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Optimal Pricing and Ordering Policies with Price Dependent Demand Linearly under Order-Size-Dependent Delay in Payments

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Abstract

In Korea, some pharmaceutical companies and agricultural machine manufacturers associate the length of the credit period with the retailer's order size. This kind of commercial practice is based on the principle of economy of scale from the supplier's point of view and tends to make retailer's order size large enough to qualify a certain credit period break. Also, the credit period allowed by the supplier makes it possible to reduce the retail price expecting that the retailer can earn more profits by the stimulating the customer's demand. Since the retailer's order size is affected by the end customer's demand, it is reasonable to determine the retail price and the order size simultaneously. In this regard, this paper analyzes the retailer's problem who has to decide his sales price and order quantity from a supplier who offers different credit periods depending on his order size. And we show that the retailer's order size large enough to qualify a certain credit period break. Also, it is assumed that the end customer's demand rate is represented by a linear decreasing function of the retail price.

Keywords: *Credit period, Order-size-dependent delay, Price, Lot-size, Linear demand function*

1. INTRODUCTION

The basic EOQ model is based on the implicit assumption that the retailer must pay for the products at the same time he receives them. However, a common practice in industry is to provide a specific delay period for the payments after the items are delivered. From this point of view, many research papers have been published which deal with the EOQ problem under a fixed credit period. Goyal [1], Chung [2] and Teng et al. [3] studied the effect of trade credit on the inventory policy. Recently, Mahata and Goswami [4] also examined the economic ordering policy of deteriorating items under trade credit. The common assumption of the previous studies is that the customer's demand is a known constant and, therefore, they disregarded the effects of credit transaction on the customer's demand. As implicitly stated by Mehta [5], an important reason why the supplier offer trade credit to the retailers is to increase the demand for the product he produces. Also, Fewings [6] stated that the advantage of credit transaction from the supplier's point of view is substantial in terms of influence on the retailer's purchasing and selling decisions. The supplier usually expects that the capital losses incurred during the credit period can be compensated by increasing the sales volume. The positive effects of trade credit on the customer's demand can be integrated into the inventory model through the consideration of retailing situations where the customer's demand is a function of the retail price. The availability of the credit period by the supplier makes it possible to reduce the retail price from a wider range of option. Since the retailer's

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order size is affected by the customer's demand, the problems of determining the retail price and the order size are interdependent and must be solved simultaneously. According to the above observations, several research papers dealt with the joint price and order size determination problem under trade credit assuming that the customer's demand is a function of selling price. Chang et al. [7], Dye and Ouyang [8], and Teng et al. [9] analyzed the problem under Day-terms supplier credit when the customer's demand is a constant price elasticity function of retail price. Also, Avinadav et al. [10] and Shi et al. [11] examined the joint price and lot size determination problem without trade credit assuming that the customer's demand is a linear decreasing function of the selling price. Recently, Shinn [12] evaluated the pricing and lot sizing policy under day terms supplier credit when the customer's demand rate is represented by a linear decreasing function of retailing price. The common assumption of the above research works is the availability of a certain fixed length of credit period offered by the supplier. However, in Korea, some pharmaceutical companies and agricultural machines manufacturers associate the length of the credit period with the retailer's total amount of purchase, i.e., they offer a longer credit period for a large amount of purchase. This kind of commercial practice is based on the principle of economy of scale from the supplier's point of view and tends to make retailer's order size large enough to qualify a certain credit period break. In this regard, Ouyang et al. [13] introduce the joint pricing and ordering problem with order-size dependent trade credit. In this regard, we extend the model presented by Shinn [12] assuming that the length of credit period is a function of the amount purchased by the retailer.

2. MODEL FORMULATION

The mathematical model of the joint price and order size determination problem is developed with the following assumptions and notations.

1) Inventory replenishments are instantaneous.
2) No shortages are allowed.
3) The demand is a linear function of retail price.
4) The supplier permits a delay in payments for the items supplied where the length of delay is a function of the total amount of purchase for retailer.
5) The sales revenue during the credit period is deposited in an interest with rate I. At the end of the credit period, the product price is paid and the retailer starts paying the capital opportunity cost of the products in stock with rate $R (R \geq I)$.

