

Modified Design Formula for Predicting the Ultimate Strength of High-tensile Steel Thin Plates

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Abstract : Methods for predicting the ultimate/buckling strength of ship structures have been extensively improved in terms of design formulas and analytical solutions. In recent years, the design strategy of ships and offshore structures has tended to emphasize lighter builds and improve operational safety. Therefore, the corresponding geometrical changes in design necessitate the use of high-tensile steel and thin plates. However, the existing design formulas were mainly developed for thick plates and mild steels. Therefore, the calculation methods require appropriate modification for new designs based on high-tensile steel and thin plates. In this study, a modified formula was developed to predict the ultimate strength of thin steel plates subjected to compressive and shear loads. Based on the numerical results, the effects of the yield stress, slenderness ratio, and loading condition on the buckling/ultimate strength of steel plates were examined, and a newly modified double-beta parameter formula was developed. The results were used to derive and modify existing closed-form expressions and empirical formulas to predict the ultimate strength of thin-walled steel structures.

Key Words : Design formulae, Thin walled plates, High-tensile steel, Slenderness ratio, Ultimate strength

1. Introduction

Steel plates and/or stiffened panels are extensively used in ship and offshore structures as strength members, such as decks or side-shell plating. Considerable research by academics and engineers over the past half-century has yielded an extensive body of literature on the buckling and ultimate strength of plates, which is too broad for a brief review. The details of the theory, mechanism, analysis solutions, design formulas, and applications are available in several textbooks (Paik and Thayamballi, 2003, 2007; Hughes and Paik, 2013) and reports (ISSC, 2012, 2015). Hence, methods for predicting the buckling/ultimate strength in terms of the design and analysis of ship and offshore structures have been well-developed. Several studies have been conducted to predict the ultimate/buckling strength of stiffened panels with various types of loading and damage using simplified (Caldwell, 1965; Smith, 1997; Paik et al., 2001; Chen, 2003; Fujibuko et al., 2005; Paik and Pedersen, 1996; Kim et al., 2009) and empirical formulation methods (Lin, 1985; Paik and Thayamballi, 1997; Khedmati et al., 2010; Zhang and Khan, 2009; Paik et al., 2013; Kim et al., 2017).

Current design strategies for ship and offshore structures tend to emphasize lighter and safer designs in response to global environmental and economic issues. Regarding these issues, the fundamental scantlings of structural members are not fully optimized in several structural design and analysis cases. Certain members are too strong, whereas others are either too weak or barely strong enough. It is clear that both weight minimization and safety maximization should be achieved simultaneously and that the strength requirements must be met. In fact, the benefits of full optimization technology for structural design have already been realized for naval ships, for which it is critical to design the structural weight in association with the functional requirements of armament (Kim and Paik, 2017).

The geometrical changes in designs have tended to necessitate the use of high-tensile steel and thin plates. The existing design and empirical formulas (Lin, 1985; Paik and Thayamballi, 1997; Zhang and Khan, 2009) for predicting the buckling/ultimate strength of stiffened panels were developed as a function of plate (b)/column (l) slenderness ratio considering mild steels ($\sigma_Y = 200 - 350$ MPa). These formulas are mainly focused on the range of thick plates and/or mild steels.

Recently, Kim et al. (2017) proposed and verified empirical formulas for predicting the ultimate strength of stiffened panels and

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Table 1. Buckling and ultimate strength of plate panels (IACS, 2006a; 2006b; 2015)

Terminology	IACS S11	ABS SafeHull	IACS CSR
Strength	$\sigma_{cr} = C \cdot \sigma_Y$	$\sigma_U = C \cdot \sigma_Y$	$\sigma_{cr} = C \cdot \sigma_Y$
C	$\begin{cases} \frac{3.6}{\beta^2} & \text{for } \beta \geq 2.68 \\ 1 - \frac{\beta^2}{14.4} & \text{for } \beta < 2.68 \end{cases}$	$\begin{cases} \frac{2.25}{\beta} - \frac{1.25}{\beta^2} & \text{for } \beta \geq 1.25 \\ 1 & \text{for } \beta < 1.25 \end{cases}$	$\begin{cases} \frac{2.14}{\beta} - \frac{0.89}{\beta^2} & \text{for } \beta \geq 1.58 \\ 1 & \text{for } \beta < 1.58 \end{cases}$

Notes: σ_U : ultimate Strength, σ_{cr} : critical (elastic-plastic) buckling stress), σ_Y : specified minimum yield stress of the material, $\beta = (\sigma_Y/E)^{0.5}$: plate slenderness ratio

conducted a statistical analysis of FEA results and existing formulas. The proposed formulas were successfully developed using a single material (313.6 MPa). Zhang (2016) presented a review and study of ultimate strength analysis methods for steel plates and stiffened panels under axial compression. Earlier, Zhang developed a formula that has been further validated and verified using a systematic non-linear FEA and a wide range of model test results. From the comprehensive validation, it is felt confident that the formula can be used to assess the ultimate strength of stiffened panels under axial compression, and it can quickly identify weak designs. However, the validation material ranged from 235 to 390 MPa according to the ship rule.

