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Technical Note

Closed-loop controller design, stability analysis and hardware implementation for fractional neutron point kinetics model

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ABSTRACT

The aim of this work is the analysis, design and hardware implementation of the fractional-order point kinetics (FNPK) model along with its closed-loop controller. The stability and closed-loop control of FNPK models are critical issues. The closed-loop stability of the controller-plant structure is established. Further, the designed PI/PD controllers are implemented in real-time on a DSP processor. The simulation and real-time hardware studies confirm that the designed PI/PD controllers result in a damped stable closed-loop response.

1. Introduction

Selection of an appropriate model is the important step in controller design. The classic nuclear point kinetics (CNPK) models describe the time evolution of neutron density inside the reactor core. The design of reactor power controllers was based on the CNPK models [1–4], which is itself based on the Fick's law, i.e., normal diffusion. To modify the Fick's law and thereby obtain the best representation of nuclear reactor dynamics, researchers presented the fractional neutron point kinetic (FNPK) model, which was based on non-Fickian assumptions [5–8]. The fractional-order models of the nuclear reactor are based on the assumption that neutron transport inside the reactor core can be better modelled as sub-diffusion [8].

Most practical control problems are nonlinear in nature and require nonlinear control to solve them. It is very difficult to design a nonlinear control for such a problem.

The PID controllers are the most popular controllers used in industry because of their simplicity, robustness, a wide range of applicability and near-optimal performance [10]. A PID controller continuously calculates an error value, which is the difference between a desired set point (SP) value and a measured value and applies to plant after tuning the gain parameters of the controller. Tuning the PID controller is nothing but finding the value of controller parameters like Kp, Ki, Kd [11]. Robustness of controller performance will totally depend upon how we tuned the gain parameters. Some methods with simple formulas use little information of process dynamics to obtain moderate performance, however they often need to be re-tuned by trial and error depending on those results. The choice of method should be based on the characteristics of the process and performance requirements. There are many variations of PID that are already implemented on different problem statement and their usefulness is also validated by various industries. The fuzzy PID technique is well-known to tune the controller parameters [12]. There are even fractional-order PID proposed in the literature [13]. The advantage of FO-PID over conventional PID is that there are more degrees of freedom as in addition to the conventional parameters like Kp, Ki, Kd, there are two more parameters γ and λ (fractional orders of the integral and derivative operator).

A nuclear reactor is a complex system and hence difficult to control and simulate in real time environment. There is a plethora of literature available on control strategies for nuclear reactor. The PID control for nuclear reactor is discussed in Ref. [3]. Sliding mode control commercial nuclear reactors has been presented in Ref. [14].

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It is important to note that recently there has been an increase in research with fractional order control of nuclear reactors [15–18]. Specifically, in the work of Davijani et al. [17] the FNPK model was applied to simulate the reactor system dynamic behaviour in order to design a fractional-order sliding mode controller to track the reference power trajectory.

The salient contributions of this work are as follows:

1 Design of PI/PD controllers for the linear FNPK model
2. Analysis of the effect of relaxation time and anomalous diffusion coefficient (α and τ for FNPK) on closed-loop system performance like settling time and peak overshoot.
3. Real-time hardware implementation of the closed-loop control with linear FNPK model and the PI/PD controllers on DSP processor.

The article is organized as follows. Next section describes the linear fractional neutron point kinetics model. Section 3 explains the integer-order approximation used in this work and its validation. The design of the PI/PD controllers and the simulation analysis is given in section 4. Section 5 presents the DSP implementation. Conclusion is given in Section 6.

2. Fractional neutron point kinetics (FNPK) model

The basic equations of fractional neutron point kinetics, denominated as FNPK model, are a system of two coupled nonlinear equations, where one of them is a differential equation of fractional-order. The basic FNPK model for a single group of delayed neutron precursors is given by Ref. [5]:

$$\begin{aligned}&\tau^{\alpha}\frac{d^{1+\alpha}n(t)}{dt^{1+\alpha}}+\tau^{\alpha}\left[\frac{1}{l}-\frac{(1-\beta)}{\Lambda}\right]\frac{d^{\alpha}n(t)}{dt^{\alpha}}+\frac{dn(t)}{dt}\\&=\frac{\rho(t)-\beta}{\Lambda}n(t)+\tau^{\alpha}\lambda\frac{d^{\alpha}C(t)}{dt^{\alpha}}+\lambda C(t)\end{aligned}\tag{1}$$

