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DOMINATION PARAMETERS IN MYCIELSKI GRAPHS

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Abstract. In this paper, we consider several domination parameters like perfect domination number, locating-domination number, open-locating-domination number, etc. in the Mycielski graph $M(G)$ of a graph G. We found upper bounds for locating-domination number of $M(G)$ and computational formulae for perfect locating-domination number and open locating-domination number of $M(G)$. We also showed that the perfect domination number of $M(G)$ is at least that of G plus 1 and that for each positive integer n, there exists a graph G_n such that the perfect domination number of $M(G_n)$ is equal to that of G_n plus n.

1. Introduction

Let G be a finite connected simple graph with vertex set $V(G)$ and edge set $E(G)$. The *neighborhood* of a vertex $v \in V(G)$, denoted by $N_G(v)$, is the set of vertices adjacent to v in G. The cardinality of the neighborhood of a vertex v is denoted by $\deg_G(v)$. The closed neighborhood of a vertex v in a graph G is denoted by $N_G[v] = N_G(v) \cup \{v\}$. For a subset S of $V(G)$, we set $N_G(S) = \bigcup_{v\in S} N_G(v)$ and $N_G[S] = N_G(S) \cup S$. The subgraph induced by a subset S of $V(G)$ is denoted by $G[S]$. We use $|X|$ for the cardinality of a set X. For other terminology not given here, we refer to [4].

A *dominating set*, abbreviated as DS, of G is a subset S of $V(G)$ satisfying $N[S] = V(G)$. The *domination number* of G, denoted by $\gamma(G)$, is the minimum cardinality of a DS for G. A *perfect dominating set*, abbreviated as PDS, of G is a subset S of $V(G)$ such that each vertex not in S is adjacent to a unique vertex in S. The *perfect domination number* of G, denoted by $\gamma_p(G)$, is the minimum cardinality of a PDS for G. A *locating-dominating set*, abbreviated as LDS, of G is a DS S with the property that for any distinct vertices u and v in $V - S$, $N(u) \cap S \neq N(v) \cap S$. The *locating-domination number*, denoted

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by $\gamma^L(G)$, of G is the minimum cardinality of a LDS for G. A *perfect locating-dominating set*, abbreviated as PLDS, of G is a PDS as well as a LDS of G. The *perfect locating-domination number*, denoted by $\gamma_p^L(G)$, of G is the minimum cardinality of a PLDS for G. An *open locating-dominating set*, abbreviated as OLDS, of G is a subset S of $V(G)$ such that for each vertex $v \in V(G)$, $N(v) \cap S$ is not empty and for any distinct vertices u and v in $V(G)$, we have $N(u) \cap S \neq N(v) \cap S$. When an OLDS exists, the *open locating-domination number*, denoted by $\gamma^{OL}(G)$, of G is the minimum cardinality of an OLDS for G. Note that a dominating set, a perfect dominating set, a locating-dominating set and a perfect locating-dominating set always exist, but an open locating-dominating set does not always exist.

For a given graph G with $V(G) = \{v_1, v_2, \ldots, v_n\}$, the *Mycielski graph* $M(G)$ of G is defined as follows:

$$V(M(G)) = \{v_1, v_2, \ldots, v_n\} \cup \{u_1, u_2, \ldots, u_n\} \cup \{w\},$$
$$E(M(G)) = \{v_iv_j, v_iu_j, u_iv_j \mid v_iv_j \in E(G)\} \cup \{u_iw \mid 1 \leq i \leq n\}.$$

We call G the *base graph* of $M(G)$. Note that the Mycielski graph $M(G)$ of G contains G itself as an isomorphic subgraph, together with $n+1$ additional vertices. For our convenience, let $V = \{v_1, v_2, \ldots, v_n\}$ and $U = \{u_1, u_2, \ldots, u_n\}$.

The Mycielski graph of a graph G was introduced by J. Mycielski for the purpose of constructing triangle-free graphs with arbitrary large chromatic number [9]. In recent years, there have been results on Mycielski graph related to several coloring problems [1, 3, 5, 7, 10] and related to domination parameters [2,6,8]. In [2], Chen and Xing computed several domination parameters for the iterated Mycielski graph of a graph and the domination number of generalized Mycielski graph of a graph was computed by Lin et al. in [6].

In this paper, we study the perfect domination number, locating-domination number and open-locating-domination number for the Mycielski $M(G)$ of a graph G.