plates have generally been based on the Euler formula and the correction equation of Johnson–Ostenfeld formulae and Perry–Robertson formulas. In ship rules from different sources, the formulations of the equations in Fig. 1 and Table 1 may appear with slightly different constants depending on the assumed structural proportional limit value. For instance, σ_{cr} (in Fig. 1) assumes a structural proportional limit of 50% of the applicable yield value. The effect of thinner and higher tensile steel plates on the current design trend should be identified and explained.

Whereas design formulas are acceptable for thick plates, they tend to overestimate the strength of thin plates. In addition, they do not completely cover the effects of material yield owing to forces that exceed the elastic buckling strength and elastic–plastic buckling strength. Therefore, the development of accurate and integrated predictive formulas for steel plate strength to guarantee the total safety of all types of ships and offshore structures is an urgent task in the shipbuilding industry.

The design of offshore structures may provide useful examples for this study’s motivation. Living quarters in offshore structures mainly use thin walls (less than 10 mm thick). In practice, problems are commonly encountered in the welding fabrication of these structures, as shown in Fig. 2, which depicts distortions owing to welding and heating processes (pairing work owing to distorted plates). Therefore, numerous researchers and engineers have focused on improving the welding and/or manufacturing process to reduce distortion or improve process control. However, these structures are generally designed with respect to the buckling strengths calculated using conventional formulas (Table 1). The distortions owing to welding and fabrication are the governing factors, whereas the strength is not a critical issue. However, the strength assessment method also influenced the failure mode. The failure mode can be determined by a

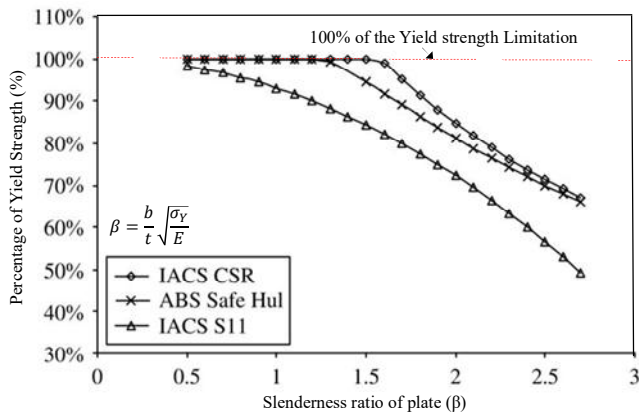


Fig. 1. Calculation methods in current industrial practice for the buckling strength of plate panels.

Existing analytical solutions to calculate the ultimate strength of steel plates can be categorized into two approaches: the membrane stress method and rigid plate theory method. These approaches have been successfully developed and used to predict the ultimate strength of steel plates. The existing design formulae for steel

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combination of the welding process and strength calculation method. The design formulas of the buckling/ultimate strength should therefore be applied to thin-walled structures to identify their failure modes clearly and accurately.

In this study, to clarify and examine the fundamental buckling and progressive collapse behavior of steel plates under axial compression and edge shear loads, a series of elastoplastic large-deflection analyses were performed, and the results were compared with the theoretical predictions of the Johnson–Ostenfeld plasticity correction including nonlinear effects. Based on the numerical results, the effects of the slenderness ratio (β), material yield strength (σ_y), and loading conditions on the buckling and ultimate strength behavior of steel plates were examined. Based on a series of nonlinear finite-element method (FEM) calculations for all edges of a simply supported plating, modified formulas were developed to predict the ultimate strength of steel plates considering the effects of the slenderness ratio and material yield strength. The aforementioned thinner structure is widely used in the living quarter and stem engine room and is also applied to the top-side wall of the offshore platform.

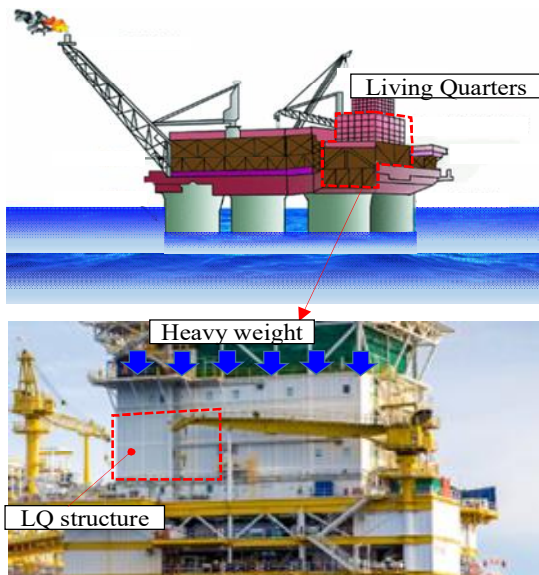


Fig. 2. Typical thin-walled panels in offshore living quarters.