For sub-diffusion process $0<\alpha<1$, where τ^{α} represents the relaxation time and α is the anomalous diffusion coefficient. The nuclear parameters are: $\beta = 0.007$, $l = 0.00024s$, $\lambda = 0.0811s^{-11}$ and $\Lambda = 0.00002s$. The concentration of precursors is given by

$$\frac{dC(t)}{dt}=\frac{\beta}{\Lambda}n(t)-\lambda C(t)\tag{2}$$

This work considers a linear fractional-order transfer function model (3) from Ref. [8]. This linear model has been derived from the nonlinear model (1)–(2) around a suitable operating point. The fractional-order open-loop transfer function (FOTF) is given by:

$$G_{CL}(s)=\frac{\left(\frac{Kn^{*}}{\Lambda}\right)(s+\lambda)}{\tau^{\alpha}\Lambda s^{2+\alpha}+\Lambda s^{2}+\tau^{\alpha}M_1 s^{1+\alpha}+\left(M_2+\frac{Kn^{*}}{\Lambda}\right)s+\tau^{\alpha}M_3 s^{\alpha}+\left(M_4+\frac{K\lambda n^{*}}{\Lambda}\right)}\tag{3}$$

where $M_1 = \Lambda\lambda + A_1\Lambda$, $M_2 = \lambda\Lambda + A_2\Lambda$, $M_3 = A_1\Lambda\lambda - \lambda\beta$, and $M_4 = A_2\Lambda\lambda - \lambda\beta$, with $A_1 = \frac{1}{l} - \frac{1-\beta}{\Lambda}$ and $A_2 = \frac{\beta}{\Lambda}$. It should be noted that the input to this model is reactivity $\rho(t)$ and the output is neutron concentration $n(t)$. The model is open-loop stable and has underdamped response as shown in Fig. 1.

3. Approximation of the FNPK model

The FNPK model (3) has fractional-order dynamics. This dynamics reflects the sub-diffusive behaviour of neutron movements in the reactor. The fractional powers of the frequency variable 's' in the transfer function arise due to the presence of fractional derivatives in the fundamental differential equations. As it is well known, the fractional integro-differential operators have infinite memory owing to their nonlocal nature [19]. This feature of the operators made them best suitable for modelling higher-order real-world systems [20]. However, this infinite dimensionality of the fractional-order operators poses a major difficulty in their simulation and hardware implementation on finite memory computational devices. This problem is overcome by employing a finite memory approximation of fractional operators. There are both time and frequency domain integer-order approximations available in the literature [21]; viz., Tustin approximation, Charef's approximation, limited-memory Grunwald-Letnikov definition, Continued fraction approximation, and Oustalup's recursive approximation. Each method has its own merits and demerits. There have been some successful attempts in the past to use the frequency domain finite-memory approximation for fractional-order transfer functions using the MATLAB routine 'invfreqs' [22]. In this work, we have used this approach to obtain an approximation for the linear FNPK model (3) owing to its accuracy and ease. Following subsections explain the detailed procedure to obtain this integer-order approximation.

3.1. Inverse frequency transform

The FNPK model (3) was simulated in MATLAB. The step response data was generated using the numerical inverse Laplace transform routine developed in Ref. [23]. The generation of frequency response data was carried out by the substitution $s = j\omega$ in (3) and obtaining the magnitude and phase data for a frequency set. The *invfreqs* (inverse frequency) routine of MATLAB provides the continuous time transfer function parameters, under the condition that the magnitude and phase response of system is known. It provides a superior algorithm that guarantees stability of the resulting linear system and searches for the best fit using a numerical, iterative scheme [21]. The procedure followed is:

S1. Obtain the magnitude and phase vectors for *Frequency* $\in [10^4, 10^{10}]$.

S2. Identify continuous-time filter parameters from frequency response data obtained from step **S1**.

S3. Integer order (IO) model validation for the frequency and time domain fitting. For frequency domain bode plot are matched and for time domain step response of fractional order system and approximated integer order system are verified. If the model is well fitted for the practical purpose the same model is used for controller design and hardware implementation, else the step **S2** is repeated with different filter order.