2. Perfect domination numbers of Mycielski graphs

For our convenience, let $\phi : U \to V$ be the function defined by $\phi(u_i) = v_i$ for each $i = 1, 2, \ldots, n$.

Lemma 2.1. *Let $M(G)$ be the Mycielski graph of G. If S is a perfect dominating set of $M(G)$, then there exists a perfect dominating set D of G such that $|S| \geq |D| + 1$. Hence $\gamma_p(M(G)) \geq \gamma_p(G) + 1$.*

Proof. It is clear that this lemma is true for any null graph G. From now on, we assume that G has at least one edge, that is, G is not a null graph. Let S be a PDS of $M(G)$. We consider the following two cases.

Case 1: $w \in S$.

Let D be the set $(S \cap V) \cup \phi(S \cap U)$. Now for any $v \in V \setminus D$, v is adjacent to unique vertex in D, and hence D is a PDS of G. Furthermore $|D| \leq |S \cap V| + |S \cap U| = |S| - 1$.

Case 2: $w \notin S$.

Since $w \in N_{M(G)}(S)$ and S is a PDS, $|S \cap U| = 1$. Let $S \cap U = \{u_i\}$. Since $N_{M(G)}(u_i) \cap U = \emptyset$ and $N_{M(G)}(u_i)$ does not contain v_i, $N_{M(G)}[S \cap V]$ contains $(U \setminus \{u_i\}) \cup \{v_i\}$. Let $D = S \cap V$. If v_i is not an element of D, then v_i is adjacent to a unique vertex in D. For any vertex $v_j \in V - (D \cup \{v_i\})$, u_j is adjacent to a unique vertex in D, which implies that v_j is adjacent to a unique vertex in D. Therefore D is a PDS of G, and $|S| = |S \cap V| + 1 = |D| + 1$.

This completes the proof. □

Lemma 2.2. *For any positive integer n, there exists a graph G_n such that $\gamma_p(M(G_n)) = \gamma_p(G_n) + n$.*

Proof. For $n = 1$, let G_1 be a null graph on V. Then $\gamma_p(G_1) = |V|$. Since V is independent in $M(G_1)$, every PDS of $M(G_1)$ must contain V. Set $S = \{w\} \cup V$. Then S is a PDS of $M(G_1)$ and hence $\gamma_p(M(G_1)) \leq |V| + 1$. Since V is not a dominating set of $M(G_1)$, we have $\gamma_p(M(G_1)) \geq |V| + 1$. So $\gamma_p(M(G_1)) = |V| + 1 = \gamma_p(G_1) + 1$.

For $n \geq 2$, let G_n be the disjoint union of $n-1$ copies of the complete graph K_2. Note that $\gamma_p(G_n) = n - 1$. Now $M(G_n)$ contains exactly $n - 1$ blocks and each of them is isomorphic to the cycle C_5 of length 5. Let $S = U \cup \{w\}$. Then S is a PDS of $M(G)$ and hence $\gamma_p(M(G_n)) \leq 2n - 1$. Let S_1 be a PDS of G_n. If S_1 contains w, then one can check that S_1 contains at least 3 vertices including w in each block and hence $|S_1| \geq 2n - 1$. Now we assume that S_1 does not contain w. Then $|S_1 \cap U| = 1$ and S_1 contains all element in V. So $|S_1| \geq 2n - 1$. This implies that $\gamma_p(M(G_n)) \geq 2n - 1$. Therefore $\gamma_p(M(G_n)) = 2n - 1 = \gamma_p(G_n) + n$. □

By combining Lemmas 2.1 and 2.2, we have the following theorem.

Theorem 2.3. *For any graph G, $\gamma_p(M(G)) \geq \gamma_p(G) + 1$ and for any positive integer n, there exists a graph G_n such that $\gamma_p(M(G_n)) = \gamma_p(G_n) + n$.*

Even though we know the existence of G_n such that $\gamma_p(M(G_n)) = \gamma_p(G_n) + n$, we do not know which graphs satisfy such a property. So, we pose the following problem.

Problem. *For each n, characterize all graphs G having the property that $\gamma_p(M(G)) = \gamma_p(G) + n$.*

An independent perfect dominating set S of V is called an *efficient dominating set* of G. Note that not every graph has an efficient dominating set. The following theorem gives a characterization of graphs G whose Mycielski graph has an efficient dominating set.

Theorem 2.4. *For a graph G, $M(G)$ has an efficient dominating set if and only if G is a null graph.*

Proof. If G is a null graph, then $\{w\}\cup V$ is an efficient dominating set of $M(G)$.

