J. Korean Soc. Math. Educ., Ser. D, Res. Math. Educ.
Vol. 24, No. 2, 83-109, June, 2021
http://doi.org/10.7468/jksmed.2021.24.2.83

The Pacing of Volume Lessons in American Elementary Textbooks Compared to Students' Development in Volume Measurement

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(Received April 29, 2021; Revised June 22, 2021; Accepted June 28, 2021)

In the early stage of lesson enactment process, teachers use textbooks and other resources to select tasks and activities. It follows that discrepancies between textbooks and research-recommended pathways for learning may lead to concerns or issues with pacing in the classroom. To explore this idea further, this study examined the alignment between three popular standards-aligned textbooks series and volume learning trajectories. The results indicated that the standards-based textbooks examined may lack attention to important topics in the pacing of volume instruction, and suggest the need to inform both pre-service and in-service teachers about the gap between textbook lessons and volume learning trajectories so that they will be able to reflect students' thinking in volume learning trajectory to their lessons.

Keywords: Textbooks, alignment, learning trajectory, volume
MESC Classification: C70
MSC2010 Classification: 97G30

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I. INTRODUCTION

Recent reform efforts in mathematics education call for using carefully selected tasks and reflecting students' thinking in mathematics instruction (National Council of Teachers of Mathematics [NCTM], 2014). One attempt to address such reform efforts has led to recent attention to learning trajectories (LT) in mathematics education. The attention was made on the areas of standards, curriculum, and instruction (Clements, Sarama, Spitler, Lange, & Wolfe, 2011; Wilson, Sztajn, Edgington, & Myers, 2015). Learning trajectories are research-based students' development of students' levels of thinking in specific mathematical topics (Clements & Sarama, 2004). With the attention given to LT, it is important to find ways to support teachers in designing lessons and selecting tasks that reflect evidence of students' mathematical thinking. Researchers believe that it is more effective, efficient, and generative for students to have learning experiences that are consistent with such natural developmental progressions compared to learning that does not follow these paths (Clements & Sarama, 2004). Previous studies have shown the benefits of LT in preparing lessons, selecting mathematical tasks, and creating richer classroom environments (Clements et al., 2011; Wilson et al., 2015).

One important component of LT is sequencing of instructional tasks that correspond to the order of LT (Clements & Sarama, 2004). With LTs as the basis, mathematical lessons, tasks and activities can be carefully selected to reflect students' thinking and avoid the fragmentation common in U.S. textbooks (Clements, 2007). One important curriculum material is textbooks. Although researchers have different views, researchers are generally in consensus that textbooks play an important role in shaping teachers' lessons (Polikoff, 2015; Remillard, Harris, & Agodini, 2014; Stein, Remillard, & Smith, 2007). If topics and tasks in mathematics textbooks are aligned to students' development steps of LT, it is likely that when teachers use those textbooks to plan their lessons, teachers will have opportunities to plan mathematical lessons that reflect students' thinking. In turn, tasks and activities can be selected to reflect students' development steps as well. In the U.S., the development of the Common Core State Standards of Mathematics (CCSSM) began with research-based learning trajectories detailing how students' mathematical understanding develops over time (Daro, Mosher, & Corcoran, 2011). Then, series of textbooks were developed to be aligned to CCSSM. Since LTs were developed based on research studies, LTs studies act as an evidence base for the Common Core State Standards. Thus, it would be important to examine the alignment between LT and textbooks.

Several studies have examined the alignment between curriculum standards and assessments (Polikoff, Porter, & Smithson, 2011) and textbooks and the Common Core

Standards (Polikoff, 2015). These studies illustrate measuring alignment between curriculum materials is an important issue in education. Because textbooks are important resources for teachers, researchers examined textbooks for different purposes, such as content, tasks, and alignment to the Common Core Standards (Polikoff, 2015; Smith, Males, & Gonulates, 2016). Another way to analyze content is to understand the alignment of content to its learning goals (National Research Council, 2004) and examining how textbooks' content reflect students' development steps in LTs is one critical area to consider when selecting textbooks (National Council of Teachers of Mathematics [NCTM], 2014).

Among the many topics in mathematics, we selected volume lessons because geometry topics connect to other areas of mathematics and to students' experiences with the physical world (Sarama & Clements, 2009). However, the findings from national assessments indicate that American students show poor performance in this domain compared to other areas (Lehrer, 2003; Mullis, Martin, Foy, & Arora, 2012). Recently, previous studies have demonstrated curricular limitations of popular American textbooks in length and area measurement topics (Smith, Males, Dietiker, Lee, & Mosier, 2013; Smith et al., 2016). Our analysis of measurement topics of volume in three additional popular American textbook series can expand our understanding of how textbooks treat measurement lessons further, as well as expand previous findings by comparing textbook coverage and sequencing to research-backed learning trajectories for volume. With the results of our analysis, we also attempt to identify any gaps between empirically supported pathways for learning and textbooks, as well as offer suggestions to textbook authors and teachers for filling those gaps. Our suggestions can be good resources to support both pre-service and in-service teachers with designing volume lessons that reflect student's thinking and development, and supplement any instructional gaps left by textbooks. The purpose of this study is to examine alignment of mathematical items in Common Core aligned textbooks to specific LT using Surveys of Enacted Curriculum (SEC), which were used in previous textbook and curriculum analyses (Polikoff, 2015; Porter, McMaken, Hwang, & Yang, 2011). Here are our research questions that we attempted to answer.

1. To what extent are the Common Core standards aligned with volume learning trajectory?
2. To what extent are items in Common Core textbooks aligned with volume learning trajectories?
3. What are the sources of misalignment, if any?