Furthermore, to clarify and examine the fundamental buckling and behavior of thin plates subjected to compression and edge shear loads, a series of nonlinear analyses were performed using well-developed nonlinear FEM modeling and analysis techniques. This study focused on the newly modified double-beta parameter

formula, which has been adopted by the International Association of Classification Societies (IACS, 2006a, 2006b, 2015). The results will be applied to predict the ultimate strength of thin high-tensile steel plates for marine applications.

2. FE model and calculation method

The geometrical configuration of a steel-plated structure is determined primarily by the function of particular structures (Paik and Thayamballi, 2003). Fig. 3 demonstrates a schematic of a typical steel-plated structure. It has the same arrangement as a continuous stiffened plate used in a living quarter (LQ) structure. The “L” is the total length including three trans frames, and “B” is the length including seven deck beams. The length of the plate is “a” and the width is “b.” This study focused on the ultimate limit state (ULS) of stiffened plate structures comprising plate elements (or plating) surrounded by support members (longitudinal stiffeners and transverse frames). Such plate elements are widely found in the deck or bottom structures of ships and offshore structures. The loading condition was applied in terms of in-plane longitudinal/transverse compressive and shear loads for comparison with the design formulas.

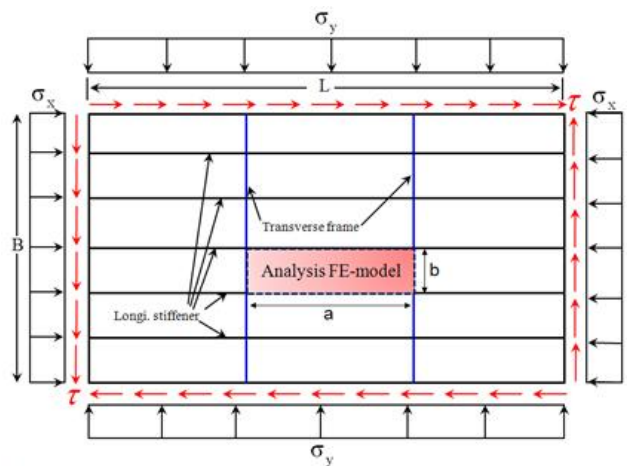


Fig. 3. Steel flat plate under biaxial load and in-plane edge shear load.

A total of 10 plate-shell elements were allocated for the plate part in the plate breadth direction, that is, between the longitudinal stiffeners. Approximately square plate elements (that is, with the unity of the ratio of element length to element breadth) were allocated along the plate length direction. For the

plate FEM model, all edges were simply supported with zero rotational restraints or were clamped with infinite rotational restraints that would keep them straight while allowing in-plane movements. Both the loading and boundary conditions were established based on prior research on FEM modeling of stiffened plate panels and unstiffened plates (Paik et al., 2008; Fujikubo and Yao, 2005).

The material and geometric nonlinear analyses were performed using a commercial FEM code, ANSYS (Ansys, 2016), a general-purpose FEM package. The arc-length method involves tracing a complex path in the load-displacement response in buckling/post-buckling regimes. This code has been well developed for FEM analysis in numerous studies and has been validated for the calculation of the nonlinear behavior of plate structures.

2.1 Initial imperfections

In advanced ship structural designs, the load-carrying capacity of the plating should be calculated by considering the initial imperfections induced by welding and cutting as the parameters of influence. Throughout this analysis, the initial buckling mode was obtained using eigenvalue analysis in the FEM considering the initial shape imperfections. Previous research results (Paik and Thayamballi, 2003; Fujikubo et al., 2005; Paik and Pedersen, 1996; Ueda and Yao, 1985; Cho et al., 2011) suggest that the following maximum values of representative initial deflections for plating in merchant vessel structures can be used to approximate W_{opl} (plate initial deflection) of the t (thickness) in Equations (1a) and (1b).

$$w_0/w_{opl} = \sum_{i=1}^M B_{0i} \sin \frac{i\pi x}{a} \sin \frac{\pi y}{b} \tag{1a}$$

$$w_{opl}/t = \begin{cases} 0.025\beta^2 & \text{for slight} \\ 0.1\beta^2 & \text{for average} \\ 0.3\beta^2 & \text{for severe} \end{cases} \tag{1b}$$

where, slenderness ratio $\beta = \frac{b}{t} \sqrt{\frac{\sigma_Y}{E}}$

For example, a slenderness ratio of 2.9 results in a plate thickness of 12 mm, and it is calculated that the initial deflection of 2.5 mm in slight, 10 mm in average, and 30 mm in severe conditions is influenced by welding.