S4. If the controller satisfies the system requirement and system is stable then it is discretized using Tustin approximation with

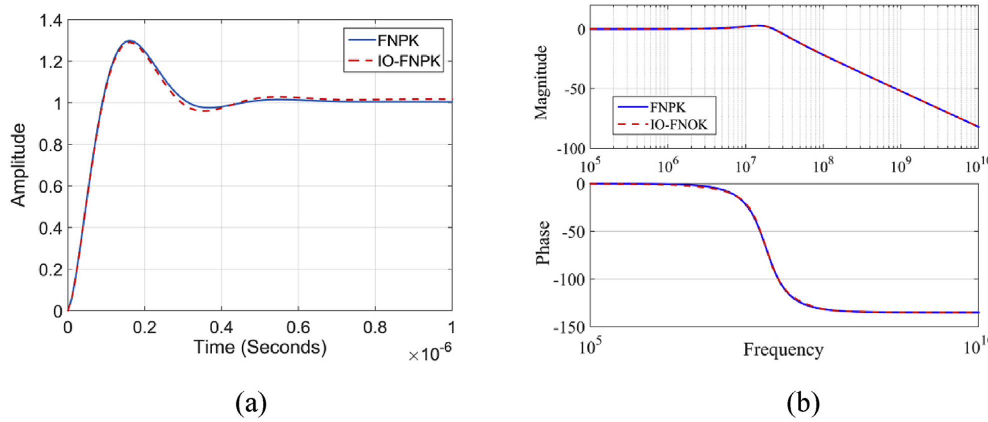

Fig. 1. (a) Step response matching (b) Frequency response matching of FNPK and Integer Order (IO) approximation with $\alpha = 0.5$ and $\tau = 10^{-3}s$.

sampling time of $T_s = 10^{-6}$. The difference equations obtained are then implemented in Digital Signal Processors (DSP) in real time environment.

The integer-order model was validated using the procedure given in the next subsection.

3.2. Validation of fractional and integer order models

This section discusses the procedure to validate the integer-order approximation for FNPK models. The simulation is done in MATLAB2014b environment. The closed-loop performance of the fractional order model (Eq. (3)) for relaxation time $\tau = 10^{-3}s$ with two anomalous diffusion coefficient values ($\alpha = 0.5$ and $\alpha = 0.25$) are studied. In order to design closed loop controller for FNPK models the plant transfer function which is a closed loop transfer function as described in Eq. (3) are approximated to:

Table 1
Error Analysis of IO approximations for $\tau = 10^{-3}s$.

Domain	Parameter	$\alpha = 0.25$	$\alpha = 0.5$
Time data	Max Error	4.39×10^{-3}	2.313×10^{-2}
	Min Error	3.92×10^{-53}	5.94×10^{-65}
	MSE	4.93×10^{-6}	1.44×10^{-4}
	Closeness	0.9987	0.9876
Bode Magnitude	Max Error	4.593×10^{-2}	0.2735
	Min Error	1.73×10^{-9}	4.62×10^{-9}
	MSE	3.05×10^{-4}	1.149×10^{-2}
	Closeness	0.9995	0.9985
Bode Phase	Max Error	0.269	1.85
	Min Error	4.21×10^{-9}	1.32×10^{-8}
	MSE	9.94×10^{-3}	0.4003
	Closeness	0.9992	0.9711

$$G(s) = \frac{-4.495 \times 10^{-7}s^{10} + 1.459 \times 10^{5}s^{9} + 2.354 \times 10^{16}s^{8} + 7.35 \times 10^{26}s^{7} + 8.274 \times 10^{36}s^{6} + 3.922 \times 10^{46}s^{5} + 8.023 \times 10^{55}s^{4} + 6.684 \times 10^{64}s^{3} + 1.955 \times 10^{73}s^{2} + 1.517 \times 10^{81}s + 1.934 \times 10^{88}}{s^{10} + 6.091 \times 10^{10}s^{9} + 1.105 \times 10^{21}s^{8} + 8.003 \times 10^{30}s^{7} + 2.499 \times 10^{40}s^{6} + 3.307 \times 10^{49}s^{5} + 1.681 \times 10^{58}s^{4} + 2.71 \times 10^{66}s^{3} + 1.10 \times 10^{74}s^{2} + 1.847 \times 10^{81}s + 1.927 \times 10^{88}} \tag{4}$$

This indicates the integer order approximation of the FNPK model for $\alpha = 0.5$ and $\tau = 10^{-3}s$; and the filter order for both numerator and denominator is 10. Fig. 1a represents the step response of the FNPK model (in blue) and integer order model (in red). Frequency domain responses of the system are illustrated in Fig. 1b. From the figures it can be deducted that the fractional order system and the integer order approximation have very close time and frequency response characteristics. Hence the approximated system can be considered to be valid system for closed loop design.