C : unit purchase cost.
S : order cost.
H : inventory carrying cost without the capital opportunity cost.
R : capital opportunity cost.
I : earned interest rate.
D : annual demand, as a function of retail price (P), $D = a - bP$, a and b are positive constants.
P : unit retail price, $P < a/b$.
Q : order size.
T : replenishment cycle time.
tc_j : credit period for the amount purchased TDC, $v_{j-1} \leq TDC < v_j$, where $tc_{j-1} < tc_j$, $j = 1,2,\cdots,m$ and $v_0 < v_1 < \cdots < v_m$, $v_0 = 0$, $v_m = \infty$.

The retailer's objective is to maximize the annual net profit $\Pi(P,T)$ from the products sales. His annual net profit consists of the following elements.

1) Annual sales revenue = DP.
2) Annual purchasing cost = DC.
3) Annual ordering cost = S/T.

4) Annual inventory carrying cost = $(TDH)/2$.

5) Annual capital opportunity cost for $v_{j-1} \le TDC < v_j$

(i) Case 1($tc_j \le T$): As products are sold, the sales revenue is used to earn interest with rate I during the credit period tc_j. And the average number of stocks earning interest during time $(0, tc_j)$ is $(Dtc_j)/2$ and the interest earned per order becomes $CItc_j(Dtc_j)/2$. When the credit is settled, the products still in stock have to be financed with rate R. Since the average number of stocks during time (tc_j, T) becomes $D(T - tc_j)/2$, the interest payable per order can be expressed as $CR(T - tc_j)D(T - tc_j)/2$. Then,

$$\text{the annual capital opportunity cost} = \frac{\frac{D}{2}(T-tc_j)(T-tc_j)CR - \left(\frac{Dtc_j}{2}\right)tc_j CI}{T} = \frac{DC(R-I)tc_j^{\,2}}{2T} + \frac{DTRC}{2} - DCRtc_j .$$

(ii) Case 2($tc_j > T$): For $tc_j > T$, the whole sales revenue is used to earn interest with rate I during the credit period tc_j. The average number of stock earning interest during time $(0, T)$ and (T, tc_j) become $DT/2$ and DT, respectively. Then,

$$\text{the annual capital opportunity cost} = -\frac{\frac{DT}{2}TCI + DT(tc_j - T)CI}{T} = \frac{DTIC}{2} - DCItc_j .$$

Then, the annual net profit $\Pi(P,T)$ can be expressed as

$\Pi(P,T)$ = Annual Sales revenue – Annual Purchasing cost – Annual Ordering cost – Annual Inventory carrying cost – Annual Capital opportunity cost.

Depending on the relative size of tc_j to T, $\Pi(P,T)$ has following two different expressions.

Case 1($tc_j \le T$)

$$\Pi_{1,j}(P,T) = DP - DC - \frac{S}{T} - \frac{TDH}{2} - \left(\frac{C(R-I)Dtc_j^{\,2}}{2T} + \frac{CRDT}{2} - CRDtc_j\right), \quad TDC \in [v_{j-1}, v_j),\ j = 1,2,\cdots,m \tag{1}$$

Case 2($tc_j > T$)

$$\Pi_{2,j}(P,T) = DP - DC - \frac{S}{T} - \frac{TDH}{2} - \left(\frac{CIDT}{2} - CIDtc_j\right), \quad TDC \in [v_{j-1}, v_j),\ j = 1,2,\cdots,m \tag{2}$$

3. DETERMINATION OF OPTIMAL POLICY

To determine the optimal retail price and order size which maximize $\Pi(P,T)$, first let's analyze the characteristics of $\Pi(P,T)$ for a fixed P. For a fixed P with P^0, $\Pi(P^0,T)$ is a concave function of T for every i and j, and there exists a unique value $T_{i,j}$, which maximizes $\Pi_{i,j}(P^0,T)$, $i = 1,2, j = 1,2,\cdots,m$ as follows;

$$T_{1,j} = \sqrt{\frac{2S + C(R-I)Dtc_j^2}{DH_1}} \quad \text{where } D = a - bP^0 \text{ and } H_1 = H + CR \tag{3}$$

$$T_{2,j} = \sqrt{\frac{2S}{DH_2}} \quad \text{where } D = a - bP^0 \text{ and } H_2 = H + CI. \tag{4}$$

$T_{i,j}$ and $\Pi_{i,j}(P^0,T)$ can be shown to have the following four properties(proofs omitted).

Property 1. $T_{1,j} < T_{1,j+1}$ holds for j = 1, 2, ..., m-1.

Property 2. $T_{2,j} = T_{2,j+1}$ holds for j = 1, 2, ..., m-1.