2.2 Material properties

The steel plates analyzed using the one-bay model had the material properties listed in Table 2. The selected material was considered applicable in shipbuilding industries, and its yield strength ranged from 235 to 800 MPa. The analytical model incorporated an idealized elastic-perfectly plastic stress-strain curve, and the strain hardening rate was set as zero. Isotropic hardening was assumed by applying von Mises yield conditions.

Table 2. Material properties for steel plate

b (mm)	840
a (mm)	3,020
Material	High-tensile steel
Yield strength (MPa)	235 - 800
Young's modulus (MPa)	206000
Poisson ratio	0.3

2.3 Parametric Study

A series of FEM analyses were performed by varying the slenderness ratio of the steel plates from 0.94 to 8.72. These slenderness ratios were obtained by varying the thickness of the plate between 6 mm and 30 mm while maintaining the breadth and length of the plate at 840 mm and 3,020 mm, respectively. The yield stress was considered to vary within 235–800 MPa. The parameters used in the series of analyses are listed in Table 3.

Table 3. Analysis scenario of steel plate

Loading condition	Thickness (mm)
Longitudinal compression	6, 8, 10, 12, 14, 16, 18, 20, 25, 30
Transverse compression	
In-plane edge shear	

Most buckling calculations of steel-plated structures have used the theoretical formulation known as the “Euler equation,” which is shown below in Equations (2a) and (2b). In general, the initial buckling calculated using the Euler equation (σ_E) is higher than the yield stress; therefore, the buckling stress must be modified with a plasticity correction. Here, the plasticity correction can be calculated using the Johnson–Ostenfeld method. In cases where the calculated Euler buckling stress is greater than half of the

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yield stress (σ_Y), the critical buckling stress is given by $\sigma_{J-O} = [1 - (\sigma_Y/4\sigma_E)]\sigma_Y$, assuming that the proportional limit is $0.5\sigma_Y$.

$$\sigma_E = k \frac{\pi^2 E}{12(1-\nu^2)} \left(\frac{t}{b}\right)^2 \quad (2a)$$

$$\sigma_{cr} = \begin{cases} \sigma_E & \text{for } \sigma_E \leq 0.5\sigma_Y \\ \sigma_Y [1 - \sigma_Y / (4\sigma_E)] & \text{for } \sigma_E > 0.5\sigma_Y \end{cases} \quad (2b)$$

As mentioned earlier, most design formulas are based on this equation. Therefore, it is instructive to compare its results with those of the FEM modeling.

3. FEA Results

Fig. 4 shows the relationship between the ultimate strength and plate thickness with varying yield stress values under longitudinal compression (Fig. 4(a)), transverse compression (Fig. 4(b)), and shear (Fig. 4(c)). The full values are listed in Table 4. The ultimate strength at the thinner plate (5–15 mm) varied considerably according to the changes in the yield stress.

Generally, the design formula for thick plates agrees well with the FEM results, whereas the strength of thinner plates is overestimated. A considerable deviation was also observed for high-tensile steel. The difference between the ultimate strength and buckling strength of the thinner plate varied considerably as the yield stress increased. An increase in the yield stress in a thin plate can easily occur via elastic buckling when subjected to in-plane compression and edge shear load, and the theoretical formula does not reflect the resulting change in the yield stress. Therefore, the current design formulas are not suitable for the design of thin and high-tensile plates.

The results implied that the current design formulas should be modified to describe thin plates and high-tensile steel. Design formulas that tend to underestimate strength result in a conservative design, which is beneficial for safety. However, the current results show the opposite trend, posing a risk of critical safety problems in the design of structures using thin and high-tensile steel.

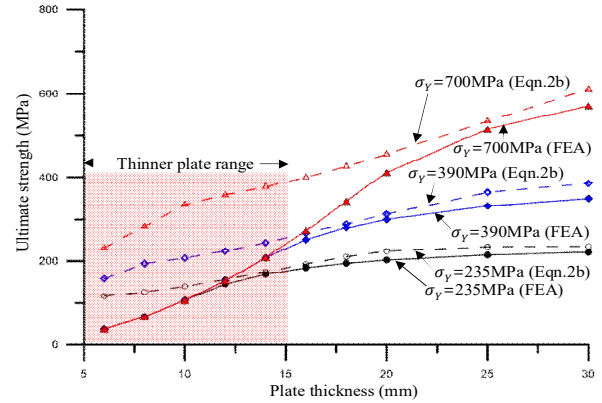


Fig. 4(a). Comparison of buckling formula and FEM results under longitudinal compression.

