

# High-Performance Tracking Controller Design for Rotary Motion Control System

Youngduk Kim<sup>\*</sup>, Su Hyeon Park<sup>\*\*</sup>, Seonghyun Ryu<sup>\*\*\*</sup>,  
Chul Ki Song<sup>\*\*\*\*</sup>, Ho Seong Lee<sup>\*\*,#</sup>

<sup>\*</sup>ROK Air Force, <sup>\*\*</sup>Department of Mechanical Convergence Engineering, Gyeongsang National University, <sup>\*\*\*</sup>New Product Process Engineering, Hyundai Mobis Co. Ltd.,  
<sup>\*\*\*\*</sup>School of Mechanical Engineering, ERI, Gyeongsang National University

## 회전운동 제어시스템을 위한 고성능 추적제어기의 설계

김영덕<sup>\*</sup>, 박수현<sup>\*\*</sup>, 류성현<sup>\*\*\*</sup>, 송철기<sup>\*\*\*\*</sup>, 이호성<sup>\*\*,#</sup>

<sup>\*</sup>대한민국 공군, <sup>\*\*</sup>경상국립대학교 기계융합공학과, <sup>\*\*\*</sup>현대 모비스(주), 신제품개발생기팀,  
<sup>\*\*\*\*</sup>경상국립대학교 기계공학부, 공학연구원

(Received 23 September 2021; received in revised form 27 September 2021; accepted 03 October 2021)

### ABSTRACT

A robust tracking controller design was developed for a rotary motion control system. The friction force versus the angular velocity was measured and modeled as a combination of linear and nonlinear components. By adding a model-based friction compensator to a nominal proportional–integral–derivative controller, it was possible to build a simulated control system model that agreed well with the experimental results. A zero-phase error tracking controller was selected as the feedforward tracking controller and implemented based on the estimated closed-loop transfer function. To provide robustness against external disturbances and modeling uncertainties, a disturbance observer was added in the position feedback loop. The performance improvement of the overall tracking controller structure was verified through simulations and experiments.

**Keywords :** Precision Servo Control(정밀서보제어), Friction Measurements(마찰측정), Robust Control(강건제어), Tracking Control(추적제어), Actuator Control(구동기제어)

## 1. Introduction

Among three main objectives of the control system design, that are stabilization, regulation and tracking, tracking control is the most challenging

task. Fig. 1 displays a control system block diagram of a rotary motion control system. Four exogenous inputs are shown in the figure that includes three external disturbances.  $w(t)$  represents external force/torque disturbance,  $d(t)$  is external position disturbance, and  $n(t)$  is electrical noise in the system electronics. The purpose of tracking control is to make the output,  $y(t)$ , to follow the reference input,

# Corresponding Author : hoslee@gnu.ac.kr

Tel: +82-55-250-7301, Fax: +82-55-250-7399

Copyright © The Korean Society of Manufacturing Process Engineers. This is an Open-Access article distributed under the terms of the Creative Commons Attribution-Noncommercial 3.0 License (CC BY-NC 3.0 <http://creativecommons.org/licenses/by-nc/3.0>) which permits unrestricted non-commercial use, distribution, and reproduction in any medium, provided the original work is properly cited.

$r(t)$ , as closely as possible under disturbances. Especially due to unmodeled friction forces, the motion control system exhibits undesirable behavior such as stick-slip limit cycles, quadrant glitches, hysteresis, and/or tracking errors due to non-linear friction components<sup>[1-2]</sup>. Lee and Tomizuka, and Lee et al. modeled nonlinear friction and implemented as a friction compensator to improve the performance of motion control system<sup>[2-3]</sup>.

In order to minimize the effect of external disturbances on the tracking control performance, a robust control scheme could be added. Ohnishi *et al.* proposed robust control scheme based on a so-called disturbance observer. The disturbance observer (DOB) can be added into a feedback loop to estimate and compensate for external disturbances and unmodeled system dynamics<sup>[4]</sup>.