Suppose that G has an edge v_iv_j and S is an efficient dominating set of $M(G)$. Assume that $w \in S$. Then $S \cap U = \emptyset$ and for any $v \in S \cap V$, $N_{M(G)}(v) \cap U = \emptyset$, which implies that v is an isolated vertex in $M(G)$. So neither v_i nor v_j belong to S and furthermore $v_i \notin N_{M(G)}(S)$ and $v_j \notin N_{M(G)}(S)$, a contradiction. Therefore $w \notin S$ and hence $|S \cap U| = 1$. Without loss of generality, we may assume that $S \cap U = \{u_1\}$. Since S is independent, $N_{M(G)}(u_1) \cap (S \cap V) = \emptyset$. It means that $v_1 \notin N_{M(G)}(S \cap V)$. From this, we have $v_1 \in S \cap V$ and hence v_1 is an isolated vertex. Since $u_i \notin S \cap U$, there is $v_k \in S \cap V$ such that $u_i \in N_{M(G)}(v_k)$. Since S is an efficient dominating set of $M(G)$, $N_{M(G)}(v_k) \cap S = \emptyset$ and it implies that u_k is not dominated by S, a contradiction. Therefore if $M(G)$ has an efficient dominating set , then G is a null graph. □

3. Locating-domination numbers of Mycielski graphs

For the study of locating domination number, we need more terminologies. For a subset D of G, we define an equivalence relation $\sim_D$ on $V(G)$ by $u \sim_D v$ if and only if $N(u)\cap D = N(v)\cap D$ and let $\Lambda^L(G) = |V(G)| - \max\{|V(G)/\sim_D| : D \text{ is a } \gamma^L(G)\text{-set of } G\}$, where D is called a $\gamma^L(G)$-set of G if D is a LDS of G satisfying $|D| = \gamma^L(G)$. Note that if D is a locating dominating set of G, then $u \nsim_D v$ for any distinct vertices u and v in $V(G) - D$.

Theorem 3.1. *For any graph G, $\gamma^L(M(G)) \leq \min\{2\gamma^L(G), \gamma^L(G) + 1 + \Lambda^L(G)\}$.*

Proof. Let $D = \{v_1, v_2, \ldots, v_\ell\}$ be a LDS of G. By a simple computation, we can see that the set $S = \{v_1, \ldots, v_\ell, u_1, \ldots, u_\ell\}$ is a LDS of $M(G)$. Hence $\gamma^L(M(G)) \leq 2\gamma^L(G)$. Next, we aim to show that $\gamma^L(M(G)) \leq \gamma^L(G) + 1 + \Lambda^L(G)$. Let $|D/\sim_D| = h$ and let $\{v_{i_1}, v_{i_2}, \ldots, v_{i_h}\}$ be the set of all representatives of $D/\sim_D$. Let k be the cardinality $|\{v \in V\setminus D : v \sim_D u \text{ for some } u \text{ in } D\}|$. Note that $k = |V \setminus D| + |D/\sim_D| - |V/\sim_D|$.

Assume that $v_{j_1}, v_{j_2}, \ldots, v_{j_k}$ are k elements in $V \setminus D$ such that $N(v_{i_s}) \cap D = N(v_{j_s}) \cap D$ for each $s = 1, \ldots, k$. Set $S = D \cup \{w\} \cup (\{u_1, u_2, \ldots, u_\ell\} \setminus \{u_{i_{k+1}}, u_{i_{k+2}}, \ldots, u_{i_h}\})$. Now one can check that S is a dominating set of $M(G)$.

