Multiview-based Spectral Weighted and Low-Rank for Row-sparsity Hyperspectral Unmixing

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Sparse unmixing has been proven to be an effective method for hyperspectral unmixing. Hyperspectral images contain rich spectral and spatial information. The means to make full use of spectral information, spatial information, and enhanced sparsity constraints are the main research directions to improve the accuracy of sparse unmixing. However, many algorithms only focus on one or two of these factors, because it is difficult to construct an unmixing model that considers all three factors. To address this issue, a novel algorithm called multiview-based spectral weighted and low-rank row-sparsity unmixing is proposed. A multiview data set is generated through spectral partitioning, and then spectral weighting is imposed on it to exploit the abundant spectral information. The row-sparsity approach, which controls the sparsity by the $l_{2,0}$ norm, outperforms the single-sparsity approach in many scenarios. Many algorithms use convex relaxation methods to solve the $l_{2,0}$ norm to avoid the NP-hard problem, but this will reduce sparsity and unmixing accuracy. In this paper, a row-hard-threshold function is introduced to solve the $l_{2,0}$ norm directly, which guarantees the sparsity of the results. The high spatial correlation of hyperspectral images is associated with low column rank; therefore, the low-rank constraint is adopted to utilize spatial information. Experiments with simulated and real data prove that the proposed algorithm can obtain better unmixing results.

Keywords: Hyperspectral unmixing, Low-rank, Multiview, Row-sparsity, Spectral weighted

OCIS codes: (100.4145) Motion, hyperspectral image processing; (300.6320) Spectroscopy, high-resolution

I. INTRODUCTION

Hyperspectral remote-sensing technology has been widely used in the military, agriculture, industry, and other fields. However, due to the limited spatial resolution of remote-sensing instruments and the complicated distribution of ground objects, hyperspectral images often contain a large number of mixed pixels. This seriously affects the processing and application of hyperspectral data. Spectral unmixing [1] refers to the identification of spectral features (endmembers) and estimation of the corresponding abundance, which can effectively solve this problem. Linear and nonlinear are the two main unmixing models. The linear unmixing model [2] has been widely used for its flexibility and simplicity. Unmixing algorithms under this model are mainly divided into three categories: statistics-based, geometry-based, and sparse-regression-based.

Sparse unmixing [3] does not require the pure-pixel assumption, and no virtual endmembers will be generated. It assumes that mixed pixels are linear combinations of endmembers in the spectral library. Numerous effective sparse-unmixing algorithms have been proposed in recent years. The sparse-unmixing algorithm via variable splitting and augmented Lagrangian (SUunSAL) [3] is a representative sparse-unmixing algorithm. However, the sparsity degree of SUunSAL is insufficient, and it is easily affected by the...
partitioning strategy is used to construct multiview hyperspectral data. Qi et al. [17] proved that the estimated abundance for virtual endmembers shows large differences in different views; therefore, virtual endmembers can be removed by setting an appropriate spectral weight. Sparse unmixing can be considered as an optimization problem of the \( l_0 \) norm. Many algorithms use convex relaxation methods to solve the \( l_0 \) norm, to avoid the NP-hard problem. However, the sparsity degree is insufficient in many scenarios. To meet the sparsity requirement, a row-hard-threshold function is introduced to solve the \( l_0 \) norm directly in this paper. In addition, the low-rank constraint is used to exploit the spatial information. Experiments with simulated and real data prove the effectiveness of the proposed MSWLRU algorithm.

**II. SPARSE UNMIXING**

Sparse unmixing assumes that mixed pixels are linear combinations of endmembers in the spectral library. Let \( Y \in \mathbb{R}^{l \times n} \) denote the observed hyperspectral image, including \( L \) spectral bands and \( n \) pixels. The mixed model is as follows:

\[
Y = AX + N,
\]

where \( A \in \mathbb{R}^{l \times m} \) denotes the preobtained spectral library containing \( m \) atoms; \( X \in \mathbb{R}^{m \times n} \), the corresponding abundance matrix; and \( N \in \mathbb{R}^{l \times n} \), the error term. The number of endmembers in the hyperspectral image is far less than that of the atoms in the spectral library. Most elements in \( X \) are zero; therefore, the abundance matrix \( X \) is sparse, and the sparsity constraint can be added to solve problem (1). In addition, the abundance sum-to-one constraint (ASC) and the abundance non-negative constraint (ANC) should also be met. To fully use the structural characteristics of \( X \), we impose only the ANC and relax the ASC. Then the sparse-unmixing model can be written as