Property 3. For any T, $\Pi_{i,j}(P^0,T) < \Pi_{i,j+1}(P^0,T)$, i = 1, 2 and j = 1, 2, ..., m-1.

Property 4. For any j, if $T_{1,j} \geq tc_j$, then$T_{2,j} \geq tc_j$, which implies that $\Pi_{2,j}(P^0,T)$ is increasing in T for $T < tc_j$. Also, if $T_{2,j} < tc_j$, then $T_{1,j} < tc_j$, which implies that $\Pi_{1,j}(P^0,T)$ is decreasing in T for $T \geq tc_j$.

Properties 1 and 2 indicate that the value of $T_{1,j}$ is strictly increasing as j increases and the value of $T_{2,j}$ is identical for every j. Also, Property 3 implies that both $\Pi_{1,j}(P^0,T)$ and $\Pi_{2,j}(P^0,T)$ are strictly increasing for any fixed value of T as j increases. From the above properties, we can make the following observations about the characteristics of the annual net profit function for T, $T \in I_j = \left\{ T \mid v_{j-1}/DC \leq T < v_j/DC \right\}$, $j = 1, 2, \cdots, m$. These observations simplifies our search process such that only a finite number of candidate values of T needs to be considered to find an optimal value T^*. Let k be the smallest index such that $T_{2,j} < tc_j$.

Observation 1. For $T \in I_j$, $j \geq k$, we can consider the following three cases for $T_{2,j}$; $T_{2,j} < \frac{v_{j-1}}{DC}$, $\frac{v_{j-1}}{DC} \leq T_{2,j} < \frac{v_j}{DC}$ and $\frac{v_j}{DC} \leq T_{2,j}$.

(i) If $T_{2,j} < \frac{v_{j-1}}{DC}$, then $T = \frac{v_{j-1}}{DC}$ provides the maximum annual net profit where $T \in I_j$.

(ii) If $\frac{v_{j-1}}{DC} \leq T_{2,j} < \frac{v_j}{DC}$, then $T = T_{2,j}$ provides the maximum annual net profit where $T \in I_j$.

(iii) If $\frac{v_j}{DC} \leq T_{2,j}$, then we do not need to consider T for $T \in I_j$ to find T^*.

Observation 2. For $T \in I_j$, $j < k$, we can consider the following four cases for $T_{1,j}$;$T_{1,j} < \frac{v_{j-1}}{DC}$, $\frac{v_{j-1}}{DC} \leq T_{1,} < \frac{v_j}{DC}$ and $\frac{v_j}{DC} \leq tc_j < T_{1,j}$.

(i) If $T_{1,j} < \frac{v_{j-1}}{DC}$, then $T = \frac{v_{j-1}}{DC}$ provides the maximum annual net profit where $T \in I_j$.

(ii) If $\frac{v_{j-1}}{DC} \leq T_{1,j} < \frac{v_j}{DC}$, then $T = T_{1,j}$ provides the maximum annual net profit where $T \in I_j$.

(iii) If $tc_j < \frac{v_j}{DC} < T_{1,j}$, then $T = v_j^-/DC$,where $v_j^- = v_j - \varepsilon$ and ε is a very small positive number, provides the maximum annual net profit where $T \in I_j$.

(iv) If $\frac{v_j}{DC} \leq tc_j < T_{1,j}$, then we do not need to consider T where $T \in I_j$ to find T^*.

Observation 3. (Search Stopping Rule)

(i) If $T = T_{1,j}$ yields the maximum annual net profit for $T \in I_j$, then $T^* \geq T_{1,j}$.

(ii) If $T = \frac{v_j^-}{DC}$ yields the maximum annual net profit for $T \in I_j$, then $T^* \geq \frac{v_j^-}{DC}$.

From the above observations, for $P = P^0$ fixed, only the elements in the set $\Omega = \{T_{i,j}(P^0)$, $\frac{v_{j-1}}{DC}$, $\frac{v_j^-}{DC}$ for $i = 1,2$ and $j = 1,2,\cdots,m\}$ become candidates for an optimal replenishment cycle time $T^*(P^0)$ where $T_{i,j}(P^0)$ is obtained by substituting P with P^0 in equations (3) and (4) . Noting that some elements of Ω can be excluded from consideration in search of $T^*(P)$, we formulate the following conditions for $T_{i,j}(P)$, v_{j-1}/DC and v_j^-/DC must satisfy to become a candidate of $T^*(P)$.