Similarly the approximation of system for different values of relaxation time τ, the anomalous diffusion coefficient α, are calculated with order of approximation taken as 10. Therefore, for all the possible combinations of τ and α this was verified, concluding that the fractional order models agree within tolerance level and can be applied for the controller design.

The validity of the integer order approximations is verified using error analysis. The system performance is compared on few error analysis techniques. The parameters taken for the modelling are maximum error, minimum error, Mean Squared Error (MSE) and closeness rating or goodness of fit parameters. The MSE gives us the numerically the order up to which the system approximation is successful (Eq. (5)), whereas the closeness rating is calculated in normalized mode with Eq. (6). First all the input data is normalized between 0 and 1. Then the ratio between system error and target value is calculated. The ratio is cumulatively added. The formulae for calculations of error performance parameter are as follow:

$$\mathrm{MSE} = \frac{1}{N}\left[\sum_{i=0}^{N}(\mathrm{Target}_i - \mathrm{Measured}_i)^2\right]^{1/2} \tag{5}$$

Table 2
Closed loop parameters for FNPK system with $\tau = 10^{-3}s$.

Parameters		Open Loop		PI Controller		PD controller	
		$\alpha = 0.5$	$\alpha = 0.25$	$\alpha = 0.5$	$\alpha = 0.25$	$\alpha = 0.5$	$\alpha = 0.25$
Gain value	Kp	–	–	0	0	86.7	12.1
	Ki	–	–	2.24×10^{6}	9.61×10^{6}	0	0
	Kd	–	–	0	0	6.01×10^{-8}	0
Closed Loop	Wgc	2.31×10^{7}	9.70×10^{7}	2.31×10^{6}	9.70×10^{6}	3.7×10^{8}	9.63×10^{8}
	PM	89.2	138	87	87.6	60	72.6
	Settling time	4.37×10^{-7}	5.57×10^{-8}	1.63×10^{-6}	3.96×10^{-7}	1.72×10^{-8}	7.49×10^{-9}
	Rise time	6.71×10^{-8}	1.17×10^{-8}	8.90×10^{-7}	2.16×10^{-7}	3.75×10^{-9}	1.45×10^{-9}
	% Overshoot	26.90%	9.1%	0%	0%	20.10%	9.7%

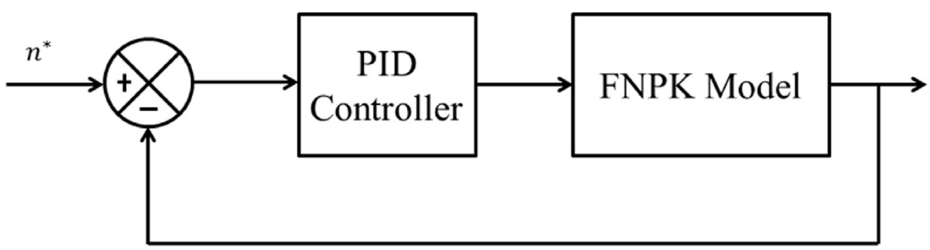

Fig. 2. Control block diagram for FNPK model.