For any $v \in V \setminus S$, $N_{M(G)}(v) \cap S$ is the disjoint union of the sets $N_G(v) \cap D$ and $N_{M(G)}(v) \cap (\{u_1, u_2, \ldots, u_\ell\} \setminus \{u_{i_{k+1}}, u_{i_{k+2}}, \ldots, u_{i_h}\})$. For any $u \in U \setminus S$, $N_{M(G)}(u) \cap S$ is the disjoint union of the sets $N_G(\phi(u)) \cap D$ and $\{w\}$. Let x and y be two distinct elements in $V(M(G)) \setminus S$. If $x, y \in V \setminus S$, then $N_{M(G)}(x) \cap S \neq N_{M(G)}(y) \cap S$ because $N_G(x) \cap D \neq N_G(y) \cap D$. If $x \in V \setminus S$ and $y \in U \setminus S$, then $N_{M(G)}(x) \cap S \neq N_{M(G)}(y) \cap S$ because $w \notin N_{M(G)}(x) \cap S$ but $w \in N_{M(G)}(y) \cap S$. Assume that x and y belong to $U \setminus S$. Note that $U \setminus S$ is the disjoint union of two sets $U \setminus \{u_1, u_2, \ldots, u_\ell\}$ and $\{u_{i_{k+1}}, u_{i_{k+2}}, \ldots, u_{i_h}\}$. If both x and y belong to $(U\setminus\{u_1, u_2, \ldots, u_\ell\})$, then $N_G(\phi(x))\cap D \neq N_G(\phi(y))\cap D$ because D is a LDS of G. If both x and y belong to $\{u_{i_{k+1}}, u_{i_{k+2}}, \ldots, u_{i_h}\}$, then we have $N_G(\phi(x))\cap D \neq N_G(\phi(y))\cap D$ because $\phi(x) \nsim_D \phi(y)$. For the last case, let $x \in (U \setminus \{u_1, u_2, \ldots, u_\ell\})$ and $y \in \{u_{i_{k+1}}, u_{i_{k+2}}, \ldots, u_{i_h}\}$. Since $\phi(y) \in D$ and $\phi(y) \nsim_D v$ for any $v \in V \setminus D$, we have $N_G(\phi(x)) \cap D \neq N_G(\phi(y)) \cap D$. Therefore S is a LDS of $M(G)$.

Since $|S| = \ell+1+(\ell-h)+k = |D|+1+(|D|-|D/\sim_D|)+(|V\setminus D|+|D/\sim_D| -|V/\sim_D|) = |D|+1+|V|-|V/\sim_D|$, we have $\gamma^L(M(G)) \leq \gamma^L(G)+1+\Lambda^L(G)$. This completes the proof. □

Next, we aim to compute the number $\gamma_p^L(M(G))$. For our convenience, let $\omega_H(G)$ be the number of components in G that are isomorphic to H for any graphs H and G.

Theorem 3.2. *For any graph G, $\gamma_p^L(M(G)) = 2|V| + \alpha(G) - 2\omega_{K_2}(G)$, where $\alpha(G) = 0$ if G has an isolated vertex and $\alpha(G) = 1$ otherwise.*

Proof. For a graph G, let V' be the set of all vertices of G not belonging to component isomorphic to K_2. If G has an isolated vertex v_i, then $S = \{w\}\cup(U-\{u_i\})\cup V'$ is a PLDS of $M(G)$ and hence $\gamma_p^L(M(G) \leq 2|V|-2\omega_{K_2}(G)$. For a graph G having no isolated vertex, $S = \{w\} \cup U \cup V'$ is a PLDS of $M(G)$. Hence $\gamma_p^L(M(G) \leq 2|V| + 1 - 2\omega_{K_2}(G)$. Therefore for any graph G, $\gamma_p^L(M(G) \leq 2|V|+\alpha(G)-2\omega_{K_2}(G)$, where $\alpha(G) = 0$ if G has an isolated vertex and $\alpha(G) = 1$ otherwise.

Now it suffices to show that $\gamma_p^L(M(G) \geq 2|V| + \alpha(G) - 2\omega_{K_2}(G)$ for any graph G. Let S be a PLDS of $M(G)$. Our discussion can be divided into the following two cases.

Case 1: $w \in S$

Suppose that there exist distinct vertices u, v in $U - S$. Then both u and v are adjacent to w and hence S is not a PLDS of $M(G)$, a contradiction. So, $|S \cap U| = |U|$ or $|U| - 1$. Assume that $|S \cap U| = |U|$. Then any vertex $v \in V$ satisfying $|N_G(v)| \neq 1$ belongs to S. This implies that $|S\cap V| \geq |V| - 2\omega_{K_2}(G)$ and hence $|S| \geq 2|V| + 1 - 2\omega_{K_2}(G)$.

For the other case, let $|S\cap U| = |U| - 1$. Without loss of generality, we may assume that $U \setminus S = \{u_1\}$.

Subcase 1.1: $\phi(u_1) = v_1$ is an isolated vertex in G.

In this case v_1 must be in S. Suppose that there exists a vertex $v \in V - S$ such that $|N_G(v)| \neq 1$. If $|N_G(v)| = 0$, namely, v is an isolated vertex, then v is not dominated by S, a contradiction. If $|N_G(v)| \geq 2$, then v is dominated by at least two vertices in $S \cap U$, which is a contradiction. Therefore any vertex $v \in V$ satisfying $|N_G(v)| \neq 1$ belongs to S. So $|S| \geq 2|V| - 2\omega_{K_2}(G)$.