II. RELATED LITERATURE

1. OPPORTUNITIES TO LEARN THAT TEXTBOOKS OFFER

Researchers generally agree that there are several stages, written or formal, intended, and implemented or enacted, in curriculum enactment process (Remillard & Heck, 2014). In most countries, "formal" or "written" curriculum is what officials recommend and expect teachers to teach or the goals and activities outlined by school policies (Stein et al., 2007). Teachers can plan their lessons in "Intended" curriculum such as lessons plans and "enacted" or "implemented" curriculum is what teachers actually teach in their classes (Remillard & Heck, 2014; Stein et al., 2007). Official State standards and CCSSM are included in written curriculum (Schmidt et al., 2001; Stein et al., 2007) and events that teachers and students experience in classes are included in enacted curriculum (Remillard & Heck, 2014). With these meanings, curriculum goes through several stages until teachers and students experience in their classrooms (Remillard, 2005; Remillard & Heck, 2014; Stein et al., 2007). Although researchers called textbooks differently "written", "formal", "intended" and "potentially implemented" curriculum (Cai & Howson, 2013; Stein et al., 2007; Valverde, Bianchi, Wolfe, Schmidt, & Houang, 2002), researchers generally agree that textbooks continue to play an important role in shaping teachers' intended and enacted mathematics curricula (Polikoff, 2015; Remillard et al., 2014; Stein et al., 2007). When teachers prepare their lessons in the early stage of curriculum enactment process, they use textbooks and other curriculum materials to select and possibly modify tasks and activities that are appropriate for particular lessons (Remillard, 2005; Remillard & Heck, 2014). In the process of planning mathematics lessons, what textbooks offer often translated as opportunities to learn (OTL) mathematics (Hong & Choi, 2014, 2018; Otten, Gilbertson, Males, & Clark, 2014; Smith et al., 2013). OTL can be defined as whether or not students have had the opportunity to study a particular topic or learn how to solve a particular type of problem (Floden, 2002; Liu, 2009). Since students can have OTL when they use textbooks (or other materials) to study and also, when they are in their mathematics classes, students' OTL can be a joint product of what curriculum materials offer and what they experience in their mathematics classes (Remillard & Heck, 2014; Stein et al., 2007). Since teachers will select and modify tasks and activities based on their belief and pedagogical orientations (Roth McDuffie, Choppin, Drake, Davis, & Brown, 2017; Son & Kim, 2015), some tasks and activities in textbooks are not going to be selected. After the lesson planning stage, when teachers enact those lessons, what is selected from the textbook can be transformed into OTL for students. In turn, limited coverage of volume measurement topics in textbooks can limit what teachers can select from those textbooks and can also possibly lead to teachers' volume measurement lessons might not reflect students' learning development well. As we mentioned, when the CCSSM were developed in the U.S., the committee considered the sequences of LT in the development process to correspond with students' mathematical understanding and development over time (Confrey, 2012). Since textbooks are one of the important steps in curriculum enactment process, aligning textbook content to LTs can potentially provide teachers with resources that reflect students' natural development and thinking in learning mathematics.

2. LEARNING TRAJECTORIES AND COMMON CORE STATE STANDARDS OF MATHEMATICS

As previously mentioned, the development of CCSSM began with research-based learning trajectories detailing how students' mathematical understanding develops over time (Daro et al., 2011). When the CCSSM were developed, the writing team thought it would be helpful to invite researchers who have been active in developing learning trajectories that cover significant elements of the school mathematics curriculum to discuss the standards writing effort. Those meetings brought writers of the standards together with researchers to discuss the potential of learning trajectories (LTs) research to act as an evidence base for the standards (Daro et al., 2011). Eventually, many of the attendees joined the CCSSM writing teams and researchers subsequently submitted trajectory examples to the writers (Confrey, 2012). According to Confrey (Confrey, 2012), although there are other voices, LT researchers' ideas contributed significantly to the final document. In the report by the CCSSM writing team shows the details of how the writing team worked with LT researchers to reflect the results of LT studies in writing CCSSM. In the report, Daro and his colleagues introduced various LTs for different mathematical topics and Sarama and Clements' work (Sarama & Clements, 2009) was introduced as a sample LT for measurement, which we are planning to use to measure alignment.

3. VOLUME LEARNING TRAJCTORY

There are three components of a LT: the learning goals, learning activities, and the thinking and learning in which students might engage (Simon, 1995). Clements and Sarama 2004) stated that generalizable LTs should be based on empirical research and defined LTs in the following way.

> we conceptualize learning trajectories as descriptions of children's thinking and learning in a specific mathematical domain and a related, conjectured route through a set of instructional tasks designed to engender those mental processes or actions hypothesized to move children through a developmental progression of levels of thinking, created with the intent of supporting children's achievement of specific goals in that mathematical domain (p. 83).

Thus, LTs are natural developmental progressions identified in empirically based models of children's thinking and learning. Therefore, LTs can be helpful in understanding how students think, curriculum design and also guiding teachers in selecting appropriate activities and tasks for their lessons (Clements & Sarama, 2004). The second component of LT is designing a sequence of instructional tasks that correspond to the order of LT (Clements & Sarama, 2004). When planning lessons, teachers select activities and tasks from textbooks and other curriculum materials. When those tasks are aligned with corresponding LTs, we can focus on the logic and progression of the students when they learn certain mathematical topics (Sztajn, Confrey, Wilson, & Edgington, 2012). Studies show that teachers are more responsive to students' mathematical thinking and had richer learning environments when they used curriculum based on LTs (Clements et al., 2011).

