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Original Article

Transient analysis of a subcritical reactor core with a MOX-Fuel using the birth-and-death model

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ABSTRACT

The operation of the nuclear reactor requires accurate and fast methods and techniques for analysing its kinetics. These techniques become even more important when the MOX-fuel is used due to the lower value of delayed neutron fraction β for ^{239}Pu. Based on a Birth-and-Death process review, the mathematical model of thermal reactor core has been proposed different from existing ones. The analytical method for thermal point-reactor parameters evaluation is described within this work. The proposed method is applied for analysis of the unsteady transient processes taking place in a thermal reactor at its start-up or shutdown power change, as well as during small accidental power variation from the rated value. Theoretical determination of MASURCA reactor core reactivity through the analysis of experimental data on neutron time spectra was made.

1. Introduction

Nuclear thermal reactors are up to now the base of modern nuclear energetic. They commonly use uranium oxide fuel enriched at several percent by ^{235}U isotope. Nevertheless up to 30% of the total energy in such reactors is produced by fission of the ^{239}Pu generated during a reactor operation [1,6,15]. Usage of MOX (Mixed-Oxide Fuel — mixture of two fissile nuclides ^{235}U and ^{239}Pu — is reasonable for power nuclear reactors and is widely studied for current and next generation nuclear reactors [8,24].

The analysis of reactor active zone parameters (such as neutron flux, multiplication factor, reactivity and etc.) is an important and time-consuming task, which requires different methods and techniques. Mostly Monte-Carlo codes are used for this purpose. Calculations via such codes generally take a significant amount of time and cant be applied for situations, when a rapid analysis is required. Here is where the analytical techniques come in useful.

In order to create and investigate analytical methods of reactor neutron kinetics analysis certain research was conducted in JIPNR — Sosny (Minsk, Belarus) [14,16,17,19,23]. These methods are based on the Birth-and-Death model, which takes roots from probability theory and Kolmogorov forward equations (also known as the Fokker–Planck equations) and Markov chains [5]. This model has already proved its suitability in biology [20], electronics [2], nuclear interactions studies [9] and other areas of science. Now it can be applied for the use of nuclear reactor physics. This work is aimed at deriving kinetics parameter of subcritical assembly core with MOX-fuel — reactivity. A new analytical method was proposed for thermal point-reactor parameters evaluation. The data produced in the "MUltiplication with an External Source-4" (MUSE-4) [3,21] experimental program on MASURCA (Saint Paul Lez Durance, France) subcritical assembly was used as reference to test the methodology.

2. Basic birth-and-death model description

Consistent probabilistic approach for analytical derivation of the average neutron population $M[N(t)]$ and its variance $D[N(t)]$ is described in classic works on math and statistics applications [10–12,22]. These are usually various versions of birth-and-death model, postulated by Kolmogorov. According to the postulates of the birth-and-death process, the forward Kolmogorov equations for the transition probabilities $P_{in}(t)$ are as follows:

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$$\begin{cases} \dfrac{dP_{i0}(t)}{dt} = & -\lambda_0(t)P_{i0}(t) + \mu_1(t)P_{i0}(0) \\ \dfrac{dP_{in}(t)}{dt} = & -\lambda_{n-1}P_{in-1}(t) - [\lambda_n(t) + \mu_n(t)]P_{in}(t) + \\ & \mu_{1+1}(t)P_{in+1}(t) \end{cases} \tag{1}$$

where $P_{in}(t)$ is a probability of a system to be in state n at time t given it was in state i $(i, n = 0, 1, 2, \ldots)$ at time 0; $\lambda_n(t)$ and $\mu_n(t)$ are averaged instant intensities of birth and death processes at state n at time t correspondingly.

Let us consider $M(t)$ as the countable amount of neutrons from the standpoint of mathematical expectation. At the initial time $t = 0$ M_0 neutrons are injected in the multiplying system. As was shown in Ref. [14] the average number of neutrons $M(t)$ at the time point t can be described with the following equation

$$M(t) = M_0 \exp\left[\frac{(\rho + \beta - \beta\exp(-t/c))t}{a - b\exp(-t/c)}\right] \tag{2}$$

with a minimized parameters for more convenient math

$c = \tau_{del}$
$b = \beta \cdot c$
$a = b + \tau_{pr}$

where ρ — reactivity of the multiplying system, β — total delayed neutron yield, τ_{del} — average delayed neutron lifetime and τ_{pr} — average prompt neutron lifetime.