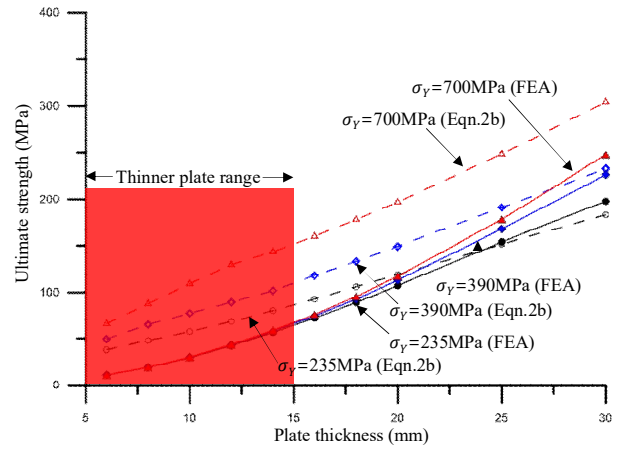


Fig. 4(b). Comparison of buckling formula and FEM results under transverse compression.

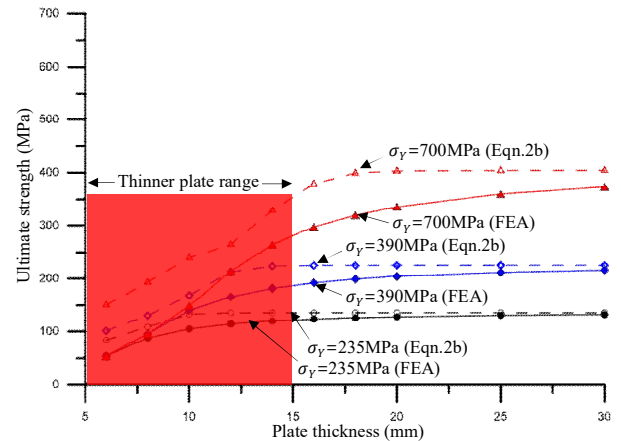


Fig. 4(c). Comparison of buckling formula and FEM results under in-plane edge shear.

Table 4(a). Ultimate strength values under longitudinal compression

t	$\sigma_Y=235$ MPa		$\sigma_Y=390$ MPa		$\sigma_Y=700$ MPa	
	Eq. 2b	FEA	Eq. 2b	FEA	Eq. 2b	FEA
6	38.34	116.25	38.34	158.20	38.34	230.65
8	68.15	126.05	68.14	194.47	68.14	283.02
10	106.4	139.67	106.4	206.92	106.49	336.18
12	144.8	156.55	153.21	223.34	153.21	358.74
14	168.7	174.70	207.57	243.37	208.44	377.25
16	184.2	193.15	250.25	266.13	272.11	399.86
18	194.8	209.99	279.51	289.53	344.17	426.06
20	202.4	222.79	300.45	313.37	411.50	455.30
25	214.1	233.54	332.57	364.40	514.99	533.89
30	220.4	234.82	350.02	385.37	571.20	611.55

Table 4(b). Ultimate strength values under transverse compression

t	$\sigma_Y=235$ MPa		$\sigma_Y=390$ MPa		$\sigma_Y=700$ MPa	
	Eq. 2b	FEA	Eq. 2b	FEA	Eq. 2b	FEA
6	10.91	37.98	10.96	49.35	10.99	65.94
8	19.22	47.71	19.37	65.06	19.48	87.96
10	29.69	57.39	30.05	77.50	30.31	109.26
12	42.14	68.47	42.88	89.41	43.43	129.56
14	56.37	80.39	57.74	101.94	58.76	143.59
16	72.14	92.87	74.47	117.72	76.22	159.84
18	89.17	106.01	92.90	133.18	95.70	177.84
20	107.15	118.80	112.84	149.16	117.09	197.01
25	153.86	151.52	167.72	190.96	178.09	248.55
30	197.80	183.10	226.47	232.84	247.93	304.05

Table 4(c). Ultimate strength values under pure shear

t	$\sigma_Y=235$ MPa		$\sigma_Y=390$ MPa		$\sigma_Y=700$ MPa	
	Eq. 2b	FEA	Eq. 2b	FEA	Eq. 2b	FEA
6	53.61	83.18	53.61	101.28	53.61	150.57
8	87.39	108.67	95.30	130.41	95.30	193.05
10	104.77	133.00	140.05	168.83	148.91	239.48
12	114.21	135.37	166.06	212.43	213.73	264.45
14	119.91	135.51	181.74	223.52	264.24	328.63
16	123.60	135.56	191.91	224.74	297.03	379.56
18	126.13	135.60	198.89	224.89	319.51	399.13
20	127.95	135.62	203.88	224.97	335.59	402.98
25	130.73	135.65	211.54	225.06	360.27	403.71
30	132.24	135.66	215.70	225.11	373.67	403.89

4. Development of modified formulation

In this study, design formulas were proposed to predict the ultimate strength of thin and thick plate panels under various loading conditions. The formulas were derived from a numerical database of 180 cases. The scope of development was used in shipbuilding and offshore structures, and additional considerations could be used in the near future. As an example, there is a representative case of a model with a yield stress of 700 MPa or more. Among the equations used to predict the ultimate strength, the range is the widest. As discussed in the previous FEM results, the ultimate strength of steel plates subjected to compressive loads can be empirically derived with fitting curves, similar to the derivation of the data in Table 1.