Sarama and Clements (2009) comprehensively summarized research findings on learning mathematics and suggested detailed learning trajectories for various mathematics content areas. They described the learning trajectories of geometry and geometric measurement at great length, carefully outlining key characteristics of the different stages of understanding as well as what students may (or may not) be able to do at each stage in the process. Some foundational concepts for volume measurement include filling a given three–dimensional space with unit cubes, comparing by counting rows of arrays and understanding layer structure (Battista & Clements, 1996; Sarama & Clements, 2009). Table 1 shows Sarama and Clements' learning trajectory (SCLT) for volume including students' developmental progression not only during school years, but also during very early childhood.

Table 1. A developmental progression for volume measurements (Sarama & Clements, 2009)

	Volume
Ages 0-3	Volume quantity recognizer
Age 4	Capacity direct comparer
Kindergarten (Age 5)	Capacity indirect comparer
Grade 1 (Age 6)	Primitive 3-D array counter
Grade 2 (Age 7)	Capacity relater and repeater Partial 3-D structurer
Grade 3 (Age 8)	3-D row and column structurer
Grade 4 (Age 9)	3-D array structurer

As seen in Table 1, the learning trajectory of volume begin from the stage of recognizing, filling three - dimensional quantities with unit and end with the stage of structuring two and three- dimensional spaces (layer) systematically. With these development steps, students can make progress from counting faces of unit cubes or double counting unit cubes to comparing, filling three – dimensional spaces and understanding how to link layer structure to volume formula (length $\times$ width $\times$ height) to compute volume.

There are other researchers that investigated how students learn volume in their early age (Battista, 2004; Battista & Clements, 1996; Vasilyeva et al., 2013). Although these studies did not specify students' age for each development level when learning volume, the results demonstrate similar challenges and difficulties, which indicate the importance of development steps in SCLT.

Battista and his colleagues introduced seven different levels of understanding volume. These seven levels are not exactly identical to Sarama and Clements' LTs but there are common challenges that students have. Table 2 describes seven levels that were found by Battista and Colleagues.

Table 2. Students' development in volume measurement by Battista

	Area and Volume Development
Level 1	Absence of units-locating and organizing-by-composites processes.
Level 2	Beginning use of the units-locating and the organizing-by-composites processes
Level 3	Units-locating process becomes sufficiently coordinated to recognize and eliminate double-counting errors
Level 4	Use of organizing-by-composites process to structure an array with maximal composites, but insufficient coordination for iteration.
Level 5	Use of units-locating process sufficient to correctly locate all units, but less-than-maximal composites employed
Level 6	Complete development and coordination of both the units-locating and the organizing-by-composites processes
Level 7	Students' spatial structuring and enumeration schemes become sufficiently abstract so that students can (a) understand the connection between numerical procedures and spatial structurings, and (b) generalize their reasoning to "packages."

Adopted from Battista, (Battista, 2004)

The results show that some students do not use layer structure and only one dimensional row or column or see faces of 3D figure so that they are not able to count the number of unit cubes correctly (Battista & Clements, 1996). In addition to Battista and colleagues' study, other studies also show that understanding unit, layer structures are challenging to elementary students (Vasilyeva et al., 2013). Students use counting faces, double counting, partial and/or complete layer structure to solve volume problems; however, when students use development steps in LT, using layer structure, they tend to answer volume problem correctly (Vasilyeva et al., 2013). While it is well documented that developmental steps in volume LT are challenging, Trends in International Mathematics and Science Study 2011 (TIMSS 2011) study shows that fundamental steps in LT are still challenging to older students. For example, 75% of 8th graders (international average) answered item M052206 incorrectly, which required students to use a same-sized unit (a book in this problem) to repeatedly to fill three-dimensional space to estimate the volume of a box (Mullis et al., 2012). More than half of 8th graders (53 % - international aeverage) have difficulties when solving item M032100 about being able to see and structure three-dimensional space that can be filled with unit cubes (Mullis et al., 2012). Additionally, 57 % of 8th graders (international average) answered item M042201 incorrectly, which required students to find the missing value for the length when the volume, the width and the height are known (Mullis et al., 2012). Since 8th graders still have challenges in solving these volume tasks, these results from TIMSS indicate that it is important for students to have OTL to be exposed to these important development steps.

III. METHODS

1. DATA SOURCES

Because many elementary mathematics textbooks are used in U.S. classrooms, we were forced to choose some series over others. Three textbooks series - *enVisionMath*, *Go Math*, and *MyMath* - are common core aligned textbooks from three major American publishers (Pearson, McGraw-Hill, and Houghton Mifflin). Polikoff (Polikoff, 2015)used the first two and examined *Math Connects* textbooks from McGraw-Hill, but, according to McGraw-Hill, the *MyMath* textbooks series are a new version of common core aligned textbooks from McGraw-Hill. Each publisher's website has a correlation study that indicates how each lesson in textbooks correlated to specific CCSSM. In fact, the first page of each lesson in three series shows the specific CCSSM that each lesson addresses. In addition to the fact that these three series were from three main publishers in America, *enVisionMath* and *Go Math* are two popular elementary textbook series in America (Dossey, Soucy McCrone, & Halvorsen, 2016; Sahm, 2015) and according to McGraw-Hill, there are 3 million *MyMath* users in America. Also, recent studies have examined measurement lessons in other popular American textbooks (Smith et al., 2013; Smith et al., 2016) so to expand our understanding of textbook coverage of measurement lessons, it is appropriate to examine common core aligned textbooks. Therefore, we decided to examine all geometry lessons related to the topics of volume from a total of 18 textbooks[2] (six textbooks ─ K through 5 ─ from each publisher), all of whose latest editions were published in 2014 or 2015, to explore learning opportunities presented in these textbooks.