The most important quantity for reactivity ρ determination is the ratio of $M(t)$ function values at two different moments t_1 and t_2:

$$\frac{M(t_1)}{M(t_2)} = \exp\left[\begin{array}{c}\dfrac{(\rho + \beta - \beta\exp[-t_1/c])t_1}{\tau_1} - \\ \dfrac{(\rho + \beta - \beta\exp[-t_2/c])t_2}{\tau_2}\end{array}\right] \tag{3}$$

where

$$\tau_1 = a - b\exp[-t_1/c]$$
$$\tau_2 = a - b\exp[-t_2/c]$$

are the neutron generation lifetimes accounting both prompt and delayed neutrons.

In some cases the logarithm of this ratio is more convenient:

$$\ln\frac{M(t_1)}{M(t_2)} = \frac{(\rho + \beta - \beta\exp[-t_1/c])t_1}{\tau_1} - \frac{(\rho + \beta - \beta\exp[-t_2/c])t_2}{\tau_2} \tag{4}$$

At small time values $t \ll \tau_{del}$ the neutron generation lifetime is

$$\tau(t) \approx \tau_{pr} + \beta t$$

Using appropriate experimental conditions and certain time period in formulas (1)–(4) simple expressions for reactivity ρ can be obtained. Of particular interest are the prompt neutrons peak measurements in subcritical facility experiments with short pulses of fast neutrons.

Such experiments have been held on MASURCA subcritical facility within the MUSE-4 experimental program in Cadarache research center for nuclear energy (France) [3,21] and others subcritical facilities studying Accelerator-Driven Subcritical (ADS) systems [4,7].

Experimental data from Ref. [3,21] is used in this work for verification of formulas (1)–(4) and their adaptation for reactivity determination of subcritical facilities operating at subritical level $\rho \sim -0.04$.

3. Experimental data

Data from the MUSE-4 program [3,21] was used. All experiments under MUSE-4 program were held on the MASURCA subcritical facility. The particular feature of this experimental setup is the MOX fuel used as a multiplying system with composition of ^{239}Pu (80%) + ^{235}U (20%). Specially developed neutron generator GEnérateur de NEutrons Pulsé Intense (GENEPI) with neutrons from (d,D) and (d,T) reactions was used as a source of neutrons.

The multiplication factor of this experimental setup was $k_{eff} = 0.96$. The effective total delayed neutron yield was $\beta_{eff} = 0.003$, the prompt neutron lifetime was $\tau_{pr} = 5.86 \cdot 10^{-7}$ sec. The average lifetime of precursor nuclei was supposed to be $\tau_{del} = 12$ sec.

To obtain statistical consistency (the neutron flux is rapidly decreasing with time thus preventing detector from reliable reading due to the noise) we used data before 60 μs. Data before 6 μs also has not been used, as the neutron pulse cutoff was made at 4 μs. Table 1 presents the experimental $\ln M(t)_{exp}$ values together with the calculated neutron generation lifetime $\tau(t)$ which is essential for reactivity estimation for MASURCA subcritical facility.

4. Reactivity determination

4.1. Reactivity determination by prompt neutron intensity decrease rate

Experimental data from Fig. 1 can be described via formula (2). In case of $t \ll c$ it can be represented as

$$M(t) \approx M_0 \exp\left[\frac{(\rho + \beta t/c)t}{\tau_{pr} + \beta t/c}\right] \tag{5}$$

Consedeirng $t_{max} \sim 6 \cdot 10^{-5}$ sec, Eq. (5) can be written as

$$M(t) \approx M_0 \exp\left[\frac{\rho t}{\tau_{pr} + \beta t}\right] \tag{6}$$

from which one can obtain ρ definition

$$\rho = \frac{(\tau_{pr} + \beta t_n)\ln[M(t)/M_0]}{t} \tag{7}$$

Introducing an integer, such as $\ln[M(t_n)/M_0] = -n$ we can reduce Eq. (7) to a more compact form

$$\rho = -n\frac{\tau_{pr} + \beta t_n}{t_n} \tag{8}$$

Easy to check that in Eq. (8) the condition $\tau_{pr} \gg \beta t_n$ is satisfied if $n \leq 3$. Than the final equation can be written

Table 1
Experimental values of $\ln M(t)_{exp}$ and calculated neutron generation lifetime.$\tau(t)$

t,μs	$\tau(t)$,μs	$\ln M(t)_{exp}$	t,μs	$\tau(t)$,μs	$\ln M(t)_{exp}$
6	0.548	8.1	25	0.605	6.5
8	0.554	7.9	30	0.620	6.3
10	0.560	7.7	35	0.635	6.0
12	0.566	7.5	40	0.650	5.7
14	0.572	7.4	45	0.665	5.5
16	0.578	7.3	50	0.680	5.3
18	0.578	7.1	55	0.695	5.0
20	0.590	7.0	60	0.710	4.8