\[
\min_{X} \frac{1}{2} \|Y - AX\|_F^2 + \lambda \|X\|_0 \quad \text{s.t. } X \geq 0,
\]

where \( \| \cdot \|_F \) denotes the Frobenius norm, \( \lambda \) is the regularization parameter, and \( \|X\|_0 \) denotes the number of nonzero elements in \( X \). Equation (2) is a single-pixel sparse-unmixing algorithm, and solving it directly is NP-hard; convex relaxation is the common method to solve this problem. Equation (3) is the representative SUNSAL algorithm, which relaxes the \( l_0 \) norm to the \( l_1 \) norm:

\[
\min_{X} \frac{1}{2} \|Y - AX\|_F^2 + \lambda \|X\|_1 \quad \text{s.t. } X \geq 0.
\]

SUnSAL entails just a small amount of calculation, but its unmixing accuracy is low. In recent years, studies of spectral information, spatial information, and sparsity have...
been the main research directions for improving the accuracy of sparse unmixing. Considering these three aspects, the MSWLRU algorithm is proposed.

III. MULTIVIEW-BASED SPECTRAL WEIGHTED AND LOW-RANK ROW-SPARSITY UNMIXING

3.1. Multiview Data Construction

The multiview hyperspectral data approach has been widely used in hyperspectral image classification, and yields promising results. However, there have been only a few attempts to apply it in hyperspectral unmixing. To exploit the rich spectral information, the AAP spectral-partitioning method is used to generate multiview data. The AAP approach, which is based on the affinity propagation (AP) [19], can automatically find the optimal number of clusters. In addition, it can remove the inconsistencies in the search process. The original hyperspectral image is first divided into q spectral groups; see [20] for more details. The spectral bands in the same group are highly similar. Then the bands are reassigned into p spectral partitions through the equal-interval-sampling band-reassignment method, similarly as in reference [21]. The spectral bands in each partition, which have less spectral similarity, are from different spectral groups. The spectral resolution of each partition is lower, but its spectral shape is consistent with that of the original hyperspectral image. Therefore, each partition contains the same endmember set. Finally, we get the p views \( Y_i \) for the original hyperspectral image, and \( Y_i \in \mathbb{R}^{L \times n} \) denotes the \( i \)th view with \( L \) bands and \( n \) pixels.

3.2. MSWLRU Algorithm

To exploit rich spectral information from the multiview hyperspectral data, a spectral weighting factor is adopted. In addition, considering spatial information and sparsity, the following unmixing model can be obtained:

\[
\min_{X} \sum_{i=1}^{p} \left( \frac{1}{2} \| Y_i - A_i X \|_F^2 + \lambda \| W^r \otimes X \|_2 + \tau \text{rank}(X) \right) \quad \text{s.t. } X \succeq 0, \tag{4}
\]

where \( X \in \mathbb{R}^{n \times n} \) and \( A_i \in \mathbb{R}^{L \times m} \) denote the estimated abundance matrix and spectral library for the \( i \)th view respectively. \( W^r \) is the spectral weighting factor. The operator \( \otimes \) denotes elementwise multiplication of two variables. \( \text{rank}(X) \) denotes the rank of \( X \). \( \lambda \) and \( \tau \) are the regularization parameters.

For multiview hyperspectral data, some characteristics for different views are quite different, such as the sensitivity to noise. Virtual endmembers may be generated by noise in some views but not in other views. In other words, the estimated abundance of the same virtual endmember is quite different in different views, though the true endmembers have similar abundance in all views. This was proved in [17]. Therefore, the virtual endmembers can be distinguished by calculating the similarity of the abundance vectors for the same endmember in different views; the lower the similarity, the more likely the endmember is virtual. Virtual endmembers should be removed during the iteration process. Therefore, let the spectral weighting factor \( W^r \) be inversely proportional to the similarity:

\[
W^{r(k+1)}(i, :) = \frac{1}{(p-1)\sum_{i=1}^{p-1} d(X^{r(k)}(i, :)) + \alpha}, \tag{5}
\]