2. APPROACHES TO ANALYZE TEXTBOOKS

When textbooks are analyzed exposition (the paragraphs, text boxes that contain definition formulas and theorems and worked examples in each lesson), worked examples (problems presented along with an explained solution) and exercise problems (mathematical items that

[2] We examined kindergarten, fourth and sixth grade textbooks but we did not find items related to volume so they are not included in Table 2.

students are expected to solve) should be examined because they can provide potentially different learning opportunities to students: teachers can use exposition to introduce mathematics content, students can see how certain problems are solved with worked examples and students can have opportunities to engage in mathematical tasks (Li, 2000; Otten et al., 2014). Previous studies examined all three areas when various textbooks were analyzed for the potential learning opportunities (Li, 2000; Otten et al., 2014). To examine how these items were counted, we have searched for other textbook analysis studies and found out that when the frequency of these items was considered, they were weighted equally (Ding, 2016; Hong, Choi, Runnalls, & Hwang, 2018, 2019; Polikoff, 2015; Smith et al., 2016). We also know that it would be possible to develop an alternative weighting scheme, such as weighting by importance, equal weighting is the most logical and defensible approach because it will be very challenging to determine which are more important than others in terms of providing learning opportunities. In all, 174 (*Envision Math*), 151 (*My Math*), and 217 (*Go Math*) items were analyzed. Table 3 describes the number of pages and lessons that we examined for this study.

Table 3. Textbooks series used in the study

Textbook Series	Publisher	Publication Date	Pages	Lessons
envision Math Series	Pearson	2015	23	6
Go Math Series	Houghton Mifflin	2015	57	4
MyMath Series	McGraw-Hill	2014	20	4
Three Series Total			100	14

3. SURVEY OF ENACTED CURRICULUM

The Webb Alignment Tool (WAT) and the Surveys of Enacted Curriculum (SEC) are two well –known ways to measure alignment (Martone & Sireci, 2009; Newton & Kasten, 2013). WAT includes a more detailed analysis of alignment than SEC, while SEC includes simpler ways to measure alignment in terms of content topics and cognitive demand (Martone & Sireci, 2009; Newton & Kasten, 2013). Our main goal in this study was to measure how items in Common Core-aligned textbooks and students' development in volume measurement agreed with each other with a particular interest in how they aligned across content topic level. When WAT and SEC were compared, SEC was deemed more appropriate to measure *content* level alignment, as WAT provided several other dimensions of alignment (Martone & Sireci, 2009). This study used the Surveys of Enacted Curriculum (SEC) which were used in previous textbook and curriculum alignment analyses (Polikoff, 2015; Porter et al., 2011) and other alignment studies (Liu & Fulmer, 2008; Liu et al., 2009). SEC gives a simple way to measure alignment. It uses two matrices, one for each curriculum material (e.g. LTs, CCSSM and textbooks in this study) and the following formula:

$$\text{Alignment Index} = 1 - \frac{\sum_{i=1}^{n}|x_i - y_i|}{2}$$

where n is the total number of cells in the table. Here, x_i is the proportion of content in cell i of document x (e. g. CCSSM) and y_i is the proportion of content in cell i of document y (e.g. LTs for volume). Both x_i and y_i can have a value between 0 and 1. If two documents are perfectly aligned, the alignment index is 1. Thus, an index closer to 1 shows better alignment between the two documents. For example, in the two frequency tables below, we can use the ratio to calculate the absolute value of the discrepancy between the ratios and compute the SEC index.

Frequency Tables

1	2
3	4

3	4
2	1

Ratio Tables

.1	.2
.3	.4

.3	.4
.2	.1

$$\text{Alignment Index} = 1 - \frac{\sum_{i=1}^{n}|x_i - y_i|}{2} = 1 - \frac{.2+.2+.1+.3}{2} = 1 - \frac{.8}{2} = .6$$

Figure 1. Example calculation of the SEC index for a pair of 2 by 2 tables.

4. CODES ESTABLISHMENT FOR VOLUMN TOPICS IN LEARNING TRAJECTORIES AND CCSSM

The SEC includes a set of mathematical topics that may be used when two educational components were compared. The SEC then compares two matrices that contain measures of the inclusion of those mathematical topics, and computes alignment based on evaluation of the agreement in the cell values of the two tables. However, we were not able to use the topics available in the SEC because those topics were not specific enough – there was only one relevant area topic, namely "Area and Volume". Thus, we needed to develop volume content codes from SCLT and CCSSM. Both SCLT and the CCSSM have volume topics for appropriate age and grade levels. During research meetings, authors discussed coding these topics. Coding the textbooks items began by establishing content codes from both the CCSSM and SCLTs for volume (see Appendix A) (Sarama & Clements, 2009). To create list of volume topics and topics that are included in the CCSSM and SCLT, the authors met several times to discuss and examine how those topics are described in CCSSM and SCLT. As we discussed SCLT and CCSSM, we noticed that both SCLT and CCSSM include several volume topics in one grade (age) level (or one standard) so we were able to establish several content codes from both SCLTs and CCSSM from one grade (age) level/standard. For example, around age 6 (1st grade), we found the following from volume SCLT.

> Partial understanding of cubes filling a space (students might count faces of cubes and also possibly double count cubes) (Sarama & Clements, 2009, p. 307)

From this, we discussed and established code 101. We’ve also discussed that it is not possible to measure students’ partial understanding with just textbook items. Thus, for code like this, we were trying to see if 1st grade textbooks give students opportunities to experience filling a space with cubes. With items about filling a space with cubes, students are able to demonstrate partial understanding, counting faces or double counting, and gradually build understanding of volume measurement. We also found around age 7 (2nd grade) the following from SCLT.

> Understands cubes as filling space but does not use layers (Sarama & Clements, 2009, p. 307)

Once we found this from SCLT, we discussed and established codes 203. Again, we wanted to see if 2nd grade textbooks give students opportunties to experience fillng a space with cubes. We’ve also examined CCSSM to see if CCSSM inlcude similar standards even if the wording is not exactly same. We had similar discussions with CCSSM as well. For example, in the fifth grade CCSSM, we found the following standard.