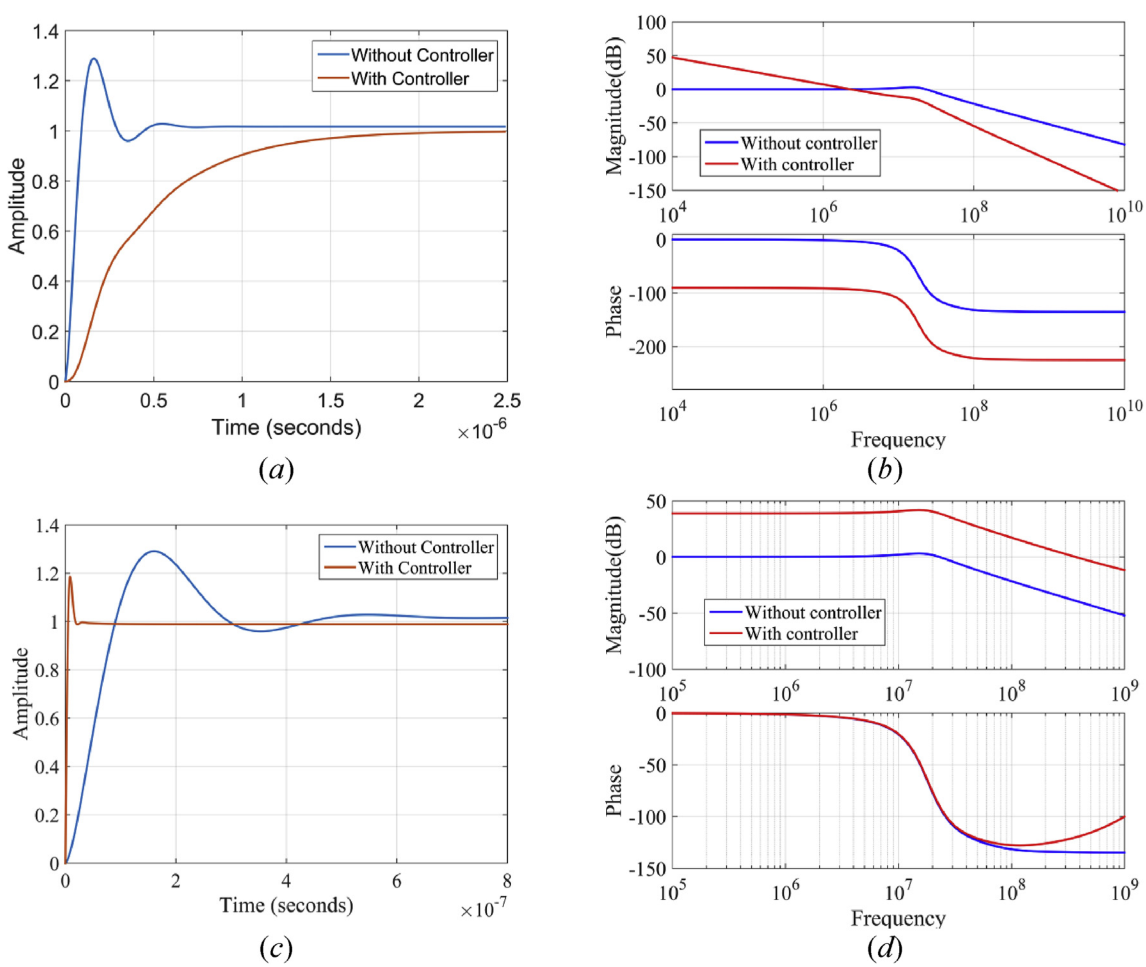

Fig. 3. Closed Loop frequency response for FNPK model with $\alpha = 0.5$ and $\tau = 10^{-3}s$: (a) Step response of PI controller (b) Frequency response of PI controller (c) Step response of PD controller (d) Frequency response of PD controller.