where \( W^{r(k+1)}(i, :) \) denotes the \( i \)th row of \( W^r \) for the \( (k+1) \)th iteration and \( \alpha > 0 \) denotes a small constant added to avoid singularities. \( d(X^{r(k)}(i, :)) \) denotes similarity calculation for the \( i \)th row between the \( e^0 \)th and \( v^b \)th views in the \( k \)th iteration. For simplicity, the Euclidean distance is used to calculate similarity and defined as

\[
d(X^{r(k)}(i, :)) = \left( (x_i - e)^T (x_i - e) + (x_i - v^b)^T (x_i - v^b) \right)^{1/2}, \tag{6}
\]

where \( x_i \) and \( e \) denote the \( i \)th row vectors of the \( e^0 \)th and \( v^b \)th views respectively. Before solving Eq. (4), rank \( (X) \) is replaced by the nuclear norm \( \| X \|_2 = \sum_{i=0}^{r(x)} \sigma_i(X) \), similarly as in [12, 13], where \( \sigma_i(X) \) denotes the \( i \)th singular value of \( X \) and \( r = \text{rank}(X) \). Then Eq. (4) becomes

\[
\min_{X} \sum_{i=1}^{p} \left( \frac{1}{2} \| Y_i - A_i X \|_F^2 + \lambda \| W^r \otimes X \|_2 + \tau \| X \|_F^2 \right) \quad \text{s.t. } X \succeq 0. \tag{7}
\]

To solve problem (7), the optimal solution for each view needs to be obtained. The optimization process of the \( v^b \)th view is as follows. First, the non-negative constraint is incorporated into the objective function, and we obtain the following equation:

\[
\min_{X} \frac{1}{2} \| Y - A X \|_F^2 + \lambda \| W^r \otimes X \|_2 + \tau \| X \|_F^2 + I_{l_{I_b}}(X^v), \tag{8}
\]

where \( l_{I_b}(X^v) \) denotes the indicator function, i.e. \( l_{I_b}(X^v) = 0 \) if \( X^v \in R^+ \) and \( l_{I_b}(X^v) = +\infty \) otherwise. Then, Eq (8) is solved under the alternating-direction method of multipliers (ADMM) framework [22]. Set \( A^v X^v = V^v, X^v = V^v, X^v = V^v, X^v = V^v, X^v = V^v \) and Eq. (8) can be written as

\[
\min_{X, Y \in \mathbb{R}^{n \times n}, X, Y \in \mathbb{R}^{L \times m}} \frac{1}{2} \| Y - V^v \|_F^2 + \lambda \| W^r \otimes V^v \|_2 + \tau \| V^v \|_2 + I_{l_{I_b}}(V^v), \tag{9}
\]

s.t. \( A^v X^v = V^v, X^v = V^v, X^v = V^v, X^v = V^v \).

Setting \( V^v = (V^v, V^v, V^v, V^v) \), \( G = [A^v, I, I, I] \), \( B = \text{diag}(-I) \). Equation (9) becomes

\[
\min_{X^v, Y^v} \frac{1}{2} \| Y^v - V^v \|_F^2 + \lambda \| W^r \otimes V^v \|_2 + \tau \| V^v \|_2 + I_{l_{I_b}}(V^v) \quad \text{s.t. } GX + BV = 0. \tag{10}
\]

The following Lagrangian function can be established using Eq. (10).
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where soft denotes the soft-threshold function $\text{soft}(y, \tau) = \text{sign}(y) \max(|y| - \tau, 0)$ [8]. Then $V_{v}^{*}$ is updated by

$$V_{v}^{*(k+1)} = \text{SVT}_{\frac{1}{\tau}} \left( X_{v}^{*(k+1)} - D_{v}^{*(k)} \right)$$

The iterative update formulas of other variables are as follows:

$$V_{v}^{*(k+1)} = \max \left( X_{v}^{*(k+1)} - D_{v}^{*(k)}, 0 \right)$$

$$D_{v}^{*(k+1)} = D_{v}^{*(k)} - A X_{v}^{*(k+1)} + V_{v}^{*(k+1)}$$

The entire flow of the proposed MSWLRLU algorithm is as follows:

Algorithm 1. Pseudocode of MSWLRLU.
1. Input: hyperspectral image $Y$, spectral library $A$.
2. Select parameters: $\rho, \lambda, \tau$.
3. Construct multiview hyperspectral data and get $p$ views
   $$[\{f\}]_{p}$$