We developed a modified form of the double-beta formula (Smith, 1977; Ueda and Yao, 1985) using a set of new coefficients (Equation (3)):

$$\frac{\sigma_U}{\sigma_Y} = \frac{C_1}{\beta^2} + \frac{C_2}{\beta} + C_3 \quad (3)$$

The coefficients C_1 and C_2 represent the functions of the double slenderness ratio, and the correction factor (C_3) represents the effect of changes in the thickness of the plate.

In this study, new design formulas, such as axial compressive load, transverse compressive load, and shear load, are expressed in the form of Eq. (3) as follows:

For longitudinal compression load:

in Case $\beta < 2.8$

$$\begin{aligned} C_1 &= 3.888 \times 10^{-4}(\sigma_Y)^2 - 2.800 \times 10^{-1}(\sigma_Y) + 46.31 \\ C_2 &= -2.150 \times 10^{-4}(\sigma_Y)^2 + 1.553 \times 10^{-1}(\sigma_Y) - 25.04 \\ C_3 &= 2.967 \times 10^{-5}(\sigma_Y)^2 - 2.143 \times 10^{-2}(\sigma_Y) + 3.893 \end{aligned} \quad (4a)$$

$\sigma_Y < 390\text{MPa}$

$$\begin{aligned} C_1 &= -5.800 \times 10^{-5}(\sigma_Y)^2 + 6.042 \times 10^{-2}(\sigma_Y) - 18.82 \\ C_2 &= 2.714 \times 10^{-5}(\sigma_Y)^2 - 2.807 \times 10^{-2}(\sigma_Y) + 9.807 \\ C_3 &= -2.943 \times 10^{-6}(\sigma_Y)^2 + 3.014 \times 10^{-3}(\sigma_Y) - 0.699 \end{aligned} \quad (4b)$$

$390\text{MPa} < \sigma_Y \leq 600\text{MPa}$

$$\begin{aligned} C_1 &= 9.005 \times 10^{-5}(\sigma_Y)^2 - 1.275 \times 10^{-1}(\sigma_Y) + 40.66 \\ C_2 &= -3.745 \times 10^{-5}(\sigma_Y)^2 + 5.289 \times 10^{-2}(\sigma_Y) - 15.55 \\ C_3 &= 3.450 \times 10^{-6}(\sigma_Y)^2 - 4.875 \times 10^{-3}(\sigma_Y) + 1.736 \end{aligned} \quad (4c)$$

$\sigma_Y > 600\text{MPa}$

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in Case $\beta \geq 2.8$

$$\begin{aligned} C_1 &= 6.126 \times 10^{-7}(\sigma_Y)^2 + 2.287 \times 10^{-3}(\sigma_Y) - 1.769 \\ C_2 &= 2.979 \times 10^{-9}(\sigma_Y)^2 - 2.794 \times 10^{-3}(\sigma_Y) + 2.898 \\ C_3 &= -1.433 \times 10^{-7}(\sigma_Y)^2 + 7.773 \times 10^{-4}(\sigma_Y) - 0.228 \end{aligned} \quad (4d)$$

For transverse compression load:

$$\begin{aligned} C_1 &= -1.338 \times 10^{-6}(\sigma_Y)^2 + 1.287 \times 10^{-3}(\sigma_Y) - 0.098 \\ C_2 &= 1.081 \times 10^{-6}(\sigma_Y)^2 - 1.039 \times 10^{-3}(\sigma_Y) + 0.759 \\ C_3 &= -1.837 \times 10^{-7}(\sigma_Y)^2 + 1.740 \times 10^{-4}(\sigma_Y) - 0.0011 \end{aligned} \quad (5)$$

For shear load:

in Case $\beta < 2.8$

$$\begin{aligned} C_1 &= -6.583 \times 10^{-4}(\sigma_Y)^2 - 4.205 \times 10^{-1}(\sigma_Y) - 60.56 \\ C_2 &= 2.838 \times 10^{-4}(\sigma_Y)^2 - 1.787 \times 10^{-1}(\sigma_Y) + 27.90 \\ C_3 &= -2.734 \times 10^{-5}(\sigma_Y)^2 + 1.667 \times 10^{-2}(\sigma_Y) - 22.10 \end{aligned} \quad (6a)$$

$\sigma_Y < 390\text{MPa}$

$$\begin{aligned} C_1 &= 3.563 \times 10^{-4}(\sigma_Y)^2 - 3.735 \times 10^{-1}(\sigma_Y) + 95.62 \\ C_2 &= -1.664 \times 10^{-4}(\sigma_Y)^2 + 1.734 \times 10^{-1}(\sigma_Y) - 41.32 \\ C_3 &= 1.666 \times 10^{-5}(\sigma_Y)^2 - 1.715 \times 10^{-2}(\sigma_Y) + 43.290 \end{aligned} \quad (6b)$$