While increasing the closed-loop bandwidth could improve tracking of time-varying desired trajectories, this option is not always feasible due to limitations of hardware. If the desired trajectory is known in advance, feedforward control is effective for precision tracking. In the design of feedforward controllers, the dynamic inverse of the controlled plant or the closed-loop system is often utilized such as the zero phase error tracking controller (ZPETC)<sup>[10-11]</sup>.

In this paper, we investigate the precise tracking control of a rotary motion control system. First, we identify nonlinear friction forces and use the model for compensating friction. We select ZPETC as the feedforward tracking controller and implement it based on the estimated closed-loop transfer function. In order to provide robustness, DOB is added in the position loop. The performance improvement of the overall tracking controller structure has been verified through simulations and experiments.

## 2. Friction Measurements, Modeling and Compensation

Fig. 2 displays Quanser QUBE Servo-2 development system based on rotary motion control. Fig. 3 shows Quanser HIL (hardware-in-the-loop) motion control development system supported by MATLAB Simulink.

A rotary motion control system in Fig. 2 is modeled as a second-order system:

$$P_n(s) = \frac{1}{s(J_e s + B_e)} \quad (1)$$

where  $P_n(s)$  represents a nominal plant model of Quanser Servo-2,  $J_e$  is the mass moment of inertia and  $B_e$  is viscous damping. Here  $J_e$  and  $B_e$  are identified as following:  $J_e = 7.143 \times 10^{-3}$  [V-s<sup>2</sup>] and  $B_e = 0.0446$  [V-s]<sup>[5,11]</sup>.

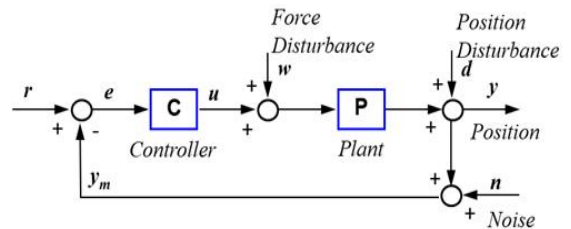
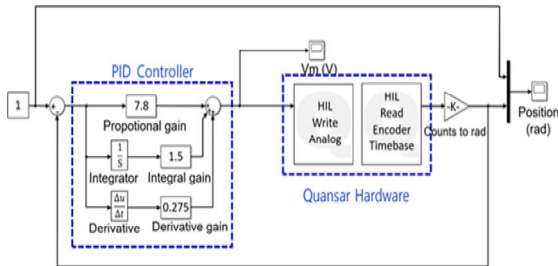


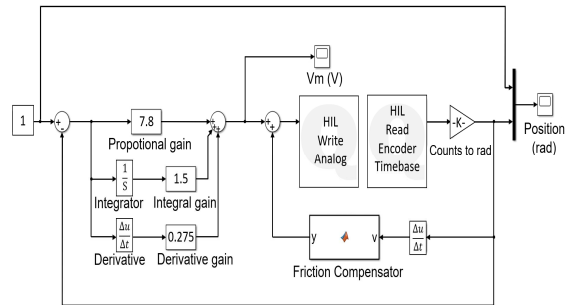
Fig. 1 Rotary motion control system: block diagram with external force/position disturbances and electrical noise



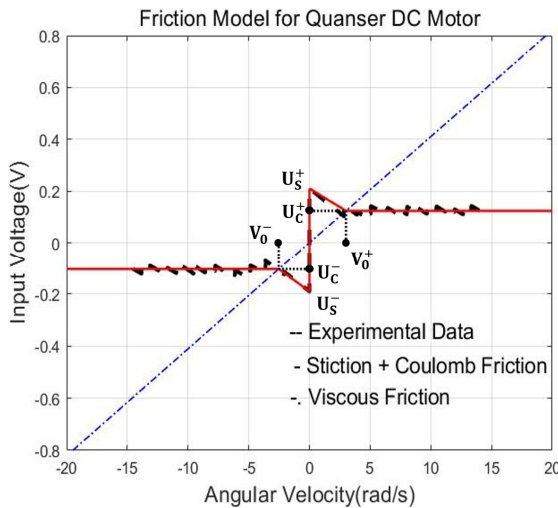
Fig. 2 Rotary motion control testbed: Quanser QUBE Servo 2 system