4. Initialization: $\mu, V_{q}^{(0)}, D_{q}^{(0)}$, $q = 1, 2, 3, 4$, and set $k = 0$

5. For the view $v = 1, ..., p$, repeat:
   $$W_{v}^{(k)}(i, \cdot) = \frac{1}{(p-1) \lambda} \sum_{i=1}^{m} \left( X_{v}^{(k)} - D_{v}^{(k)} \right)(i, \cdot) + \sigma$$
   $$X_{v}^{(k+1)}(i, \cdot) = [A^{T} A + \lambda]^{-1} \left( A^{T} V_{v}^{(k+1)} + D_{v}^{(k+1)} + V_{v}^{(k+1)} + D_{v}^{(k+1)} + V_{v}^{(k+1)} + D_{v}^{(k+1)} \right)$$
   $$V_{v}^{*(k+1)} = \text{SVT}_{\frac{1}{\tau}} \left( X_{v}^{*(k+1)} - D_{v}^{*(k)} \right)$$
   $$V_{v}^{*(k+1)}(i, \cdot) = \text{soft} \left( \text{sign}(y) \max(|y| - \tau, 0) \right)$$
   $$D_{v}^{*(k+1)} = D_{v}^{*(k)} - A X_{v}^{*(k+1)} + V_{v}^{*(k+1)}$$
   $$D_{v}^{*(k+1)} = D_{v}^{*(k)} - X_{v}^{*(k+1)} + V_{v}^{*(k+1)}$$

6. Until the stopping criterion is met

After the iteration stops, $p$ abundance matrices of different views are obtained. They are similar, and any of them can be used as the final abundance estimation matrix. In addition, appropriate abundance fusion strategies can be adopted to achieve better results. For simplicity, the simple averaging method is used to obtain the final abundance matrix, similarly as in [17]. In other words, the mean of $p$ abundance matrices is used as the final result.

IV. EXPERIMENTAL RESULTS AND ANALYSIS

To validate the effectiveness of the proposed MSWLRLU algorithm, simulated and real hyperspectral data are used for experiments. The experimental results of several state-of-the-art algorithms (SSMCSU, $S^{2}$WSU-W1, CSUnL0,
SUnSAL-TV, and CLSUnSAL) are also reported for comparison. The famous USGS spectral library, whose spectral range is from 0.4 to 2.5 µm, is used for all experiments; it has a total of 498 substances and 224 bands. The metrics root-mean-square error (RMSE) and signal-to-reconstruction error (SRE) are used to evaluate the unmixing effect of these algorithms. Let \( x_i \) be the true abundance vector and \( \hat{x}_i \) be an estimate of \( x_i \). The metric RMSE can be expressed as follows:

\[
\text{RMSE} = \sqrt{\frac{1}{mn} \sum_{i=1}^{n} \| x_i - \hat{x}_i \|^2},
\]

where \( n \) denotes the number of pixels and \( m \) the number of endmembers. The smaller the RMSE, the higher the unmixing accuracy. The metric SRE can be defined by

\[
\text{SRE (dB)} = 10 \log_{10} \left( \frac{E(\|x_i\|^2)}{E(\|x_i - \hat{x}_i\|^2)} \right),
\]

where \( E(\cdot) \) denotes the expectation function. The higher the SRE (in dB), the higher the unmixing accuracy.

### 4.1. Simulated Data Experiment 1

The size of simulated-data cube DC1 is 64 × 64, which has been used in [17, 23]. Five atoms are randomly selected from the spectral library as the endmembers (Em) of this experiment, namely, Anorthite HS349.3B, Howlite GDS155, Andradite WS487, Almandine WS475, and Uralite HS345.3B. The abundance satisfied the ASC and ANC. Figure 1 shows the true abundance map for each endmember. For simplicity, a subset \( A_1 \subseteq \mathbb{R}^{224 	imes 240} \) of the USGS spectral library is used. The angle between any endmembers in \( A_1 \) is greater than 4.44 degrees. To be more realistic and detect the robustness of each algorithm to noise, DC1 is contaminated with white Gaussian noise for three signal-to-noise ratio (SNR) levels: 20, 30, and 40 dB.