> A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010).

From this, we established codes 501. Since the SCLTs suggest not only the order of development of volume topics, but also the ages for doing so, these recommended ages were matched against typical corresponding grade levels. Thus, kindergarten codes were matched with age 5, grade 1 progressions were matched with age 6, etc. For example, around the age of 8 (third grade), according to area SCLT, students are able to compute the number of cubes in one row and multiply by the number of layers to get the total. Once we observed these topics from SCLT, we examined the CCSSM to determine whether fifth grade CCSSM include a similar standard even if the wording is not exactly the same.

Codes derived from SCLTs (LT codes) helped us examine how textbooks paced learning compared to the learning progressions developed for volume. We also created codes from the CCSSM for learning volume as these codes could inform us of early implementation of later standards or revisiting prior standards in students' learning progression. Some codes are derived from both SCLTs and CCSSM because we determined that some topics are about identical volume concepts even if the wording is not exactly same. Both the CCSSM codes and LT codes were then combined to create a comprehensive list of codes for volume; we expected these codes to show how standards-based textbooks paced student learning trajectories. Appendix A[3] illustrates our established content codes for volume from the CCSSM and learning trajectories.

5. CODING EXAMPLES

After creating of a list of volume content codes from SCLT and CCSSM (Table in Appendix A), we examined textbook pages to see if we can develop new or significant codes that were not found from SCLT and CCSSM. If found, these codes would have represented a further sign of misalignment with the CCSSM and LTs, but would have also required coding if they contributed significantly to the textbook's development of volume measurement. However, we did not find any new codes from textbook items, and so the content codes in Table in Appendix A are the final set of comprehensive codes used for our analysis.

We then coded items from the three Common Core-aligned textbook series to explore how mathematical items in volume lessons in the three series reflected developmental steps in the SCLT and CCSSM. First, to identify volume lessons, we first examined the table of contents of each textbook. We first included those lessons that contained the term "volume" in the title.

[3] The first number in each code represents its corresponding grade level. For example, the first number, 2, in the code 202 indicates this code is from the second grade topic.

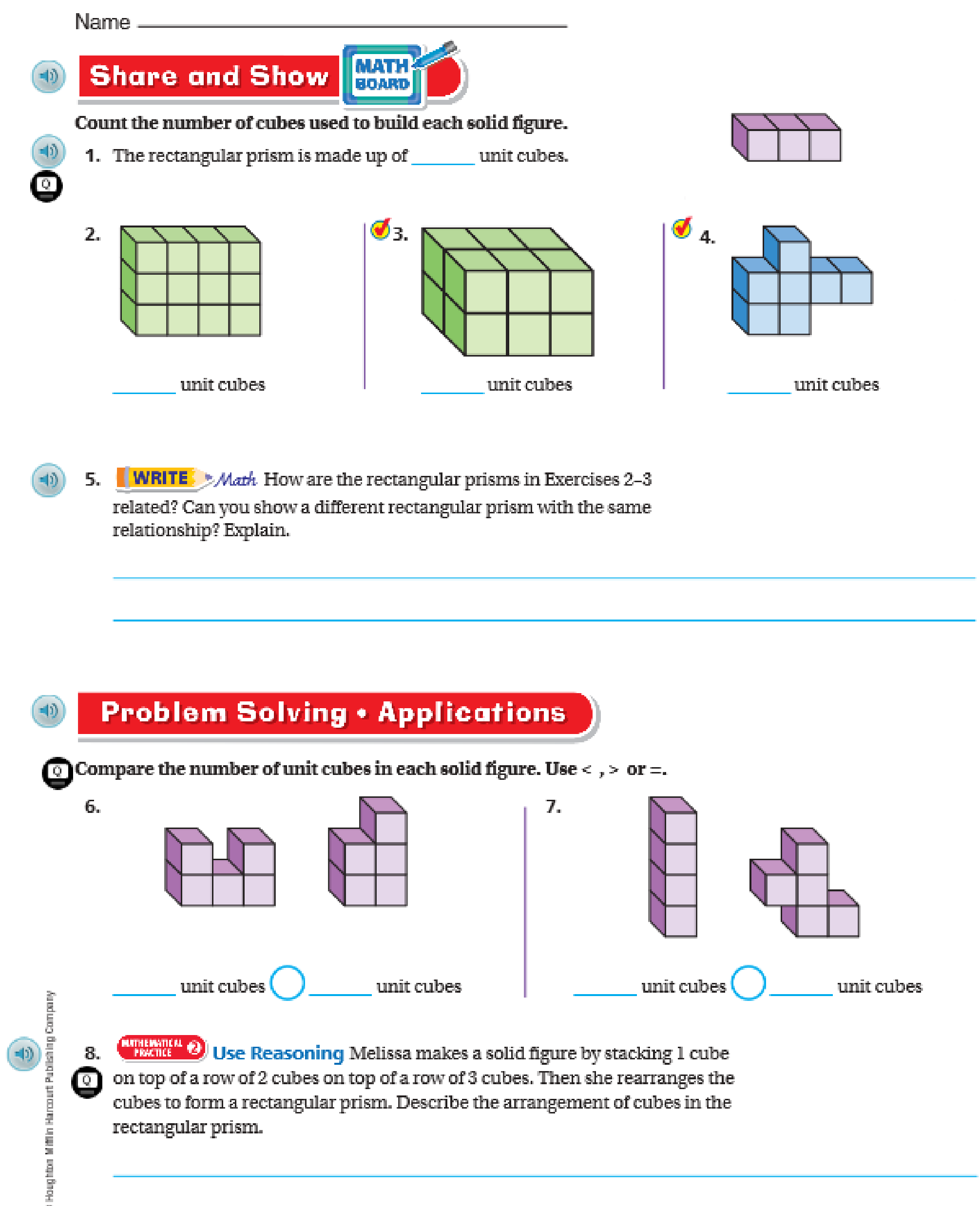
We then conducted a manual search to identify lessons that did not have the term “volume” in the title, but were related to volume measurement. Some keywords such as “space – filling”, “layers” and “building three dimensional shapes” were used. All pages with at least one instance of such content were included. Then, we needed to decide on a unit of analysis. The coded unit of analysis was a single sentence or problem; it also could be two or more consecutive sentences or paragraph for worked example and exposition. Figure 2 and Table 4 show one example of unit of analysis we used in this study. Since we gave equal weight to these, there are eight items to be coded. Once we found these items from a textbook, we looked at each code (e. g. Appendix A) and assigned codes for each item. For example, items in Figure 2 are all related to measuring volume by counting unit cube so they are coded accordingly (503).