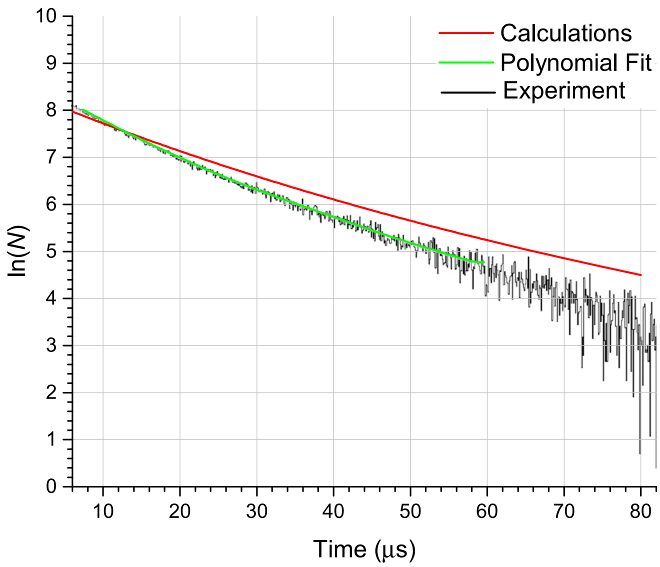

Fig. 1. $\ln(N)$ time spectra in the fuel zone of MASURCA for sub-critical level $k_{eff} = 0.960$[3].

$$\rho = -n\tau_{pr}/t_n \tag{9}$$

Using data from MUSE-4 experimental program [3,21] shown in Table 2 and value of the prompt neutron lifetime from Ref. [14,16] one can obtain reactivity values from Eq. (9). The corresponding calculation results and relative difference ρ_{calc}/ρ_{exp} to the experimental value $\rho_{exp} = -0.04$ are also shown in Table 2.

4.2. Reactivity determination by prompt neutrons peak

We described [17] the experimental data presented on Fig. 1 by formula (2) corresponding to birth-death model as in previous section. In this section we compare experimental values $\ln M(t)_{exp}$ with theoretical ones obtained from formula (6) using defined ρ and τ_{pr} values.

The papers [3,21] give value $\rho = -0.04$ and $\tau_{pr} = 5.86 \cdot 10^{-7}$ sec. If we prolong time spectra from data on Fig. 1 to the origin we can estimate $M_0 \approx 4320$ ($\ln M_0 \approx 8.37$). Then finally we obtain theoretical $M(t)_{th}$ as

$$M(t)_{th} \approx 4320 \exp\left[-\frac{0.04t}{5.86 \cdot 10^{-7} + 0.003t}\right] \tag{10}$$

and

$$\ln M(t)_{th} \approx 8.37 - \frac{0.04t}{5.86 \cdot 10^{-7} + 0.003t} \tag{11}$$

The comparison between $\ln M(t)_{th}$ values calculated with Eq. (11) and experimental values $\ln M(t)_{exp}$ are shown in Table 3 up to 60 μs. The ratio $\ln M(t)_{th}/\ln M(t)_{exp}$ is denoted as Δ. Extended calculation up to 80 μs were also made and included in the table without comparing with experimental data due to the large detection error.

Table 2
Reactivity calculations using experimental values of the logarithmic time spectrum at corresponding time points.

$M(t)_{exp}$	t, μs	ρ_{calc}	Δ
8.37	0	N/A	N/A
7.37	14	−0.041	1.03
6.37	27	−0.043	1.08
5.37	47	−0.037	0.93

Table 3
Comparison between $\ln M(t)_{th}$ and.$\ln M(t)_{exp}$

t,μs	$\tau(t)$,μs	$\ln M(t)_{th}$	$\ln M(t)_{exp}$	Δ
6	0.598	7.97	8.1	0.98
8	0.604	7.84	7.9	0.99
10	0.610	7.72	7.7	1.00
12	0.616	7.59	7.5	1.01
14	0.622	7.47	7.4	1.01
16	0.628	7.34	7.3	1.01
18	0.634	7.24	7.1	1.02
20	0.640	7.12	7.0	1.02
25	0.655	6.84	6.5	1.05
30	0.670	6.58	6.3	1.04
35	0.685	6.33	6.0	1.06
40	0.700	6.09	5.7	1.07
45	0.715	5.85	5.5	1.06
50	0.730	5.63	5.3	1.06
55	0.745	5.42	5.0	1.08
60	0.760	5.21	4.8	1.09
65	0.775	5.04	N/A	N/A
70	0.790	4.85	N/A	N/A
75	0.805	4.67	N/A	N/A
80	0.820	4.50	N/A	N/A