$390\text{MPa} < \sigma_Y \leq 600\text{MPa}$

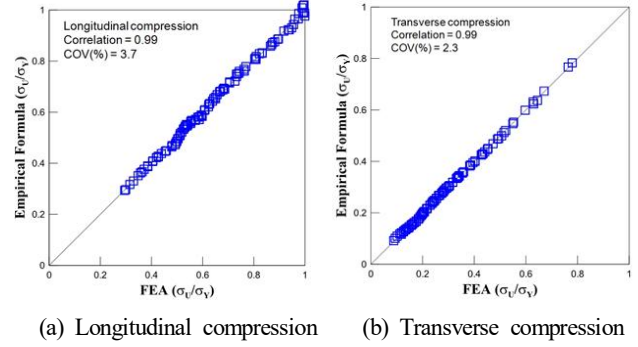
$$\begin{aligned} C_1 &= -1.317 \times 10^{-4}(\sigma_Y)^2 + 1.888 \times 10^{-1}(\sigma_Y) - 66.63 \\ C_2 &= 5.555 \times 10^{-5}(\sigma_Y)^2 - 7.955 \times 10^{-2}(\sigma_Y) + 30.78 \\ C_3 &= -5.300 \times 10^{-6}(\sigma_Y)^2 + 7.570 \times 10^{-3}(\sigma_Y) - 2.628 \end{aligned} \quad (6c)$$

$\sigma_Y > 600\text{MPa}$

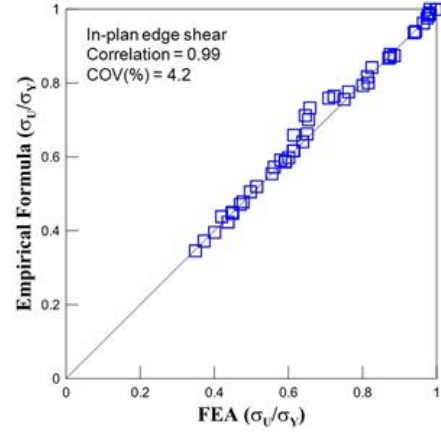
in Case $\beta \geq 2.8$

$$\frac{\sigma_U}{\sigma_Y} = 1 \quad (6d)$$

The accuracy of the proposed formula for each value of the slenderness ratio was assessed by comparing the calculated results with the corresponding FEM results, as shown in Fig. 5. As indicated, the correlation ratio and the percentage of covariance of the error between the empirical formula and FEM were 0.99 and 3.7%, respectively, for longitudinal compression, 0.99% and 2.3%, respectively, for transverse compression, and 0.99% and 4.2%, respectively, for shear load. Thus, the proposed formula provides a reasonable estimation of the ultimate strength of steel plates in comparison with FEM.



(a) Longitudinal compression (b) Transverse compression



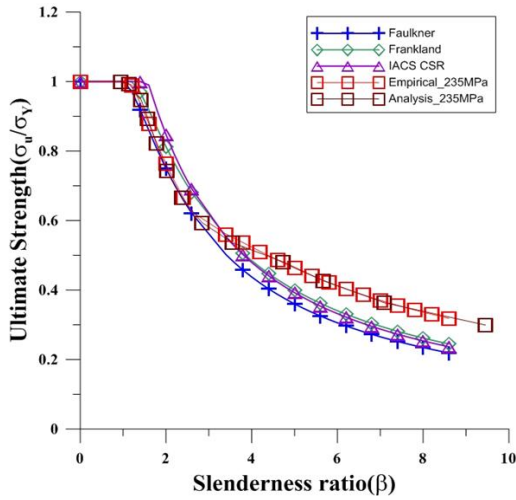
(c) In-plane edge shear load

Fig. 5. Correlation of the empirical formula with ultimate strength of steel plates obtained using FEM.

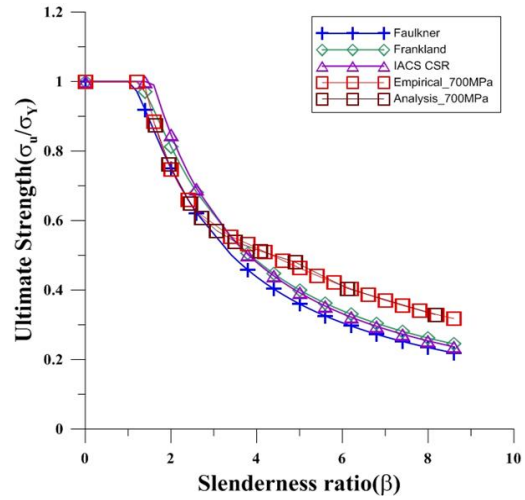
As depicted in Fig. 6, the new empirical formula was also compared with the current design formulas (see Table 1) for steel under longitudinal compressive loads. Fig. 6 depicts the relationships between the dimensionless ultimate strength and the slenderness ratio for varying yield stress values, as predicted by different methods.