(Cond. 1): The conditions of $T_{i,j}(P)$ to be a candidate for $T^*(P)$.

$$T_{1,j}(P) \geq tc_j \text{ and } \frac{v_{j-1}}{DC} \leq T_{1,j}(P) < \frac{v_j}{DC} \quad \text{for Case 1} \tag{5}$$

$$T_{2,j}(P) < tc_j \text{ and } \frac{v_{j-1}}{DC}, \leq T_{2,j}(P) < \frac{v_j}{DC} \quad \text{for Case 2} \tag{6}$$

(Cond. 2): The conditions of $\frac{v_{j-1}}{DC}$ to be a candidate for $T^*(P)$.

$$\frac{v_{j-1}}{DC} \geq tc_j \text{ and } \frac{v_{j-1}}{DC} \leq T_{1,j}(P) \quad \text{for Case 1} \tag{7}$$

$$\frac{v_{j-1}}{DC} < tc_j \text{ and } \frac{v_{j-1}}{DC} > T_{2,j}(P) \quad \text{for Case 2} \tag{8}$$

(Cond. 3): The conditions of $\frac{v_j^-}{DC}$ to be a candidate for $T^*(P)$.

$$\frac{v_j}{DC} > tc_j \text{and } \frac{v_j}{DC} \leq T_{1,j}(P) \quad \text{for Case 1} \tag{9}$$

For $T_{i,j}(P)$ to be a candidate of $T^*(P)$ in Case 1, $T_{1,j}(P)$ should be included on $[v_{j-1}/DC, v_j/DC)$, and also $T_{1,j}(P) \geq tc_j$ must be satisfied. For v_{j-1}/DC to be a candidate of $T^*(P)$ in Case 1, $\Pi_{1,j}(P,T)$ must be decreasing at v_{j-1}/DC. In other words, the conditions $\frac{v_{j-1}}{DC} > T_{1,j}(P)$ and $\frac{v_{j-1}}{DC} > tc_j$ must hold. For v_{j-1}/DC to be a candidate of $T^*(P)$ in Case 1, $\Pi_{1,j}(P,T)$ must be increasing at v_j^-/DC and so, the conditions $v_j/DC \leq T_{1,j}(P)$ and $\frac{v_j}{DC} > tc_j$ must be satisfied. The conditions for Case 2, inequalities (6) and (8), are justified in a similar way. Now, let us consider $T_{1,j}(P) \geq tc_j$ in inequality (5). Because the demand rate D is also a function of retail price P, the inequality can be rewritten as

$$T_{1,j}(P) = \sqrt{\frac{2S + C(R-I)(a-bP)tc_j^2}{(a-bP)(H+CR)}} \geq tc_j \ . \tag{10}$$

Rearranging inequality (10),

$$P \geq \frac{a}{b} - \frac{2S}{b(H+CI)tc_j^2} \ . \tag{11}$$

It is self evident that for any $P \geq \frac{a}{b} - \frac{2S}{b(H+CI)tc_j^2}$, the inequality $T_{1,j}(P) \geq tc_j$ holds. Similarly, $\frac{v_{j-1}}{DC} \leq T_{1,j}(P)$ in inequality (5) can be rewritten as

$$\frac{v_{j-1}}{DC} \leq \sqrt{\frac{2S + C(R-I)Dtc_j^2}{D(H+CR)}} \ . \tag{12}$$

For $R > I$, we have the following quadratic inequality of D;

$$f(D) = (R-I)C^3 tc_j^2 D^2 + 2SC^2 D - (H+CR)v_{j-1}^2 \geq 0 \ . \tag{13}$$

Because $(R-I)C^3 tc_j^2 > 0$ and the discriminant of $f(D)$ is positive, we have following two real roots;

$$D = \frac{-2SC^2 \pm \sqrt{4S^2C^4 + 4C^3(R-I)tc_j^2(H+CR)v_{j-1}^2}}{2C^3(R-I)tc_j^2} \ . \tag{14}$$

And therefore, the solution of inequality $f(D) \geq 0$, are as follows;

$$D \leq \frac{-2SC^2 - \sqrt{4S^2C^4 + 4C^3(R-I)tc_j^2(H+CR)v_{j-1}^2}}{2C^3(R-I)tc_j^2} \text{ or } D \geq \frac{-2SC^2 + \sqrt{4S^2C^4 + 4C^3(R-I)tc_j^2(H+CR)v_{j-1}^2}}{2C^3(R-I)tc_j^2} \ . \tag{15}$$