### 4.2. Simulated Data Experiment 2

A more complicated dataset (DC2) containing 100 × 100 pixels is used for this experiment, similarly as in [11, 24]. The spectral library \( A_1 \) is also used, and nine spectral signatures are randomly chosen from it as the endmembers: Anorthite HS349.3B, Howlite GDS155, Andradite WS487, Almandine WS475, Uralite HS345.3B, Palygorskite CM46, Tremolite HS18.3, Spessartine WS480, and Jarosite GDS99. Their corresponding abundances are created using fractals and are shown in Fig. 2. It can be seen from the abundance maps that each endmember has a more scattered distribution, and DC2 is more in line with the real distribution of objects. In addition, DC2 is also contaminated with white Gaussian noise with SNRs of 20, 30, and 40 dB. The value of each parameter is constantly adjusted to obtain optimal performance.

Tables 1 and 2 present the SRE and RMSE values, required time, and optimal parameter values of each algorithm, for DC1 and DC2 respectively. It can be clearly seen that the proposed MSWLRU algorithm has the highest SRE values and the lowest RMSE values for all SNRs.
FIG. 2. True abundance maps of endmembers (Em) in DC2.

TABLE 1. SRE (dB), RMSE, Time (s), and parameters of each algorithm for DC1

<table>
<thead>
<tr>
<th>SNR (dB)</th>
<th>Criteria</th>
<th>CLSUnSAL</th>
<th>SUnSAL-TV</th>
<th>CSUnL0</th>
<th>$S^3$WSU-W1</th>
<th>SSMCSU</th>
<th>MSWLRU</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>SRE (dB)</td>
<td>5.4018</td>
<td>6.8983</td>
<td>6.7622</td>
<td>6.7918</td>
<td>13.5346</td>
<td>15.1580</td>
</tr>
<tr>
<td></td>
<td>RMSE</td>
<td>0.0235</td>
<td>0.0198</td>
<td>0.0201</td>
<td>0.0201</td>
<td>0.0092</td>
<td>0.0077</td>
</tr>
<tr>
<td></td>
<td>Time (s)</td>
<td>5.5711</td>
<td>47.5032</td>
<td>28.2420</td>
<td>20.0744</td>
<td>35.1486</td>
<td>49.2351</td>
</tr>
<tr>
<td></td>
<td>Parameters</td>
<td>$\lambda = 3.5$</td>
<td>$\lambda = 0.05$</td>
<td>$a_0 = 0.04$</td>
<td>$\lambda = 0.2$</td>
<td>$s = 3$</td>
<td>$\lambda = 0.06$, $\tau = 0.03$</td>
</tr>
<tr>
<td>30</td>
<td>SRE (dB)</td>
<td>9.8149</td>
<td>16.1704</td>
<td>23.5234</td>
<td>25.5173</td>
<td>26.4262</td>
<td>29.0632</td>
</tr>
<tr>
<td></td>
<td>RMSE</td>
<td>0.0142</td>
<td>0.0068</td>
<td>0.0029</td>
<td>0.0023</td>
<td>0.0021</td>
<td>0.0015</td>
</tr>
<tr>
<td></td>
<td>Time (s)</td>
<td>5.9694</td>
<td>47.1560</td>
<td>26.6856</td>
<td>19.9804</td>
<td>33.6523</td>
<td>50.1546</td>
</tr>
<tr>
<td></td>
<td>Parameters</td>
<td>$\lambda = 0.8$</td>
<td>$\lambda = 0.005$</td>
<td>$a_0 = 0.01$</td>
<td>$\lambda = 0.02$</td>
<td>$s = 3$</td>
<td>$\lambda = 0.01$, $\tau = 0.007$</td>
</tr>
<tr>
<td>40</td>
<td>SRE (dB)</td>
<td>15.4563</td>
<td>24.1759</td>
<td>33.0446</td>
<td>35.7366</td>
<td>36.3590</td>
<td>39.2224</td>
</tr>
<tr>
<td></td>
<td>RMSE</td>
<td>0.0074</td>
<td>0.0027</td>
<td>0.0009</td>
<td>0.0007</td>
<td>0.0006</td>
<td>0.0004</td>
</tr>
<tr>
<td></td>
<td>Time (s)</td>
<td>8.0689</td>
<td>46.9690</td>
<td>33.3440</td>
<td>28.8953</td>
<td>38.5416</td>
<td>49.2974</td>
</tr>
<tr>
<td></td>
<td>Parameters</td>
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<td>$\lambda = 0.003$</td>
<td>$a_0 = 0.0005$</td>
<td>$\lambda = 0.005$</td>
<td>$s = 3$</td>
<td>$\lambda = 0.003$, $\tau = 0.001$</td>
</tr>
</tbody>
</table>
UnSAL algorithm has the worst unmixing effect among all algorithms. The unmixing effect of SUnSAL-TV is better than that of CLSUnSAL but worse than those of other algorithms. Both CSUnL0 and S\textsuperscript{2}WSU-W1 algorithms achieve better unmixing performance than the classic CLS-UnSAL and SUnSAL-TV algorithms. However, CSUnL0 and S\textsuperscript{2}WSU-W1 are severely affected by noise, and their unmixing accuracy is low when the SNR is 20 dB. Both MSWLRU and SSMCSU algorithms adopt the multi-view hyperspectral data, which greatly improves their antinoise ability. Therefore, MSWLRU and SSMCSU can still obtain good unmixing when the SNR is 20 dB. The SSMCSU algorithm adopts the multi-view spectral and spatial weighting factors, and obtains promising results. However, the sparsity of SSMCSU is insufficient. The proposed MSWLRU algorithm uses the spectral weighting factor and low-rank constraint to exploit spectral and spatial information. In addition, to meet the sparsity requirement MSWLRU introduces the row-hard-threshold function to solve the $\ell_{2,0}$ norm directly. Therefore, the MSWLRU algorithm obtains better unmixing than SSMCSU.