The empirical formulas of Frankland (Frankland, 1940) and Faulkner (Faulkner, 1975) and the IACS criterion (IACS, 2006a, 2006b, 2015) provide overly conservative estimates of the ultimate strength of thinner plates.

Therefore, the current design formulas cannot be applied to steel plates below a certain thickness (those with a slenderness ratio greater than 4.0). In comparison, the results from the numerical simulation were slightly less conservative. The proposed modified design formulas agreed well with the FEM results over the full range of possible slenderness ratios for various high-tensile steels in ships and offshore structures.

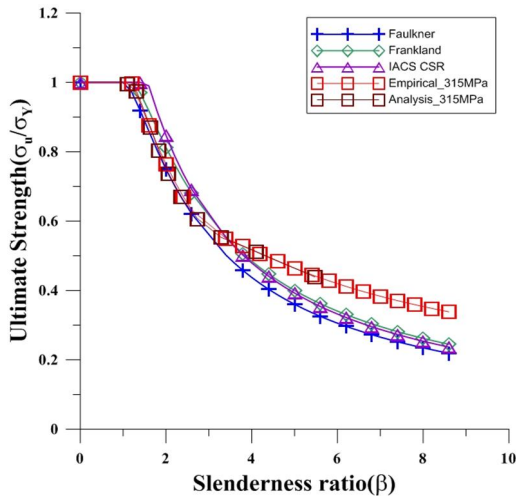


(a) Yield stress = 235 MPa

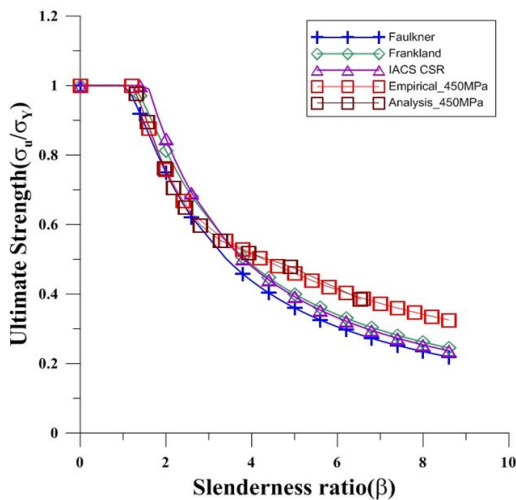


(b) Yield stress = 700 MPa

Fig. 6. Comparison of empirical ultimate strength formulas and present results under longitudinal compression.



(b) Yield stress = 315 MPa



(c) Yield stress = 450 MPa

5. Conclusion and remarks

The objective of this study was to clarify and examine the fundamental ultimate buckling/elastoplastic collapse behavior and ultimate strength of steel plates under a variety of loading conditions (longitudinal and transverse compressions and in-plane edge shear load). Based on the calculated results, the effects of the slenderness ratio, yield stress, and loading condition on the buckling and ultimate strength were examined. A simple formulation was then developed as an efficient method, particularly to predict the ultimate strength of thinner plates. The following conclusions were drawn:

- The theoretical formula indicated large discrepancies between the ultimate strength and buckling strength of steel plates under in-plane compression and edge shear load compression because it did not consider changes in the yield stress.
- For a thin plate of a high-strength material, elastic buckling could occur much earlier than for low-strength materials, and the theoretical formula thus provided a highly conservative prediction for the ultimate strength of such a plate. A more sophisticated formulation was required to provide reasonable predictions of the ultimate strength in these cases.
- The ultimate strength of a steel plate significantly reduced when the slenderness ratio exceeded 4.0. In these cases, a large discrepancy existed between the results of the FEM calculations and other methods.

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- For a steel plate under transverse compression, the buckling and ultimate strengths were underestimated as the thickness decreased, mainly because the wrong collapse pattern was assumed.
- A modified design formula developed for a steel plate can provide a reasonable estimate of the ultimate strength of the plate under a variety of loading conditions (longitudinal/transverse compressive load and in-plane edge shear load).
- The ultimate strengths predicted by the modified empirical design formula showed a good correlation with the FEM results within 4.2%.
- This will provide useful information for the current developed and existing empirical formulas for predicting the ultimate strength of stiffened plate panels.

In future studies, it will be necessary to develop an empirical formula for the buckling and ultimate strength considering the influence of residual stress from welding. The aforementioned stress is a particularly important factor in thinner plates, and experimental verification is also needed to clarify the buckling behavior. The results of this study can be used as a basis for rapidly predict the buckling and ultimate strengths during the initial design stage.

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