Because the demand rate D is also a function of retail price P, the inequalities (15) can be rewritten as

$$P \geq \frac{a}{b} + \frac{2SC^2 + \sqrt{4S^2C^4 + 4C^3(R-I)tc_j^2(H+CR)v_{j-1}^2}}{2bC^3(R-I)tc_j^2} \text{ or } P \leq \frac{a}{b} + \frac{2SC^2 - \sqrt{4S^2C^4 + 4C^3(R-I)tc_j^2(H+CR)v_{j-1}^2}}{2bC^3(R-I)tc_j^2} \ . \tag{16}$$

For $R = I$,

$$\frac{v_{j-1}}{DC} \leq \sqrt{\frac{2S}{D(H+CR)}} \ . \tag{17}$$

Also, because the demand D is a function of retail price P, the inequality can be rewritten as

$$P \leq \frac{a}{b} - \frac{(H+CR)v_{j-1}^2}{2bSC^2} \ . \tag{18}$$

Also, from $T_{1,j}(P) < \frac{v_j}{DC}$ in inequality (5), we have

$$\frac{a}{b} + \frac{2SC^2 - \sqrt{4S^2C^4 + 4C^3(R-I)tc_j^2(H+CR)v_j^2}}{2bC^3(R-I)tc_j^2} < P < \frac{a}{b} + \frac{2SC^2 + \sqrt{4S^2C^4 + 4C^3(R-I)tc_j^2(H+CR)v_j^2}}{2bC^3(R-I)tc_j^2} \quad \text{for } R > I. \tag{19}$$

$$P > \frac{a}{b} - \frac{(H+CR)v_j^2}{2bSC^2} \quad \text{for } R = I \tag{20}$$

In a similar way, other price ranges are obtained from inequalities (6) to (9) and they are

$$P < \frac{a}{b} - \frac{2S}{b(H+CI)tc_j^2} \text{ from } T_{2,j}(P) < tc_j \ . \tag{21}$$

$$P > \frac{a}{b} - \frac{(H+CI)v_j^2}{2bSC^2} \text{ from } T_{2,j}(P) < \frac{v_j}{DC} \ . \tag{22}$$

$$P \leq \frac{a}{b} - \frac{(H+CI)v_{j-1}^2}{2bSC^2} \text{ from } T_{2,j}(P) \geq \frac{v_{j-1}}{DC} \ . \tag{23}$$

$$P \geq \frac{a}{b} - \frac{v_j}{bCtc_{j+1}} \text{ from } tc_{j+1} \leq \frac{v_j}{DC} \ . \tag{24}$$

$$P > \frac{a}{b} - \frac{v_j}{bCtc_j} \text{ from } tc_j < \frac{v_j}{DC} \ . \tag{25}$$

We conclude that $T_{1,j}(P)$ determined with P value which satisfies all the three inequalities (11), (16) and (19) can be a candidate of $T^*(P)$ for $R > I$. Utilizing the price ranges in inequalities (11) to (25), we find the following price intervals which correspond to conditions (Cond. 1), (Cond. 2) and (Cond. 3).

(PI-1): Price Interval on which $T_{i,j}(P)$ becomes a candidate for $T^*(P)$.

$$PI1_j = \left\{ P | P \geq \frac{a}{b} - \frac{2S}{b(H+CI)tc_j^2} \right\} \cap \left\{ P | \ P \geq \frac{a}{b} + \frac{2SC^2 + \sqrt{4S^2C^4 + 4C^3(R-I)tc_j^2(H+CR)v_{j-1}^2}}{2bC^3(R-I)tc_j^2} \text{ or } P \leq \frac{a}{b} + \frac{2SC^2 - \sqrt{4S^2C^4 + 4C^3(R-I)tc_j^2(H+CR)v_{j-1}^2}}{2bC^3(R-I)tc_j^2} \right\} \cap \left\{ P | \ \frac{a}{b} + \frac{2SC^2 - \sqrt{4S^2C^4 + 4C^3(R-I)tc_j^2(H+CR)v_j^2}}{2bC^3(R-I)tc_j^2} < P < \frac{a}{b} + \frac{2SC^2 + \sqrt{4S^2C^4 + 4C^3(R-I)tc_j^2(H+CR)v_j^2}}{2bC^3(R-I)tc_j^2} \right\} \quad \text{for Case 1}(R > I) \tag{26}$$