**Fig. 3 Quanser HIL (hardware-in-the-loop) motion control development system based on MATLAB Simulink**

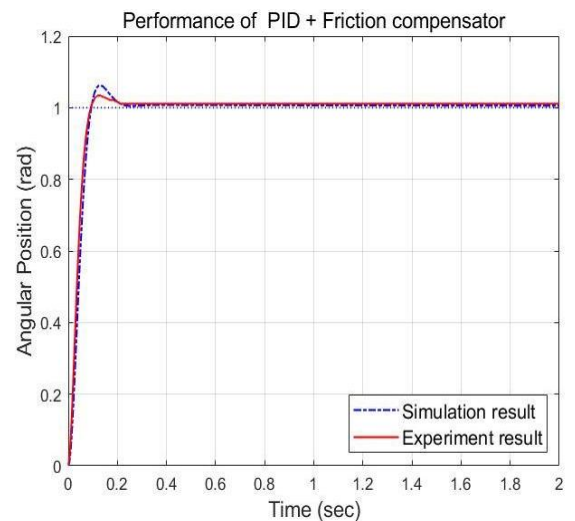


**Fig. 5 Quarc real-time control system setup for friction compensation experiments**



**Fig. 4 Friction force versus angular velocity: modeling of friction force with viscous friction, stiction and Coulomb friction components**

We have selected PID control gains based on nominal performance requirements as following:  $K_p = 7.8$ ,  $K_i = 1.5$  and  $K_d = 0.275$ . In order to enhance the performance of the PID controller, we have conducted experiments for friction measurements and constructed a nonlinear friction model as shown in Fig. 4. This nonlinear friction model is added into a nominal plant model shown in Eq. (1) as an augmented plant model. The friction model is also utilized as nonlinear friction compensation for the rotary motion control system as shown in Fig. 5<sup>[3,9]</sup>.



**Fig. 6 Comparison of step responses with friction compensation: simulation and experiment**

Simulation and experimental results based on the setup shown in Fig. 5 are presented in Fig. 6. By including friction compensation, the steady-state error of the experimental system has been significantly reduced to 1.11%, while the result from simulation exhibits almost zero steady-state error. This is the anticipated result, since the friction compensator should completely cancel the nonlinear friction model.

### 3. Design of Feedforward Tracking Controller

### 3.1 Zero Phase Error Tracking Controller (ZPETC)

Theoretically, if the closed-loop transfer is unity, we can achieve the perfect tracking without any additional control effort. Even with a high bandwidth PID-controller, precise tracking control is hardly achievable due to distortion caused by the closed-loop dynamics. If the desired trajectory that the motion control system should follow is known a priori, a feedforward controller can be designed such a way that the distortion could be compensated by mimicking the dynamics of feedforward controller that is equivalent to the inverse of the closed-loop dynamics. In this way, the overall system transfer function from the input to the output becomes close to the unity. Fig. 7 depicts a feedforward tracking controller used in this study.

The discretized closed-loop transfer function is given by

$$G_d(z) = \frac{y(z)}{r(z)} = \frac{P(z)C(z)}{1+P(z)C(z)} = \frac{z^{-d}\beta(z^{-1})}{\alpha(z^{-1})} \quad (2)$$

where

$$\alpha(z^{-1}) = \alpha_0 + \alpha_1 z^{-1} + \alpha_2 z^{-2} + \dots + \alpha_p z^{-p},$$

$$\beta(z^{-1}) = \beta_0 + \beta_1 z^{-1} + \beta_2 z^{-2} + \dots + \beta_q z^{-q}.$$

$\beta(z^{-1})$  can be rewritten as  $\beta(z^{-1}) = \beta^u(z^{-1})\beta^a(z^{-1})$

where  $\beta^u(z^{-1})$  contains uncancelable zeros and some zeros close to  $z = -1$ , and  $\beta^a(z^{-1})$  contains cancelable zeros.