Table 4. Coding examples

Item number	Code
1	503
2	503
3	503
4	503
5	503
6	503
7	503
8	503

6. CODING RELIABILITY

Each textbook includes worked examples, exercise problems, and exposition. As Polikoff (2015) did, we gave equal weight to these. After discussion about the established codes, two authors coded about 20% of the textbook items to check inter-rater reliability. The coders gave, at most, three codes to each item, because an item can possibly involve multiple standards in the CCSSM as well as stages in the learning trajectory. After comparing codes for sample items and finding an acceptable high inter-rater reliability, the authors coded all textbook items jointly to produce a final set of tables for analysis, resolving coding differences of individual items. To determine reliability, we applied a generalizability theory D study (Alkhrausi, 2012). This technique produced a reliable coefficient of 0.964. The reliability coefficient is greater than coefficients found in previous research with the same technique (Polikoff, 2015). In all, 174 (*Envision Math*), 151 (*My Math*), and 217 (*Go Math*) items were analyzed.



Name ________

Share and Show MATH BOARD

Count the number of cubes used to build each solid figure.

1. The rectangular prism is made up of ______ unit cubes.

2. ______ unit cubes

3. ______ unit cubes

4. ______ unit cubes

5. WRITE Math How are the rectangular prisms in Exercises 2–3 related? Can you show a different rectangular prism with the same relationship? Explain.

Problem Solving • Applications

Compare the number of unit cubes in each solid figure. Use <, > or =.

6. ______ unit cubes ◯ ______ unit cubes

7. ______ unit cubes ◯ ______ unit cubes

8. MATHEMATICAL PRACTICE 2 **Use Reasoning** Melissa makes a solid figure by stacking 1 cube on top of a row of 2 cubes on top of a row of 3 cubes. Then she rearranges the cubes to form a rectangular prism. Describe the arrangement of cubes in the rectangular prism.

Figure 2. Sample textbook page (*Go Math*, 2015 5th, p. 665).

IV. RESULTS

1. ALIGNMENT BETWEEN LT AND CCSSM

Table 5 indicates that the volume topics that students' progress through are not aligned to what they are expected to learn in the CCSSM. We found two explanations for this. Topics were either not covered at all in the CCSSM or they were misaligned because grade level (age level) expectations from SCLT and the CCSSM differed.

Table 5. Main alignment indices for LT and CCSSM

	Learning trajectory: Volume
Common Core: Volume	0.00

Table 6 reveals that the alignment between textbook items in volume topics and SCLT. Remarkably, volume items in textbooks are not aligned except for *Go Math* (about 1.4% alignment). Clearly, the opportunities that textbooks offer and how students learn volume according to SCLTs were not aligned. In addition to examining the alignment indices, we need to explore the sources of these misalignments.

Table 6. Alignment indices for LT and Textbooks

	Go Math	My Math	Envision Math
Learning Trajectory: Volume	0.0138	0.00	0.00

2. SOURCE OF MISALIGNMENT

Table 7 displays the frequency of volume learning trajectory codes in each textbook. Some codes are not included in these textbook series (e.g., 201, 202, 401, and 403). These codes are "use simple units to fill containers", "understands fewer larger than smaller objects needed to fill a given container", "multiplicatively iterates squares in a row or column to determine area", and "explain how multiplication leads to a measure of volume". These codes were important foundational ideas to understanding volume concepts as counting unit cubes to fill a given three-dimensional space and understanding partial array structure are the key to understanding volume concepts. When textbooks include items for LT codes, they are not aligned in terms of grade or age level. For examples, two LT codes from grades 3 and 4 (e. g. 302 and 402) were included in grade 5 textbook in all three series. Also, four LT codes are included in only one textbook series (e.g., 101, 203, 204, and 303) and only two LT codes are covered in all three textbook series (302 and 402). Again, the main questions are "Are those codes critical in developing volume concepts for students?" and "What will be the impact of having misaligned volume codes in lesson planning?"