$$PI1_j = \left\{ P | P \geq \frac{a}{b} - \frac{2S}{b(H+CI)tc_j^2} \right\} \cap \left\{ P | \ \frac{a}{b} - \frac{(H+CR)v_j^2}{2bSC^2} < P \leq \frac{a}{b} - \frac{(H+CR)v_{j-1}^2}{2bSC^2} \right\} \quad \text{for Case 1}(R = I) \tag{27}$$

$$PI1_j = \left\{ P | P < \frac{a}{b} - \frac{2S}{b(H+CI)tc_j^2} \right\} \cap \left\{ P | \ \frac{a}{b} - \frac{(H+CI)v_j^2}{2bSC^2} < P \leq \frac{a}{b} - \frac{(H+CI)v_{j-1}^2}{2bSC^2} \right\} \quad \text{for Case 2} \tag{28}$$

(PI-2): Price Interval on which v_{j-1}/DC becomes a candidate for $T^*(P)$.

$$PI2_j = \left\{ P | P \geq \frac{a}{b} - \frac{v_{j-1}}{bCtc_j} \right\} \cap \left\{ P | \ \frac{a}{b} + \frac{2SC^2 - \sqrt{4S^2C^4 + 4C^3(R-I)tc_j^2(H+CR)v_{j-1}^2}}{2bC^3(R-I)tc_j^2} < P < \frac{a}{b} + \frac{2SC^2 + \sqrt{4S^2C^4 + 4C^3(R-I)tc_j^2(H+CR)v_{j-1}^2}}{2bC^3(R-I)tc_j^2} \right\} \quad \text{for Case 1}(R > I) \tag{29}$$

$$PI2_j = \left\{ P | P \geq \frac{a}{b} - \frac{v_{j-1}}{bCtc_j} \right\} \cap \left\{ P | \ P > \frac{a}{b} - \frac{(H+CR)v_{j-1}^2}{2bSC^2} \right\} \quad \text{for Case 1}(R = I) \tag{30}$$

$$PI2_j=\left\{P|P<\frac{a}{b}-\frac{v_{j-1}}{bCtc_j}\right\}\cap\left\{P|\,P>\frac{a}{b}-\frac{(H+CI)v_{j-1}^2}{2bSC^2}\right\} \quad \text{for Case 2} \tag{31}$$

(PI-3): Price Interval on which v_j^-/DC becomes a candidate for $T^*(P)$.

$$PI3_j=\left\{P|P>\frac{a}{b}-\frac{v_j}{bCtc_j}\right\}\cap\left\{P|P\geq\frac{a}{b}+\frac{2SC^2+\sqrt{4S^2C^4+4C^3(R-I)tc_j^2(H+CR)v_j^2}}{2bC^3(R-I)tc_j^2} \text{ or } P<\frac{a}{b}+\frac{2SC^2-\sqrt{4S^2C^4+4C^3(R-I)tc_j^2(H+CR)v_j^2}}{2bC^3(R-I)tc_j^2}\right\}$$

$$\text{for Case 1}(R>I) \tag{32}$$

$$PI3_j=\left\{P|P>\frac{a}{b}-\frac{v_j}{bCtc_j}\right\}\cap\left\{P|\,P<\frac{a}{b}-\frac{(H+CR)v_j^2}{2bSC^2}\right\} \quad \text{for Case 1}(R=I) \tag{33}$$

The price intervals in equalities (26) to (33) that we present have a significant role to solve the model. For example, we consider (PI-1). If $P\in PI1_j$, $T_{i,j}(P)$ satisfies condition **(Cond. 1)** and then it becomes a candidate for $T^*(P)$. Substituting T with $T_{i,j}(P)$ in $\Pi_{i,j}(P,T)$, we have a maximizing problem $\Pi_{i,j}(P,T_{i,j}(P))$ that is a single variable function of retail price *P*. Let $\Pi^0{}_{i,j}(P) = \Pi_{i,j}(P,T_{i,j}(P))$, $i=1,2$ and $j=1,2,\cdots,m$. Note that $\Pi^0{}_{i,j}(P)$ is valid only on the interval $P\in PI1_j$. Similarly, if $P\in PI2_j$, then v_{j-1}/DC satisfies condition **(Cond. 2)**. Substituting T with v_{j-1}/DC in $\Pi_{i,j}(P,T)$, we have a single variable function $\Pi_{i,j}(P,v_{j-1}/DC)$, $i=1,2$ and $j=2,3,\cdots,m$ because v_{j-1}/DC is a function of *P*. Also, if $P\in PI3_j$, then v_j^-/DC satisfies condition **(Cond. 3)**. Substituting *T* with v_j^-/DC in $\Pi_{1,j}(P,T)$, also we have a single variable function $\Pi_{1,j}(P,v_j^-/DC)$, $j=1,2,\cdots,m-1$ because v_j^-/DC is a function of *P*. So, an optimal solution (P^*,T^*) which maximizes $\Pi(P,T)$ is found by searching over $\Pi^0{}_{i,j}(P)$, $\Pi_{i,j}(P,v_{j-1}/DC)$ and $\Pi_{1,j}(P,v_j^-/DC)$, and