The feedforward controller,  $F(z)$ , shown in Fig. 7 is designed by the following equation:

$$F(z) = \frac{r_d(z)}{r(z)} = \frac{z^d \alpha(z^{-1}) \beta^u(z)}{[\beta^u(1)]^2 \beta^a(z^{-1})} \quad (3)$$

where  $\beta^u(z)$  is obtained by replacing every  $z^{-1}$  in  $\beta^u(z^{-1})$  by  $z$ . The controller shown in Eq. (3) is proposed by Tomizuka and known as ZPETC (zero phase error tracking controller)<sup>[10]</sup>. Here  $[\beta^u(1)]^2$  is a

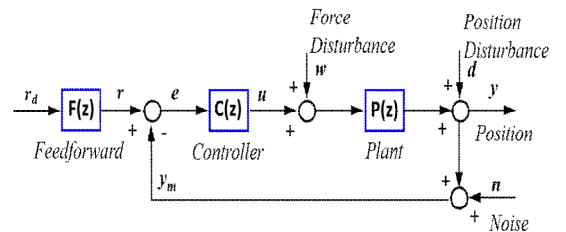
gain factor that makes the low frequency magnitude of the transfer function close to one. The zero phase tracking controller,  $F(z)$  in Eq. (3), cancels the cancelable zeros and poles of  $G_c(z)$  in Eq. (2) mutually and eliminates the phase shift caused by uncancelable zeros. This means that the phase shift of the overall transfer function is zero at all frequencies<sup>[10]</sup>.

### 3.2 Implementation of ZPETC

In order to implement ZPETC in the discrete-time domain, we need to discretize the plant transfer function  $P(s)$  and the controller  $C(s)$  into an equivalent digital form,  $P(z)$  and  $C(z)$  respectively, as shown in Fig. 7 and Eq. (2). Here  $P(z)$  is the zero-order-hold (ZOH) transformation of the plant,  $P(s)$ , shown in Eq. (1).

$$P(z) = \frac{c_1 z + c_2}{(z-1)(z-p)} \quad (4)$$

where  $p = \exp\left(-\frac{T_s B_e}{J_e}\right)$ ,  $c_1 = \frac{1}{B_e} \left[ T_s - \frac{J_e}{B_e} (1-p) \right]$ ,  $c_2 = -\frac{1}{B_e} \left[ T_s p - \frac{J_e}{B_e} (1-p) \right]$ , and  $T_s$  is the sampling time. The discrete-time equivalent PID-controller is obtained by utilizing the backward transformation for the derivative term and the bilinear transformation for the integral term.



**Fig. 7 Feedforward tracking controller structure implemented in the digital domain**

$$C(z) = K_p + K_d \frac{z-1}{T_s z} + K_i \frac{T_s(z+1)}{2(z-1)} \quad (5)$$

Poles and zeros of the closed-loop transfer function,  $G_d(z)$  of Eq. (2), are denoted as  $d_1, d_2, d_3, d_4, n_1, n_2,$  and  $n_3$ . These values are given in Table 1 for the sampling time of  $T_s = 0.001$ s. We identify one uncancelable zero at  $n_1 = -0.997920873$  that is close to  $z = -1$ . ZPETC in Eq. (3) is determined as

$$\begin{aligned} \alpha(z^{-1}) &= (1-d_1z^{-1})(1-d_2z^{-1})(1-d_3z^{-1})(1-d_4z^{-1}), \\ \beta^a(z^{-1}) &= mc_1(1-n_2z^{-1})(1-n_3z^{-1}), \\ \beta^u(z) &= (1-n_1z), \text{ and } m = K_p + \frac{K_d}{T_s} + \frac{K_i T_s}{2}. \end{aligned}$$

Since the implementation of ZPETC in the discrete-time domain is not trivial, Tomizuka and Sun have proposed the simplified realization of ZPETC<sup>[11]</sup>.