Subcase 1.2: $\phi(u_1) = v_1$ is not an isolated vertex in G.

In this case v_1 must be in S. In deed if $v_1 \notin S$, then there exists a unique $u_2 \in (S \cap U)$ such that v_1 and u_2 are adjacent. Since S is a PLDS,

$\phi(u_2) = v_2 \notin S \cap V$ and then there exists a unique $u_3 \in (S \cap U)$ such that v_2 and u_3 are adjacent. Since S is a PLDS, $\phi(u_3) = v_3 \notin S \cap V$. So $N_{M(G)}(v_1) \cap S = \{u_2\} = N_{M(G)}(v_3) \cap S$, which contradicts the fact that S is a perfect locating-domination set.

For any $v \in N(v_1) \cap V$, $v \notin S$ because v is adjacent to u_1 belonging to $N_{M(G)}(w)$. This implies $|N(v_1)| = 1$. If $|N_G(v)| \geq 2$, then $N_{M(G)}(v) \cap S$ contains both v_1 and some vertex in U, a contradiction. So the component C of G containing v_1 is isomorphic to K_2. Any vertex $v' \in V \setminus V(C)$ satisfying $|N_G(v')| \neq 1$ belongs to S. This implies that $|S \cap V| \geq |V| - 2\omega_{K_2}(G) + 1$ and hence $|S| \geq 2|V| + 1 - 2\omega_{K_2}(G)$.

Case 2: $w \notin S$.
In this case, $|S \cap U| = 1$. Without loss of generality, we may assume that $S \cap U = \{u_1\}$.

Subcase 2.1: $\phi(u_1) = v_1$ is an isolated vertex in G.
In this case v_1 must be in S. For any $v \in (S \cap V) \setminus \{v_1\}$, we have $|N_G(v)| = 1$.

Suppose that $V \setminus S$ is not empty. For any $v_i \in V \setminus S$, there is a unique $\tilde{v} \in (S \cap V)$ such that v_i and $\tilde{v}$ are adjacent. This means that $N_{M(G)}(v_i) \cap S = N_{M(G)}(u_i) \cap S = \{\tilde{v}\}$, a contradiction. Therefore $S \cap V = V$ and each component of $G - \{v_1\}$ is isomorphic to K_2. So $|S| = |V| + 1 = 2|V| - 2\omega_{K_2}(G)$.

Subcase 2.2: $\phi(u_1) = v_1$ is not an isolated vertex in G.
In this case v_1 must be in $S \cap V$. In deed, if $v_1 \notin S \cap V$, then there exists a unique $v_2 \in (S \cap V)$ such that v_1 and v_2 are adjacent. Since $u_2 \notin S$, there exists a unique $v_3 \in (S \cap V)$ such that v_3 and u_2 are adjacent. This implies that $N_{M(G)}(v_1) \cap S = \{v_2\} = N_{M(G)}(u_3) \cap S$, which contradicts the fact that S is a perfect locating-dominating set.

Since v_1 is not isolated, there exists a vertex v_2 in $N_G(v_1)$. Since S is a PLDS of $M(G)$, $v_2 \in S$. Suppose that there is a vertex $v_i \in V \setminus \{v_1, v_2\}$ adjacent to v_1 or v_2. If v_i and v_1 are adjacent, then $N_{M(G)}(u_2) \cap S = N_{M(G)}(u_i) \cap S = \{v_1\}$, a contradiction. If v_i and v_2 are adjacent, then $|N_{M(G)}(u_2) \cap S| \geq 2$ or $N_{M(G)}(v_i) \cap S = N_{M(G)}(u_i) \cap S = \{v_2\}$ depending on $v_i \in S$ or not. In any cases, a contradiction occurs.

Therefore the component containing v_1 and v_2 is isomorphic to K_2. By similar reason with Subcase 2.1, other components of G are isomorphic to K_2 and $S \cap V = V$. Therefore $|S| = |V| + 1 = 2|V| + 1 - 2\omega_{K_2}(G)$.

In any cases, we showed that $\gamma_p^L(M(G) \geq 2|V| + \alpha(G) - 2\omega_{K_2}(G)$, where $\alpha(G) = 0$ if G has an isolated vertex and $\alpha(G) = 1$ otherwise.