As seen from Table 2 there is a good agreement within several percent between calculated and experimental data for $t < 40\mu s$. For higher t values the discrepancy is slightly bigger but it is still consistent with experimental errors. In general, we conclude that birth-death model can reasonably well describe experimental data from Fig. 1 without any corrections of multiplying system parameters. Nevertheless, the expressions for $M(t)_{th}$ in Eq. (10) and $\ln M(t)_{th}$ in Eq. (11) does not account the uncertainty of τ_{pr} value which determines $M(t)$ corresponding to Eq. (6). Thus the method of reactivity determination by prompt neutrons peak considerably relies on the prompt neutron lifetime value τ_{pr} in the multiplying system.

4.3. Reactivity determination by logarithm of ratio of average neutron numbers

For current method it is easy to obtain quite simple reactivity expression. If we consider $t_1 \ll c$ and $t_2 \ll c$ Eq. (6) may be rewritten as

$$M(t) \approx M_0 \exp\left[\frac{\rho t}{\tau(t)}\right] \tag{12}$$

and the expression for reactivity will be written in the following form:

$$\rho = \frac{\ln\left[M(t_1)_{exp}/M(t_2)_{exp}\right]}{t_1/\tau_1(t) - t_2/\tau_2(t)} \tag{13}$$

This equation can be derived from (4) directly. Nevertheless it is important to see all hidden parameters, that could be possible used during transient processes analysis.

One precursor group approximation assumes $\tau(t) \approx \tau_{pr} + \beta t$ and then Eq. (13) takes its final form:

$$\rho = \frac{\ln\left[M(t_1)_{exp}/M(t_2)_{exp}\right]}{t_1/(\tau_{pr} + \beta t_1) - t_2/(\tau_{pr} + \beta t_2)} \tag{14}$$

Table 4 shows estimations of reactivity value by formula (14) in

Table 4
Reactivity estimations by several time points.

$t_1, \mu s$	$\tau_1, \mu s$	t_1/τ_1	$t_2, \mu s$	$\tau_2, \mu s$	t_2/τ_2	$t_1 - t_2, \mu s$	$\ln \frac{M(t_1}{M(t_2)}$	ρ_{th}
8	0.60	13.25	14	0.62	22.51	−6	0.39	−0.042
10	0.61	16.39	20	0.64	31.25	−10	0.64	−0.043
10	0.61	16.39	60	0.76	78.95	−50	2.92	−0.047
25	0.66	38.17	60	0.76	78.95	−35	1.97	−0.048
12	0.62	19.48	30	0.67	44.78	−18	1.17	−0.046
18	0.63	28.39	50	0.73	68.49	−32	1.87	−0.047
average								−0.045

several time points. The multiplying system parameters are the same: $\beta = 0.003$ and $\tau_{pr} = 5.86 \cdot 10^{-7}$ sec. It is necessary to remark that current method is more efficient when the time scale discretized by neutron generation lifetime is used [13,14,18,19].

5. Conclusions

A theoretical background of several methods for reactivity determination subcritical facility with multiplication factor $k_{eff} \sim 0.96$ was proposed. These methods rely on the excitation of a multiplying system by short fast neutron pulses.

Physical phenomena under these conditions are simple and well known. The new approach of current work is based on the particle birth-death model proposed in general form by Kolmogorov, described in details in Refs. [12] and adapted for describing thermal nuclear reactor in Refs. [14–16,18,19].

The correctness of suggested methods was checked by comparison between calculated values of reactivity and the experimental ones for the same multiplying system.

Data was retrieved from experiments on MASURCA subcritical facility in Cadarache (France) driven by the GENEPI neutron generator [3,21]. The effective value of multiplication factor for these experiments was $k_{eff} \sim 0.96$ ($\rho = -0.04$), effective total delayed neutron yield $\beta_{eff} = 0.003$, prompt neutron lifetime $\tau_{pr} = 5.86 \cdot 10^{-7}$ sec (several orders of magnitude lower than for thermal neutron reactor with ^{235}U fuel only). The values of reactivity in Table 4 calculated with suggested methods by logarithm of average neutron numbers ratio are in good agreement (within 10% if points with a wide time interval, where the error is rather large, are elided) with the experimental data [3,21].

More investigations on developing reactivity determination method within the Birth-and-Death mathematical model framework will be made in future. The study will be focused on adopting this model to different types of nuclear reactors and installations with fuel, moderator and reflector of different types. More attention will be paid to the delayed neutron influence on the reactivity value within the proposed model.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared influence the work reported in this paper.

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