$$\max_{P,T}\Pi(P,T) = \max\left[\max_{\substack{P\in PI1_j\\ i,j}}\Pi^0{}_{i,j}(P),\ \max_{\substack{P\in PI2_j\\ i,j}}\Pi_{i,j}\left(P,\frac{v_{j-1}}{DC}\right),\ \max_{\substack{P\in PI3_j\\ j}}\Pi_{1,j}\left(P,\frac{v_j^-}{DC}\right)\right]. \tag{34}$$

Now, we want to find an optimal retail price (P^*) and replenishment cycle time (T^*) which maximizes $\Pi(P,T)$ in (34). Recognizing that the above single variable functions have very complicated structure, the functions can be solved approximately by using numerical search method. Then, we present the following solution algorithm to determine the optimal retail price and replenishment cycle time.

Solution Algorithm

Step 1. For each T_0, $T_0\geq tc_j$, its optimal retail price $P_{1,j}$ is determined from the corresponding price intervals.

1.1. Determine $P_{1,j}$ which maximizes $\Pi_{1,j}{}^0(P)$ among the price intervals: $P\in PI1_j$ and $P\leq a/b$ with $T_0=T_{1,j}(P)$, $j=1,\ 2,\ \cdots,m$.

1.2. Determine $P_{1,j}$ which maximizes $\Pi_{1,j}(P,v_{j-1}/DC)$ among the price intervals: $P\in PI2_j$ and $P\leq a/b$ with $T_0=v_{j-1}/DC$, $j=2,\ 3,\ \cdots,m$.

1.3. Determine $P_{1,j}$ which maximizes $\Pi_{1,j}(P,v_j^-/DC)$ among the price intervals: $P\in PI3_j$ and $P\leq a/b$ with $T_0=v_j^-/DC$, $j=1,2,\ \cdots,m-1$.

Step 2. For each T_0, $T_0<tc_j$, its optimal retail price $P_{2,j}$ is determined from the corresponding price intervals.

2.1 Determine $P_{2,j}$ which maximizes $\Pi_{2,j}{}^0(P)$ among the price intervals: $P\in PI1_j$ and $P\leq a/b$ with $T_0=T_{2,j}(P)$, $j=1,\ 2,\ \cdots,m$.

Table 1. Results of Step 1

j	$P \in PI1_j$	$P_{1,j}$	$T_{1,j}(P_{1,j})$	$\Pi(P,T)$	$P \in PI2_j$	$P_{1,j}$	v_{j-1}/DC	$\Pi(P,T)$	$P \in PI3_j$	$P_{1,j}$	v_j^-/DC	$\Pi(P,T)$
1	[6.92, 7.99]	6.92	0.37	5071	-	-	-	-	[4, 6.92]	5.53	0.16	7490
2	[4.51, 6.98]	5.49	0.25	7642	[6.98, 7.99]	6.98	0.39	4921	[4, 4.51]	4.51	0.23	6435
3	∅	-	-	-	[5.33, 7.99]	5.48	0.32	7738[a]	-	-	-	-

[a] Optimal solution for Case 1. Also, this solution is the global optimal solution with its annual net profit $7,738.

2.2 Determine $P_{1,j}$ which maximizes $\Pi_{2,j}(P, v_{j-1}/DC)$ among the price intervals: $P \in PI2_j$ and $P \leq a/b$ with $T_0 = v_{j-1}/DC,\ j = 2,\ 3,\ \cdots, m$.

Step 3. Select the optimal retail price (P^*) and replenishment cycle time (T^*) and which gives the maximum annual net profit among the values obtained in the previous steps.

