. And, the value of *alpha* was set to 1.01, which was set close to the normal extended Kalman filter. Also, the noise added to the observation state V_T was set to 0.01 $[V^2]$.

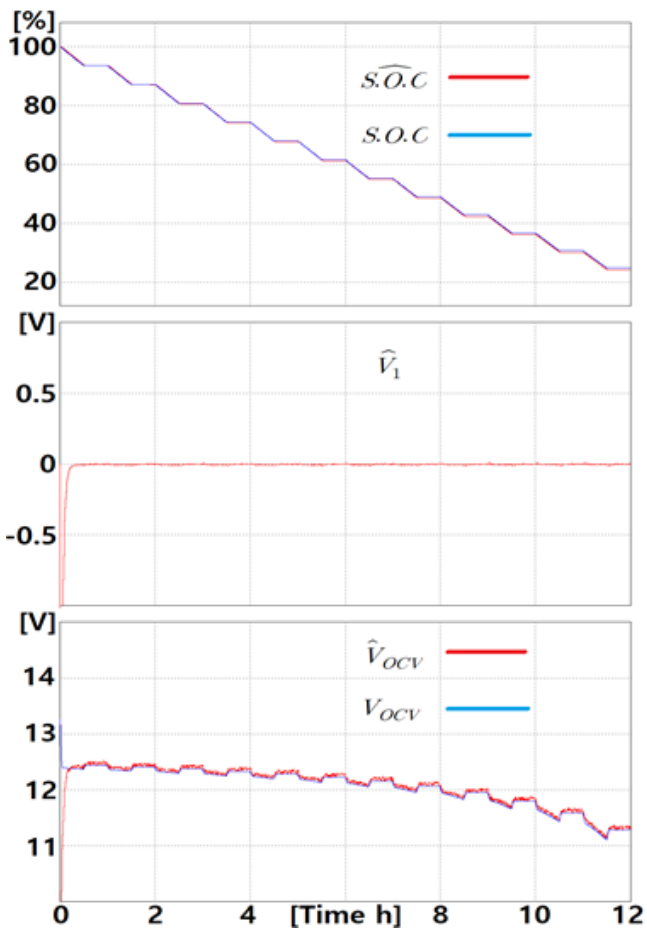

FIGURE 3. Estimated battery state when current 1 [A] discharge.

Fig. 3 shows the state estimation result when the battery model from Eq. (22) to Eq. (23) is discharged with a current of 1A. In the simulation, a current of 1A was discharged for 30 minutes and no current flowed for 30 minutes. The top part of Fig. 3 is the result of the estimation of the SOC. The SOC is created as a result of integrating only the current charged in the battery, and the effect of noise added to the input current has a major influence on the estimation error. As a result, it is judged that the error increases as the discharge time increases, and the estimation error increases as the current exhibits AC characteristics. In order to reduce the error, it can be seen that as the cutoff frequency of the low pass filter is decreased in order to reduce the error, the time delay of the estimated value occurs. The middle in Fig. 3 is an estimate of the voltage across the impedance Z of the battery electrical estimation model in Fig. 1 and is a hypothetical value. The bottom of Fig. 3 is an estimate of the OCV of the battery. In the Extended Kalman filter, the battery open state voltage was estimated as a constant of the section. It was confirmed that the OCV of the battery model decreased according to the SOC. It can be confirmed that the error between the battery model and the estimate is estimated by keeping it within 0.1 [V].

5. RESULTS

In this paper, we proposed the design method for estimating the SOC of lead acid batteries with strong nonlinear characteristics. The state estimator designed by converting the system model in the continuous time domain into a discrete time system. It has the advantage of improving the accuracy of estimation by designing an extended Kalman filter that is robust against noise and can take into account the model error of the battery model that is a nonlinear system. An additional effort is needed to apply the model for simulation to the actual battery model and tune it with each parameter.

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Yongho Jeon received M.Sc. and Ph.D. from KwangWoon University. Since 2009 he has been at JungWon University. His research interests include discrete mathematic, fractional calculus and air craft control.

Department of Aviation Maintenance, JungWon University, 85, Munmu-ro, Goesan-gun, Chungbuk, 28024, Korea.
e-mail: waterjliar@jwu.ac.kr