Table 7. Frequency of volume learning trajectory codes in each textbook

Textbook	Learning Trajectory Code											
	K01	101	201	202	203	204	301	302	303	401	402	403
Go Math Grade 2	0	1	0	0	3	6	3	0	0	0	0	0
Go Math Grade 3	17	0	0	0	0	0	0	0	0	0	0	0
Go Math Grade 4	0	0	0	0	0	0	0	0	0	0	0	0
Go Math Grade 5	0	0	0	0	0	0	9	4	1	0	3	0
Total	17	1	0	0	3	6	12	4	1	0	3	0
My Math Grade 2	0	0	0	0	0	0	0	0	0	0	0	0
My Math Grade 3	0	0	0	0	0	0	0	0	0	0	0	0
My Math Grade 4	0	0	0	0	0	0	0	0	0	0	0	0
My Math Grade 5	0	0	0	0	0	0	0	4	0	0	15	0
Total	0	0	0	0	0	0	0	4	0	0	15	0
Envision Math Grade 2	0	0	0	0	0	0	0	0	0	0	0	0
Envision Math Grade 3	1	0	0	0	0	0	0	0	0	0	0	0
Envision Math Grade 4	0	0	0	0	0	0	0	0	0	0	0	0
Envision Math Grade 5	0	0	0	0	0	0	19	5	0	0	19	0
Total	1	0	0	0	0	0	19	5	0	0	19	0
Three Series Total	18	1	0	0	3	6	31	13	1	0	37	0

LT codes that were covered in only one textbook series were: "Filling a space with cubes", Cubes "fill a space, but does not use layers", "counting cubes", and "compute by rows and using multiplication to get the total number of cubes". The learning trajectory for volume indicates that students need understanding of the structure of layers of cubes in order for them to understand volume measurement (Sarama & Clements, 2009). In order to make this progress, students need to experience filling a three-dimensional space (LT code 201), understand cubes fill a space (but not using layers) (LT code 203), count cubes (unsystematically) (LT code 204) and compute the total by counting only rows and multiplying (LT code 303). Some of these LT codes for ages 8 and 9 (grade 3 and grade 4, respectively – codes 302, 303, and 402) were not introduced until the fifth grade textbook. SCLT for these ages focused heavily on learning how to structure three-dimensional arrays using rows, columns, and layers, a fundamental component of understanding the notion of volume (Sarama & Clements, 2009). Because of these results, students' experience with these textbooks was not similar to natural progression described in volume SCLT.

In summary, some early fundamental ideas of volume are entirely missing or only limited opportunities are provided. Since those early topics are challenging to elementary students, instructional and curricula attention is needed. As Confrey (Confrey, 2012) mentioned, it is not ideal to say that there is a complete isomorphic relationship between LT, the CCSSM, and textbooks, but our analysis shows that when students are using these textbooks, it will be likely that teachers might not be able to include SCLT's development steps, which can lead to challenges in developing a conceptual understanding of volume concepts.

3. DISCUSSION

In this study, we examined alignment between common core aligned textbooks and volume learning trajectory by Sarama and Clements. The results show that, there were significant gaps in the textbook development of volume concepts when compared to SCLT (sample LT used in CCSSM development). Foundational topics such as understanding of layer structures from SCLT were either skipped entirely, introduced later than recommended, or collapsed into one single grade. Since those topics in SCLTs are found to be challenging to some elementary students in later grades (Battista, 1999; Battista & Clements, 1996; Mullis et al., 2012), our results all indicate issues with the pacing of content in the textbooks. When volume topics are introduced in these textbooks series, students may not have opportunities to understand volume conceptually, or may lack necessary prior learning experiences; this can lead to issues with the concepts and difficulties later on. The most prominent gap in progression was found in the third, fourth, and fifth grade textbooks concerning volume, important stages included in the learning trajectory for ages 8 and 9 (grades 3 and grade 4, respectively) were not introduced until the fifth grade textbook. While some of these topics were introduced in the fifth grade textbook, they were treated sparingly and incompletely. For example, only two codes were covered in all three textbooks series. This indicated that although the textbooks attempted to include some prerequisite knowledge about the concept of volume, it may not be enough to fully develop the complex idea of three-dimensional array structures. Without several years to develop ideas of volume, students may not be able to acquire a conceptual understanding of structuring three-dimensional space, once more indicating possible future struggles. These results are significant because the results from other studies also indicate that not including these development steps in SCLT could be an issue with learning volume measurement (Battista, 1999, 2004; Battista & Clements, 1996; Vasilyeva et al., 2013). As we described earlier, TIMSS 2011 results show that many 8th graders still have challenges in answering volume tasks (Mullis et al., 2012) and not including fundamental steps in LT can be one possible reason for those challenges. To understand volume measurement tasks correctly, students need to be able to organize layer structure (Vasilyeva et al., 2013). Without items about counting cubes, teachers might not be able to assess students' level of understanding. For example, counting faces of cubes, double counting and partial layer counting won't be demonstrated with the items in three textbooks series until later grades. Not including these tasks and activities can lead to fourth and fifth grade students not being able to use or understand layer structure (Battista, 2004; Battista & Clements, 1996; Vasilyeva et al., 2013). Assessing students' thinking (partial understanding) is critical in lesson planning process (National Council of Teachers of Mathematics [NCTM], 2014); however, with misaligned items in three textbook series, students might have only limited opportunities to demonstrate partial understanding. SCLT for volume also indicates that students need a partial understanding of layers of cubes before they understand the total structure (e.g. LT codes 201, 203, 204 and 303) (Sarama & Clements, 2009) so if they are asked to count the total number of cubes in Figure 2 with limited experiences of counting a partial array of cubes, it is difficult for them to count them correctly. For example, a study shows when a student (fifth grader) was asked to count the total number of cubes in Figure 3, she provided the total of 105 (Battista, 2004). As we can see from the Figure 3, she counted some cubes more than once because this student counted faces of cubes instead of cubes. Both SCLT and other studies indicated that this is a common error for students who only have partial understanding of volume measurement (Battista, 1999, 2004; Sarama & Clements, 2009; Vasilyeva et al., 2013).

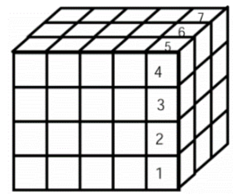


Figure 3. One student's attempt to count the number of cubes (Battista, 2004, p. 198).