### 3.3 Desired Trajectory and Tracking Performance of ZPETC

The desired trajectory,  $r_d(t)$ , for tracking is designed as the fifth order polynomial that is continuous up to the third-order derivative at the start time,  $t = 0$ , and transition (or end) time,  $t = T_r$ .

$$r_d(t) = A \left[ 6 \left( \frac{t}{T_r} \right)^5 - 15 \left( \frac{t}{T_r} \right)^4 + 10 \left( \frac{t}{T_r} \right)^3 \right] \quad (6)$$

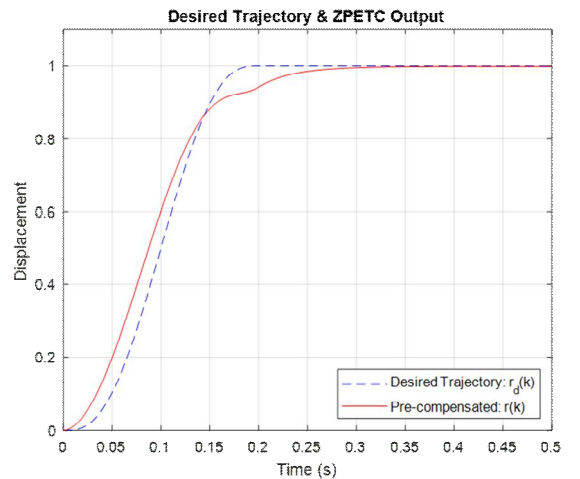
where  $A$  is an amplitude and  $T_r$  is the transition time. Fig. 8 shows the desired trajectory,  $r_d(k)$ , and the output of the ZPETC,  $r(k)$ , for  $A = 1$ rad,  $T_r = 0.2$ s and the sampling time,  $T_s = 1$ ms. Here  $r(k)$  is calculated by Eq. (3) and shows the pre-shaped trajectory of  $r_d(k)$  that will compensate the phase shift effect of the closed-loop transfer function,  $G_c(z)$ .

Fig. 9 demonstrates the performance of various tracking controllers to follow the desired trajectory given by Eq. (6). For the PID controller only, overshoot is observed during the transient period and

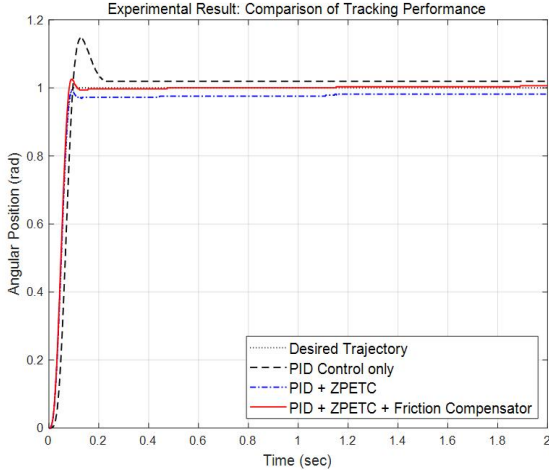
long tail of steady-state errors. For PID+ZPETC is effective in following the desired trajectory during the transient period. However, steady-state errors remain visible due to friction. In order to further improve the tracking performance, we add the friction compensator for this tracking controller, that is, PID+ZPETC+FC(friction compensator). The experimental performance demonstrates the faster rise time, reasonable overshoot and minimum steady-state errors.

**Table 1 Poles and zeros of  $G_d(z)$  in Eq. (2) for  $T_s = 0.001$ s**

Poles	$d_1 = 0.99806178$ $d_2 = 0.97707159386 + 0.0240100555i$ $d_3 = 0.97707159386 - 0.0240100555i$ $d_4 = 0.02007169128$
Zeros	$n_1 = -0.997920873$ $n_2 = 0.999806389$ $n_3 = 0.972604398$



**Fig. 8 Desired trajectory of the fifth order polynomial,  $r_d(k)$ , and the output of ZPETC,  $r(k)$**



**Fig. 9 Tracking performance comparison of various controllers: PID, PID+ZPETC, PID+ZPETC+ Friction Compensator**

#### 4. Disturbance Observer: A Class of Robust Feedback Controller

With a reasonably well identified and modeled plant, ZPETC performs superbly. However, its performance is not robust against unmodeled system dynamics, plant parameter variations and external disturbances. In order to enhance the performance of the tracking controller based on ZPETC, we propose to utilize a disturbance observer (DOB) – a robust feedback controller. Theories behind DOB have been well established and its implementation for motion control applications is shown in Fig. 10<sup>[4-9]</sup>.