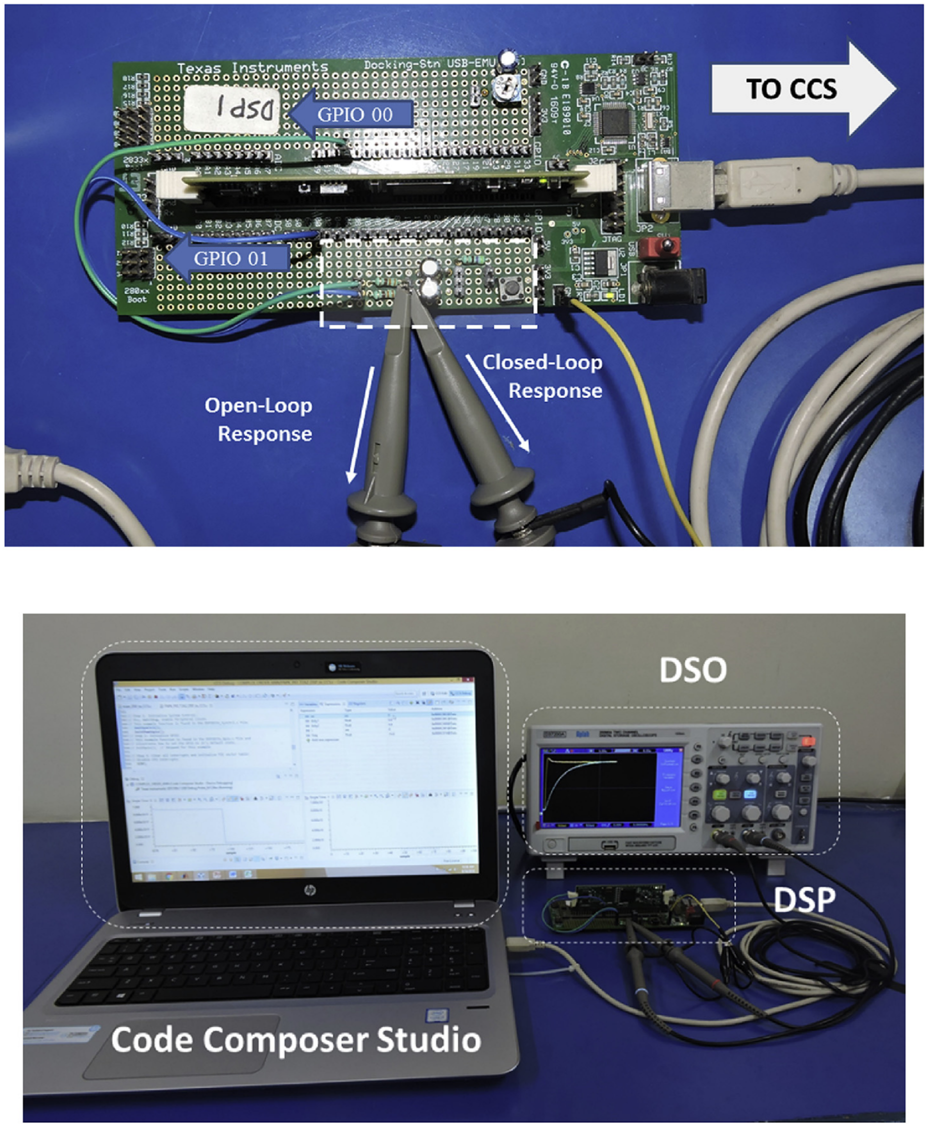

Fig. 4. Hardware setup.

$$\text{Closeness} = \frac{1}{N}\sum_{i=0}^{N}\left(1 - \left|\frac{\text{Target}_{0i} - \text{Measured}_{0i}}{\text{Target}_{0i}}\right|\right) \quad (6)$$

where the subscript 0 in Eq. (6) represents a normalized value. The ideal value of MSE should be equal to zero and that of closeness function should be close to 1.

The index of 1 in closeness indicates the perfect fit. The error results are presented in Table 1. According with these results the maximum error between FNPK models and their approximations is small in every case. Now, the MSE values are sufficiently small, and for all the system the value of closeness parameter is close to 1. From the closeness function value, it can be concluded that the integer order approximations represent the dynamics of the FNPK model in both time and frequency domain.

4. Controller design

This section discusses the closed loop control design of FNPK models using frequency domain analysis. The controller design and analysis were done in MATLAB 2014b environment. The effect of the controller on the time domain indices like settling time, overshoot, rise time and frequency domain parameters like gain margin, phase margin, gain cross-over frequency are calculated and analysed.

Fig. 2 represents the control block diagram for proposed system. The output of FNPK model is neutron concentration as given in Eq. (3). The difference between FNPK output and reference value n^* is given as input to the PID controller. It generates the control signal for the stable performance. The controller is tuned accordingly such that the response of the nuclear model is stalled.

Combinations of controllers are designed for the current system for various performance parameters. For a given system it was observed that the PI controller can be tuned to reduce the response time of the FNPK system whereas PD controllers can be used to improve the settling time. Frequency domain loop shaping technique is used to find out the controller parameters of PID controllers. The gain cross-over frequency of bode response of model decides the settling time of closed loop systems. The higher the cross-over frequency values faster the response in proportional. By proper placement of pole/zero provides the control over the closed