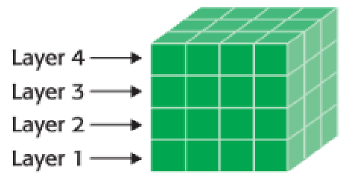
Figure 4 illustrates typical volume items from fifth grade textbooks. Although the items include the idea of counting cubes to figure out volume, students were expected to correctly count the total number of cubes with limited previous experiences of partial layer structure, critical to understanding volume (Battista, 2004; Sarama & Clements, 2009). Since these items were found in fifth grade textbooks, students were expected to do these when their prior experiences about counting cubes and volume were limited with three textbook series. If item like Figure 4 or items that show a partial layer structure (e. g. one or two layers of cubes) was included earlier (even if they are not able to count them correctly), students could have early experiences of counting cubes and seeing the partial structure of layers, which can help them seeing the global structure of three dimensional cubes. As a matter of fact, there were no volume items in the fourth grade textbooks even though students can have a partial understanding of volume, according to SCLT and other studies (Battista, 1999, 2004; Vasilyeva et al., 2013).

V. CONCLUSION

Overall, the three textbook series did not match the pacing of SCLTs for volume. With the issues that we found, when teachers prepare volume lessons with these textbook series, it is likely that their lessons do not reflect development steps of SCLT and it might more likely push students towards a procedural understanding, without addressing the conceptual groundwork. With all of our findings, we are also not claiming that textbooks are the single cause of students' difficulties in the future. It will be less problematic, however, if misaligned topics and activities were not fundamental concepts to understand volume. Since those topics and activities are fundamental ideas in volume, it will be important to fill the gaps between what textbooks offer and corresponding LTs when teachers and students are using these textbooks.

Try It

Use centimeter cubes to build the rectangular prism shown. Complete the table for each layer.



Layer	Length (cm)	Width (cm)	Height (cm)	Number of Cubes	Volume (cubic cm)
1					
2					
3					
4					

How many cubes were used to build the prism? ______________

What is the volume? ______________

Figure 4. One volume item from one textbook series (*MyMath*, 2014 f, p. 950).

Teachers will likely be called to fill in some of the "gaps" present in the progression of concepts. Confrey (Confrey, 2012) called this "bridging standards" to indicate that there are gaps between the CCSSM and LTs. We know that what is in textbooks might not be fully implemented by teachers because there are factors that influence such implementation (Son & Kim, 2016). Thus, it is important to make connections across concepts and grade levels more explicit for teachers to help them lead their classrooms based on what students have already learned, especially in districts where a series of textbooks is adopted over several years. For example, teachers need to know that even if textbooks do not cover, it is important to include activities that students are able to do in the early ages to address those early development steps. Informing teachers about the gaps can be done by professional development or designing appropriate content courses for teacher education programs. A previous study showed how LT can be used in professional development of teachers, and the results suggest that it can help them understand students' thinking of a concept and select appropriate activities and tasks (Wilson et al., 2015). With teachers' limited knowledge in measurement, it is critical to provide professional development opportunities for teachers (Hong & Runnalls, 2020, 2021; Runnalls

& Hong, 2019a, 2019b). Awareness of students' learning in terms of LT enables teachers to better recognize students' progress and eventually can help students progress toward high-level learning.

Finally, this study was limited to Sarama and Clements' (Sarama & Clements, 2009) LT and three textbook series in the U.S. Since our study was limited to three textbook series, we need to examine other common core aligned textbooks and textbooks from other countries to see learning opportunities in those textbook series. Although previous studies showed that students in other countries also have similar challenges and difficulties (Curry, Mitchelmore, & Outhred, 2006; Mullis et al., 2012), there is still a need to conduct more studies to investigate students' (both U.S. and other countries) natural progressions in learning volume to add, remove, or modify existing LT. Adding, removing, and modifying curriculum standards and textbooks should follow as these processes are recommended in the Curriculum Research Framework (CRF) (Clements, 2007). Then, textbooks can be also revised and teachers can be trained to understand LTs that more students experience when they learn volume.

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Appendix A

Established Codes for Volume

Code	Source	Grade	Description
K01	LT	K	Compares two containers using a third & transitive reasoning
101	LT	1	Filling a space with cubes
201	LT	2	Use simple units to fill containers
202	LT	2	Use fewer larger than smaller objects to fill a given container
203	LT	2	Cubes fill a space, but does not use layers
204	LT	2	Counts cubes
301	LT	3	Understand shapes as composites of individual shapes and layers (unit of... units)
302	LT	3	Counts by rows, then uses addition or skip counting
303	LT	3	Computes by rows, using multiplication
304	CCSSM	3	Measure/estimate liquid volume using standard units
305	CCSSM	3	Use basic operations to solve 1-step word problems with mass/volume; same unit
401	LT	4	Multiplicatively iterates squares in a row or column to determine area
402	LT	4	Can compute volume of prisms
403	LT	4	Can explain how multiplication leads to measure of volume
404	CCSSM	4	Use basic operations to solve word problems; multi-unit
405	CCSSM	4	Represent quantities using diagrams that feature a scale
501	CCSSM	5	Unit cube has one cubic unit of volume and is used to measure volume
502	CCSSM	5	Solid figure packed using *n*-unit cubes has volume *n*-cubic units
503	CCSSM	5	Measure volume by counting unit cubes
504	CCSSM	5	Relate volume to multiplication and addition
505	CCSSM	5	Solve real world and mathematical problems involving volume
506	CCSSM	5	Find volume of right rectangular prism by packing
507	CCSSM	5	Show that prism volume is same as that found by multiplying
508	CCSSM	5	Represent 3fold products as volumes (e.g. associative property)
509	CCSSM	5	Apply formulas to find volume in real world problems
510	CCSSM	5	Recognize volume as additive
511	CCSSM	5	Find volume of composite figures by adding, and apply to real world problems
900			N/A