The key parameter of DOB is the  $Q$ -filter and it is included in DOB blocks denoted as  $B_1$  and  $B_2$ <sup>[2,9]</sup>.

$$Q(s) = \frac{3(\tau s) + 1}{(\tau s)^3 + 3(\tau s)^2 + 3(\tau s) + 1} \quad (7)$$

$$B_1(s) = \frac{1}{1 - Q(s)} \quad (8)$$

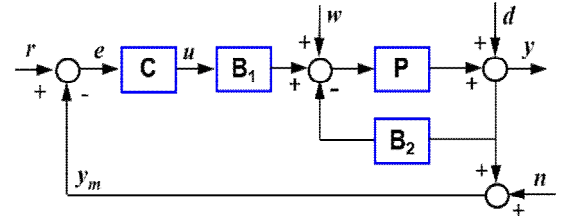
$$B_2(s) = \frac{Q(s)}{[1 - Q(s)]P_n(s)} \quad (9)$$

where  $P_n(s)$  the nominal plant model as shown in

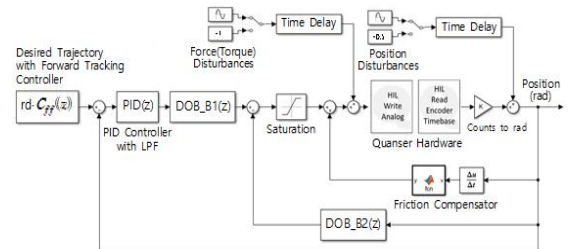
Eq. (1) and  $\tau$  is the time constant that determines the time- and frequency-domain characteristics of the  $Q$ -filter. Based on the experimental optimization, we select  $\tau = 0.04s$  for DOB implementation and further performance evaluation of the tracking controller<sup>[9]</sup>.

#### 5. Robust Tracking Controller Design and Its Performance Evaluation

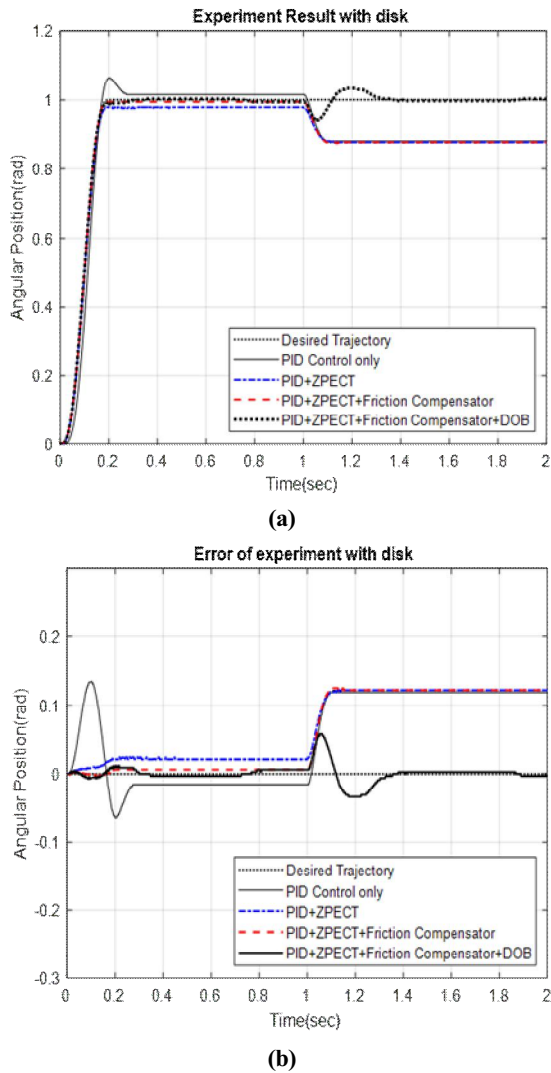
As we see in Fig. 9, the PID controller alone can not satisfactorily follow the desired trajectory given in Eq. (6). The feedforward tracking controller based on ZPETC pre-compensates phase shift due to the closed-loop transfer function and performs almost perfectly when the precise system identification is obtained. In order to provide the robustness to PID+ZPETC tracking control, the disturbance observer (DOB) has been added to the feedback loop.