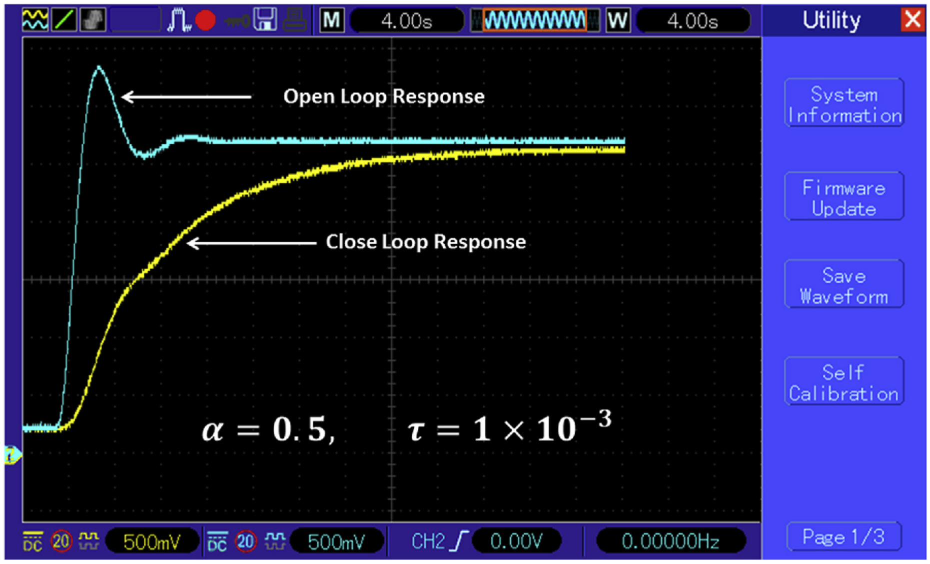

Fig. 5. Time response of FNPK system $\alpha = 0.5$.

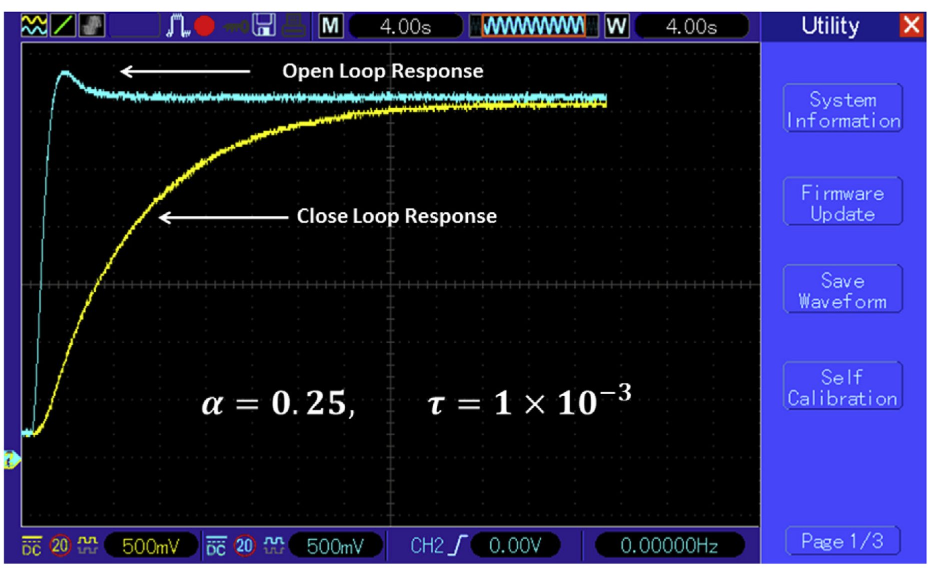

Fig. 6. Time response of FNPK system $\alpha = 0.25$.

loop performance.

The two types of controller were designed for each system.

C1. Proportional-Integral Control: This type of control is used to increase the settling time of a closed loop system making the system slower.

C2. Proportional-Derivative Control: This type of control is used to decrease the settling time of a closed loop system making the system faster.

4.1. Closed-loop FNPK model

This section discusses the closed loop performance of the FNPK system under study. The system was first approximated using the inverse frequency technique as was discussed in Section 3. The control algorithm is then designed for the PI/PD controller.

The performance of the closed loop system under study, FNPK model with $\alpha = 0.5$ and $\alpha = 0.25$ for $\tau = 10^{-3}s$, is analysed. Both PI and PD controllers were designed for their performance analysis, the parameters are given in Table 2.