4. NUMERICAL EXAMPLE

To illustrate the solution algorithms, the following problem is considered.

(1) S = \$ 50, C = \$ 3, R = 15% (= 0.15), I = 10% (=0.10), H = \$ 0.1.

(2) Supplier's credit schedule:

Total amount of purchase	Credit period
$0 \leq TDC < \$1{,}500$	$tc_1 = 0.1$
$\$1{,}500 \leq TDC < \$3{,}000$	$tc_2 = 0.2$
$\$3{,}000 \leq TDC$	$tc_3 = 0.3$

In order to solve the problem, the computer program written in C language was developed. And the solution procedure with $a = 10{,}000$, $b = 1{,}250$ that is, $D = 10{,}000 - 1{,}250P$ and $P \leq 10{,}000/1{,}250\ (= 8)$ generates the optimal solution (P^*, T^*) through the following steps.

Step 1.

1.1. Searching $P_{1,j}$ numerically which maximizes $\Pi_{1,j}{}^0(P)$ among the price intervals: $P \in PI1_j$ and $P \leq a/b$ with $T_0 = T_{1,j}(P),\ j = 1,\ 2,\ 3$, we obtain $P_{1,j}$ as listed in Table 1.

1.2. Searching $P_{1,j}$ numerically which maximizes $\Pi_{1,j}(P, v_{j-1}/DC)$ among the price intervals: $P \in PI2_j$ and $P \leq a/b$ with $T_0 = v_{j-1}/DC,\ j = 2,\ 3$, we obtain $P_{1,j}$ as listed in Table 1.

1.3. Searching $P_{1,j}$ numerically which maximizes $\Pi_{1,j}(P, v_j^-/DC)$ among the price intervals: $P \in PI3_j$ and $P \leq a/b$ with $T_0 = v_j^-/DC,\ j = 1, 2$, we obtain $P_{1,j}$ as listed in Table 1.

Step 2.

2.1 Searching $P_{2,j}$ numerically which maximizes $\Pi_{2,j}{}^0(P)$ among the price intervals: $P \in PI1_j$ and $P \leq a/b$ with $T_0 = T_{2,j}(P),\ j = 1,\ 2,\ 3$, we obtain $P_{2,j}$ as listed in Table 2.

2.2 Searching $P_{2,j}$ numerically which maximizes $\Pi_{2,j}(P, v_{j-1}/DC)$ among the price intervals: $P \in PI2_j$ and $P \leq a/b$ with $T_0 = v_{j-1}/DC,\ j = 2,\ 3$, we obtain $P_{2,j}$ as listed in Table 2.

Step 3. From the results of Step 1 and Step 2, an optimal solution (P^*, T^*) becomes (5.48, 0.32) with its maximum annual net profit of $7,738.

Table 2. Results of Step 2

j	$P \in PI1_j$	$P_{1,j}$	$T_{1,j}(P_{1,j})$	$\Pi(P,T)$	$P \in PI2_j$	$P_{1,j}$	v_{j-1}/DC	$\Pi(P,T)$
1	∅	-	-	-	-	-	-	-
2	∅	-	-	-	∅	-	-	-
3	[0.00, 4.80]	4.80	0.25	7160	∅	-	-	-

[a] Optimal solution for Case 2(Annual net profit = $7,160).

5. CONCLUSION

One of the important determinants of the length of the credit period is the size of the account. It is generally known that the free credit period tends to be shorter on large shipments than on smaller ones, probably because the supplier sells the larger quantities at lower prices. Rather than giving some price discount for larger amount of purchase, some manufacturers in Korea offer a longer credit terms. The supplier’s policies tend to make the retailer's order size larger by inducing him to qualify for a longer credit period in his payment. For a retailer who benefits from the supplier's credit term, it is common that he lowers the retail price to a certain degree expecting that he can earn more profits by stimulating the customer demand. This paper deals with an optimal pricing and ordering policy of retailer when the demand of the product can be represented by a linear decreasing function of the retail price, and the length of delay in payment is a function of the retailer's total amount of purchase. After formulating the mathematical model, we propose the solution procedure that leads to an optimal pricing and ordering policy. With an example problem, the validity of the algorithm is examined. The results show that the annual net profit could be increased through a wise selection of both the retail price and order size.

There are several interesting opportunities for future researches in this subject. The model can be extended to the case of perishable product. While this paper focuses on one type of product, the case of joint ordering of multiple products for different credit terms could be suggested.

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