**Fig. 10 DOB implemented in the position feedback-loop for this study**



**Fig. 11 Robust tracking controller structure and experimental setup: PID controller +  $C_{ff}(z)$  (ZPETC) + DOB + Friction Compensator (FC)**



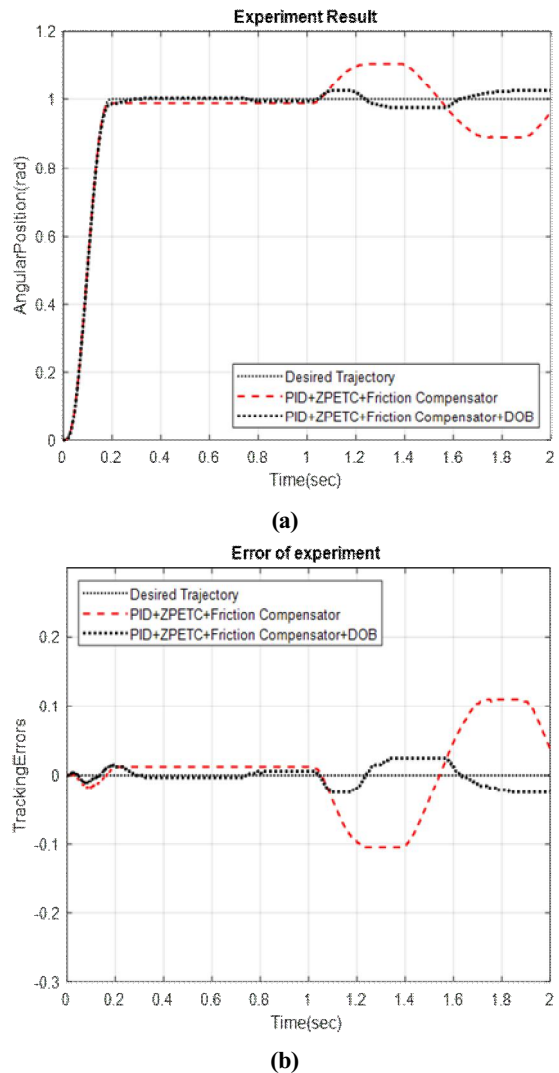
**Fig. 12 Comparison of controller performance under step torque disturbance and reduced inertia: (a) tracking performance; (b) tracking errors**

In order to further enhance the performance of PID+ZPETEC+DOB control, the friction compensator has been added to reduce the adverse effect of nonlinear friction components as shown in Section 2.

The overall robust tracking control scheme is shown in Fig. 11. The superior performance of the combined tracking controller has been proven

through experiments by utilizing Quanser Qube Servo 2 rotary motion control system.

In Fig. 12, experimental tracking performance of four different controller combinations under internal and external disturbances is displayed: 1) unity step torque disturbance exerted at  $t = 1$  s, and 2) the inherent friction effect of the motion control system.



**Fig. 13 Comparison of controllers under sinusoidal torque disturbance: (a) tracking performance; (b) tracking errors**

As we see in Fig. 12(b), DOB performs well in compensating external step disturbance and enhances the robustness of the tracking control system based on ZPETC.

The proposed controller of PID+ZPETC+FC+DOB demonstrates the best overall tracking performance. However, slight transient tracking errors below 0.3s can be visible due to modeling uncertainty.

Disturbance rejection capability of the disturbance observer against external sinusoidal torque disturbance is demonstrate in Fig. 13. DOB effectively compensated the 1Hz sinusoidal torque disturbance. Addition of the friction compensator has further improved the tracking performance, although there are remaining minor tracking errors during the transient period and the steady state due to unmodeled dynamics.