Both cases were analysed, but only the case for $\alpha = 0.5$ is presented: The frequency response of closed loop system is shown in Fig. 3b and d. For PI controller, the settling time of system is reduced by decreasing the gain crossover frequency of the closed loop system. The phase margin obtained is 87°, which implies that the system will not have any overshoots. The Gain margin obtained is 47.5 dB providing good immunity towards noisy signal. The time performance of the system is presented in Fig. 3. The blue represents the open loop system step response and red represent the closed loop response. The settling time of the open loop system was $4.37 \times 10^{-7}s$ and the closed loop system is $1.63 \times 10^{-6}s$. For PD controller, the settling time is increased to a value of $1.72 \times 10^{-8}s$ and the phase margin is 60°. For $\alpha = 0.25$ it can be deducted (Table 2) that for the stability only I or only P controller is required. If we try to create a PD controller the system becomes unstable.

5. DSP implementation

This section focuses on hardware implementation and analysis of the proposed control based on the FNPK model. In order to validate the feasibility of designed controller in a real time environment the transfer function of plant and controller are discretized with a proper time sampling suitable with hardware speed. First, the plant model is verified for its step response. The model obtained from step **S3** of the procedure (Section 3.1) is discretized and equations are implemented into the hardware. The hardware platform used is a TMS320F28335 Digital Signal Processor (DSP) boards (see Fig. 4). TMS320F28335 is a Digital Signal Processor by Texas Instruments with High-Performance Static CMOS Technology. The clock speed is 150 MHz with 32-Bit CPU embedded with IEEE-754 Single-Precision Floating-Point Up to 6 Event Capture Inputs Unit (FPU), allowing floating point operations.

Initially the open loop step performance of the approximated model on hardware is verified. The Code Composer Studio (CCS) is used for online debugging and live tuning. Once the step responses are matched the performance of the closed loop system for reference tracking is observed.

The step response of simulation and hardware system, as well as the reference tracking of closed loop system for variation in setpoint are depicted in Fig. 4 for $\alpha = 0.5$ and $\alpha = 0.25$ with $\tau = 10^{-3}s$, respectively.

The system consists of a DSP development board with real time computing system with Code Composer Studio (CCS) v6.1 installed. The CCS enables real time debugging and monitoring facility to the system. The system runs at 150 MHZ clock frequency, hence the discretization of 10^{-6} s is possible. The ePWM1A and ePWM1B ports are used for open-loop and closed-loop output. The results are displayed on a digital oscilloscope. Even though the system can run at very high speed, in order to see the results on oscilloscope the sampling time of system is changed to 0.5s. The limitation on DSP allows DAC signal of maximum voltage of 3.3 V. Hence the interpolation of data is performed in order to see visualize the results. The equation for the interpolation of data is as follows:

$$\text{Voltage}_{out} = \frac{3.3 \times value}{Max(value)} \tag{7}$$

GPIO 00 is generating the open-loop response and is connected to the blue channel of oscilloscope. The GPIO 01 is connected to the yellow channel of oscilloscope and is representing the closed-loop response of the system. Both GPIO pins are enabled in PWM peripheral with frequency of 15 kHz.

Fig. 5 represents the time response of the FNPK system with $\alpha = 0.5$ and Fig. 6 with $\alpha = 0.25$. The blue signal represents the open-loop response of the system whereas the yellow represents the closed-loop response with PI controller. In these figures it can be observed two cases, the first case corresponds to the reactor with relatively low sub-diffusivity (Fig. 5), and the second case corresponds to relatively high subdiffusivity (Fig. 6). The suddiffusivity behaviour is evident in an open-loop response, where for $\alpha = 0.5$ it shows a damped response. However, for $\alpha = 0.25$ presents an initial overshoot due to high suddiffusivity. The hardware implementation shows that the closed-loop system is stable with minimal overshoot and better control over settling time of the system.

6. Conclusions

This technical note reports the exercise of design and hardware implementation of PI/PD controllers for the inherently unstable linear fractional-order neutron point kinetics (FNPK) model. The closed-loop PI/PD controllers were designed for the suitable time performance as per the system requirement. The closed-loop system was found to be stable with minimal overshoot and better control over settling time of the system. The system approximation using frequency-domain identification was good enough to be implemented on hardware in real-time environment. The proposed closed-loop models with PI/PD controllers were implemented and verified on the DSP platform.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.net.2020.07.026.

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