Figures 3 and 4 show the unmixing abundance maps and difference maps between the estimated and true abundances for DC1 and DC2 respectively, for different SNRs. For space considerations, only the second endmember is selected for comparison; the unmixing results for other endmembers are similar to those for Em2. It can be seen from Fig. 3 that the estimated abundance maps for CLSUnSAL and SUnSAL-TV are quite different from the real ones, for different SNRs. Their difference maps also show that the unmixing error is larger than that for other algorithms. The estimated abundance maps for the MSWLRU, SSMCSU, S\textsuperscript{2}WSU-W1, and CSUnL0 algorithms are similar when the SNR is 40 dB. However, it can be clearly seen that the unmixing effect of MSWLRU is better than those of other algorithms, for other SNRs. Only MSWLRU and CSUnL0 can get clear yellow circles for Em2 with an SNR of 30 dB. However, the background of MSWLRU is clearer and purer than that of CSUnL0. In addition, it can still be seen from the difference maps that the unmixing error for other algorithms is larger than for MSWLRU. Figure 4 shows that the unmixing effect of CLSUnSAL and SUnSAL-TV on DC2 is still poor. It is difficult to distinguish the unmixing effect through the estimated abundance maps for other algorithms. However, it can still be seen from the difference maps that MSWLRU has the highest unmixing accuracy. Therefore, the simulated data experiments prove the validity of the proposed MSWLRU algorithm.

### 4.3. Computational Cost Analysis

To analyze the complexity of the proposed MSWLRU algorithm, all algorithms are run in MATLAB R2018a on a laptop equipped with an Intel Core 5 processor (2.3 GHz main frequency) and 12 GB of memory. The tolerance and maximum number of iterations for all algorithms are set to $10^{-4}$ and 1000 respectively. Tables 1 and 2 present the running times of all algorithms for DC1 and DC2. It can be seen that CLSUnSAL takes the least time among all algorithms. MSWLRU takes more time than SSMCSU, S\textsuperscript{2}WSU-W1, CSUnL0, and CLSUnSAL. The time required by MSWLRU is similar to that of SUnSAL-TV. Therefore, MSWLRU has similar computational complexity to SUnSAL-TV.

### 4.4. Convergence Analysis

It is worth noting that the MSWLRU algorithm features...
the $l_2,0$ norm and low-rank constraint, so it is difficult to prove the convergence of MSWLRU directly. Figure 5 shows the change in SRE with number of iterations during the unmixing process of DC1 and DC2. It can be seen from Fig. 5 that the SRE no longer changes when the number of iterations reaches a certain value. Therefore, the maximum number of iterations and the residual can be used as iteration-stop conditions of MSWLRU. In our experiments, the maximum number of iterations and the tolerance for MSWLRU are set to 1000 and $10^{-4}$ respectively.