## 6. Conclusion

In this study, the robust tracking control scheme based on the zero phase error tracking controller (ZPETC) and the disturbance observer (DOB) is presented. The superior performance of the proposed algorithm is confined by both MATLAB simulations and experimental results.

- 1) Nonlinear friction forces have been measured and modeled. The nonlinear model is used as an augmented plant and inserted into the velocity loop as an add-on compensator specifically designed for canceling nonlinear friction components. This scheme makes the simulation results agree well with experiments. Hence control system design engineers can do speedy design works by relying more on realistic simulations than expensive experiments.
- 2) The disturbance observer (DOB) is effective against step torque/force disturbances, sinusoidal disturbances below cutoff frequency, and unmodeled linear friction.
- 3) ZPETC (Zero Phase Error Tracking Control)

performs well with a precisely identified plant model. Since ZPETC is based on inverse dynamics, it is sensitive to unmodeled dynamics, modeling errors, and external disturbances. Hence DOB is an effective complement for providing the overall system robustness. For further enhancement of overall tracking control performance against nonlinear friction forces, the friction compensator (FC) is added into the velocity feedback loop. The high performance tracking controller structure of PID+ZPETC+DOB+FC is implemented and its superior performance has been experimentally demonstrated.

## Acknowledgment

This work was supported by the fund of research promotion program, the Office of Academy and Industry Collaboration, Gyeongsang National University, and the fund of 2020 Undergraduate Research Program (URP), Korea Foundation for the Advancement of Science and Creativity.

## References

1. Armstrong-Helouvry, B., Dupont, P., and Canudas de Wit, C., "A Survey of Model, Analysis Tools and Compensation Methods for the Control of Machines with Friction," *Automatica*, Vol. 30, No. 7, pp. 1083-1138, 1994.
2. Lee, H. S. and Tomizuka, M., "Robust Motion Controller Design for High-Accuracy Positioning Systems," *IEEE Transactions on Industrial Electronics*, Vol. 43, No. 1, pp. 48-55, 1996.
3. Lee, H. S., Jung, S., and Ryu, S., "Performance Enhancement of Motion Control Systems Through Friction Identification and Compensation," *Journal of the Korean Society of Manufacturing Process Engineers*, Vol. 19, No. 6, pp. 1-8, 2020.
4. Ohnishi, K., Shibata, M., and Murakami, T., "Motion control for advanced mechatronics,"



- IEEE/ASME Transactions on Mechatronics, Vol. 1, No. 1, pp. 56-67, 1996.
5. Itagaki, H. and Tsutsumi, M., "Control System Design of a Linear Motor Feed Drive System Using Virtual Friction," Precision Engineering, pp. 1-12, 2013.
  6. Chen, W. H., "Disturbance Observer Based Control for Nonlinear Systems," IEEE/ASME Transactions on Mechatronics, Vol. 9, No. 4, pp. 706-710, 2004.
  7. Ryoo, J. R., Doh, T. Y., and Chung, M. J., "Robust Disturbance Observer for the Track-following Control System of an Optical Disk Drive," Control Engineering Practice, Vol. 12, No. 5, pp. 577-585, 2004.
  8. Sariyildiz, E. and Ohnishi, K., "Stability and Robustness of Disturbance-Observer-Based Motion Control Systems," IEEE Transactions on Industrial Electronics, Vol. 62, No. 1, pp. 414-422, 2015.
  9. Lee, H. S. and Ryu, S., "Design of a Robust Motion Controller for a Rotary Motion Control System: Disturbance Compensation Approach," Microsystem Technologies, Vol. 27, pp. 2293-2302, 2021.
  10. Tomizuka, M., "Zero Phase Error Tracking Algorithm for Digital Control," ASME Journal of Dynamic System, Measurement and Control, Vol. 109, No. 1, pp. 65-68, 1987.
  11. Tomizuka, M. and Sun, L., "Simplified Realization of Zero Phase Error Tracking," ASME Journal of Dynamic System, Measurement and Control, Vol. 143, No. 3, pp. 031008(1-6), 2021.