Therefore, $\gamma_p^L(M(G) = 2|V| + \alpha(G) - 2\omega_{K_2}(G)$. □

Now, we aim to compute the open locating-domination numbers of the Mycielski graphs when an OLDS of G exists.

Theorem 3.3. *For any graph G which has an OLDS, $\gamma^{OL}(M(G)) = \gamma^{OL}(G) + 2$.*

Proof. Let S_1 be an OLDS of G such that $|S_1| = \gamma^{OL}(G)$. Now for any $u \in U$, one can check that the set $S_1 \cup \{u, w\}$ is an OLDS of $M(G)$. Hence $\gamma^{OL}(M(G)) \leq \gamma^{OL}(G) + 2$. Now it suffices to show that $\gamma^{OL}(M(G)) \geq \gamma^{OL}(G) + 2$.

First, we aim to show that for an OLDS S of $M(G)$, $|S| \geq \gamma^{OL}(G) + 1$ and $S \cap V$ is an OLDS of G. Let $D = S \cap V$. By a simple computation, we can see that $N_G(v_i) \cap D = N_{M(G)}(u_i) \cap D$ and $N_{M(G)}(u_i) \cap S = (N_{M(G)}(u_i) \cap D) \cup A$ for each $i = 1, 2, \ldots, |V|$, where $A = \{w\}$ if $w \in S$ and $A = \emptyset$ otherwise. From this, we can deduce that D is an OLDS of G. By considering S as a $\gamma^{OL}(M(G))$-set, we have $\gamma^{OL}(M(G)) \geq 1 + \gamma^{OL}(G)$ because $N_{M(G)}(w) \cap (S \cap U) \neq \emptyset$.

Now it suffices to show that $\gamma^{OL}(M(G)) \neq \gamma^{OL}(G) + 1$. Suppose that there exists an OLDS S_1 of $M(G)$ such that $|S_1| = \gamma^{OL}(G) + 1$. Since $S_1 \cap V$ is an OLDS of G and $N_{M(G)}(w) \cap (S_1 \cap U) \neq \emptyset$, one can say that $S_1 \cap V$ is a $\gamma^{OL}(G)$-set, $w \notin S_1$, and $|S_1 \cap U| = 1$. Let $S_1 \cap U = \{u_1\}$. Now $N_{M(G)}(u_1) \cap S_1 = N_G(v_1) \cap D = N_{M(G)}(v_1) \cap S_1$. This contradicts the fact that S_1 is an OLDS of $M(G)$). It completes the proof. □

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References

[1] G. J. Chang, L. Huang, and X. Zhu, *Circular chromatic numbers of Mycielski's graphs*, Discrete Math. **205** (1999), no. 1-3, 23–37. https://doi.org/10.1016/S0012-365X(99)00033-3

[2] X. Chen and H. Xing, *Domination parameters in Mycielski graphs*, Util. Math. **71** (2006), 235–244.

[3] G. Fan, *Circular chromatic number and Mycielski graphs*, Combinatorica **24** (2004), no. 1, 127–135. https://doi.org/10.1007/s00493-004-0008-9

[4] T. W. Haynes, S. T. Hedetniemi, and P. J. Slater, *Fundamentals of domination in graphs*, Monographs and Textbooks in Pure and Applied Mathematics, 208, Marcel Dekker, Inc., New York, 1998.

[5] M. Larsen, J. Propp, and D. Ullman, *The fractional chromatic number of Mycielski's graphs*, J. Graph Theory **19** (1995), no. 3, 411–416. https://doi.org/10.1002/jgt.3190190313

[6] W. Lin, J. Wu, P. C. B. Lam, and G. Gu, *Several parameters of generalized Mycielskians*, Discrete Appl. Math. **154** (2006), no. 8, 1173–1182. https://doi.org/10.1016/j.dam.2005.11.001

[7] D. D.-F. Liu, *Circular chromatic number for iterated Mycielski graphs*, Discrete Math. **285** (2004), no. 1-3, 335–340. https://doi.org/10.1016/j.disc.2004.01.020

[8] D. A. Mojdeh and N. J. Rad, *On domination and its forcing in Mycielski's graphs*, Sci. Iran. **15** (2008), no. 2, 218–222.

[9] J. Mycielski, *Sur le coloriage des graphs*, Colloq. Math. **3** (1955), 161–162. https://doi.org/10.4064/cm-3-2-161-162

[10] C. Tardif, *Fractional chromatic numbers of cones over graphs*, J. Graph Theory **38** (2001), no. 2, 87–94. https://doi.org/10.1002/jgt.1025

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