4.5. Impact of the Parameters

The MSWLRU algorithm first uses the AAP spectral-partitioning strategy to construct multiview hyperspectral data, and then performs unmixing on these data. The two steps have a certain degree of independence, so the influence of their corresponding parameters on the unmixing effect can be analyzed separately. When analyzing some
parameters, other parameters are set to appropriate values. Take DC1 as an example. Figure 6 presents the influence of parameters on the performance of MSWLRU for DC1, with a SNR of 30 dB. It can be seen that the number of views \( p \) has a greater impact on the unmixing performance. The multiview method can make full use of spectral information and improve the antinoise ability. However, as the number of views increases, the spectral bands in each view gradually lose the spectral shape of the original hyperspectral image. Therefore, there should be a balance for the parameter \( p \), which is set to 2 or 3 in this study. After determining the parameter \( p \), let SRE be a function of parameters \( \lambda \) and \( \tau \), and continuously adjust their values to obtain the best performance.

4.6. Experiment with Real Data

The well-known Cuprite dataset is used in this experiment. It was collected by airborne visible/infrared imaging.
spectrometer in 1997 and has been widely used for hyperspectral unmixing experiments. A subset containing $250 \times 191$ pixels of the Cuprite dataset is used for simplicity, similarly as in reference [25–27]. It has a total of 224 bands, but after removing the bands with large water absorption and low SNR (1–2, 105–115, 150–170, 223–224), only 188 bands are left. The USGS spectral library $A \in \mathbb{R}^{224 \times 498}$ is used. Figure 7 shows the distribution of different substances in the dataset. It was produced in 1995 by the USGS and it can be used as a reference to qualitatively evaluate the unmixing performance. The parameters of SSMCSU, $S^2$WSU-W1, CSUnL0, SUnSAL-TV and CLSUnSAL are set ac-

**FIG. 5.** Convergence curves for MSWLRLU: (a) for DC1, and (b) for DC2.

**FIG. 6.** SRE (in dB) as a function of parameters in MSWLRLU, for DC1 with a SNR of 30 dB: (a) for varying number of views, and (b) for the varying parameters $\lambda$ and $\tau$. 
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Table 3 shows the required time and parameter values for each algorithm. Three representative minerals (alunite, buddingtonite, and chalcedony) are chosen from the Cuprite dataset [28]. Their abundance maps estimated by each algorithm are compared to those generated by the USGS software Tricorder 3.3\(^2\), which are shown in Fig. 8.

It can be clearly seen from Fig. 8 that the estimated abundance maps for SUnSAL-TV have more blurred edges and oversmoothing phenomenon than those for other algorithms. In addition, the background of SUnSAL-TV is not as clear and pure as those of other algorithms, especially for the mineral buddingtonite. The estimated abundance maps for the mineral chalcedony for the CLSUnSAL and

---

**TABLE 3.** Parameters and times corresponding to each algorithm, for the Cuprite dataset

<table>
<thead>
<tr>
<th>Parameters</th>
<th>CLSUnSAL</th>
<th>SUnSAL-TV</th>
<th>CSUnL0</th>
<th>S(^2)WSU-W1</th>
<th>SSMCSU</th>
<th>MSWLRLU</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time (s)</td>
<td>198.2</td>
<td>1223.4</td>
<td>630.2</td>
<td>424.7</td>
<td>843.6</td>
<td>1231.5</td>
</tr>
</tbody>
</table>

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**FIG. 7.** Distribution map for different substances in the Cuprite dataset [28].
CSUnL0 algorithms are quite different from that of the software Tricorder 3.3. The MSWLRU, SSMCSU, and S²WSU-W1 algorithms show similar unmixing effects, but it can still be seen that MSWLRU has the highest abundance for the mineral chalcedony and is closest to the true distribution. In other words, MSWLRU has the highest unmixing accuracy for the Cuprite dataset. Therefore, these real data also validate the effectiveness of the proposed MSWLRU algorithm.

V. CONCLUSION

In this paper, a novel algorithm called MSWLRU is proposed. The MSWLRU algorithm focuses on spectral information, spatial information, and sparsity at the same time, to improve the accuracy of unmixing. The AAP spectral-partitioning strategy is used to construct multiview hyperspectral data, and spectral weighting is imposed on it to exploit the abundant spectral information. The multiview unmixing method can also greatly improve antinoise performance. To meet the sparsity requirement, MSWLRU introduces the row-hard-threshold function to solve the $l_{2,0}$ norm directly. In addition, the low-rank constraint is used to explore spatial information. Experiments using simulated and real data validate the effectiveness of the proposed MSWLRU algorithm.

REFERENCES


