RIEMANN SOLITONS ON CERTAIN TYPE OF KENMOTSU MANIFOLD

Manoj Ray Bakshi*, Kanak Kanti Baishya, and Ashoke Das

Abstract. The object of the present paper is to investigate the nature of Riemann solitons on generelized D-conformally deformed Kenmotsu manifold with hyper generalized pseudo symmetric curvature conditions.

1. Introduction

Let the symbols ∇ and ∇^d stand for the Riemann connection and the generalized D-conformally deformed connection respectively. Also, let R, S, Q, r and R^d, S^d, Q^d, r^d respectively stand for curvature tensor, Ricci tensor, Ricci operator, scalar curvature with respect to ∇ and ∇^d respectively. In this study, we consider an almost contact metric manifold $(M^{2n+1}, \phi, \xi, \eta, g)$ that consists of a $(1,1)$-tensor field ϕ, a vector field ξ and a 1-form η called respectively the structure endomorphism, the characteristic vector field and the contact form. In a recent paper, the authors ([2]) has introduced a new type of space called hyper generalized weaky symmetric manifold. Then the authors studied ([8]) hyper generalized pseudo Q-symmetric semi-Riemanian manifold. In Section 3 of this paper we extend this concept to generalized D-conformally deformed structure of a $(2n+1)$-dimensional Kenmotsu manifold.

A $(2n+1)$-dimensional Kenmotsu manifold is said to be hyper generalized pseudo symmetric (which will be abbreviated hereafter as $[H(GPS)_n, \nabla]$) if it admits the equation

$$
\begin{aligned}
&(\nabla_X \bar{R})(Y, U, V, \ W) \\
={}& 2\alpha(X)\bar{R}(Y, U, V, \ W) + \alpha(Y)\bar{R}(X, U, V, \ W) \\
&+ \alpha(U)\bar{R}(Y, X, V, \ W) + \alpha(V)\bar{R}(Y, U, X, \ W) \\
&+ \alpha(W)\bar{R}(Y, U, V, \ X) + 2\beta(X)(g \wedge S)(Y, U, V, W) \\
&+ \beta(Y)(g \wedge S)(X, U, V, W) + \beta(U)\,(g \wedge S)(Y, X, V, W) \\
&+ \beta(V)\,(g \wedge S)(Y, U, X, W) + \beta(W)\,(g \wedge S)(Y, U, V, X),
\end{aligned}
$$
(1)

where

$$(g \wedge S)(Y, U, V, W) = g(Y, W)S(U, V) + g(U, V)S(Y, W)$$
$$\text{(2)} \qquad\qquad\qquad -g(Y, V)S(U, W) - g(U, W)S(Y, V),$$

and α, β being non-zero 1-forms defined as $\alpha(X) = g(X, \theta_1)$ and $\beta(X) = g(X, \theta_2)$.

Ricci flow was first introduced by R. S. Hamilton ([17]) in 1982 which generalizes the notion of Riemann flow ([20], [19]). Keeping the tune with Ricci soliton, Hirica and Udriste ([18]) introduced and studied Riemann soliton. The Riemann flow is an evolution equation for metrics on a Riemannian manifold defined as follows

$$\frac{\partial}{\partial t} G(t) = -2R(g(t)), \qquad t \in [0, I],$$

where $G = \frac{1}{2} g \circledast g$, $\circledast$ is the Kulkarni-Nomizu product and R is the Riemann curvature tensor associated to the metric g. For $(0, 2)$-tensors A and B, the Kulkarni-Nomizu product $(A \circledast B)$ is given by

$$(A \circledast B)(Y, U, V, Z) = A(Y, Z)B(U, V) + A(U, V)B(Y, Z)$$
$$\text{(3)} \qquad\qquad\qquad -A(Y, V)B(U, Z) - A(U, Z)B(Y, V).$$

Recently, the present authors studied the Riemann solitons in the frame of $(LCS)_n$-manifolds ([4]) and α-cosymplectic manifolds ([5]). The Riemann soliton is a smooth manifold M together with Riemannian metric g that satisfies

$$\text{(4)} \qquad\qquad 2R + (g \circledast \mathcal{L}_W g) = 2\kappa(g \circledast g),$$

where W is a potential vector field, $\mathcal{L}_W$ denotes the Lie-derivative along the vector field W and κ is a constant. A Riemann soliton is called expanding, steady and shrinking when $\kappa < 0$, $\kappa = 0$ and $\kappa > 0$ respectively.

We organize our present paper as follows: After Introduction, in Section 2, we briefly recall some known results for Kenmotsu manifolds and generalized D-conformally deformed of a Kenmotsu manifold and established some properties of the deformed Kenmotsu manifold. In Section 3, we discuss the properties of a generalized D-conformally deformed Kenmotsu manifold under hyper generalized pseudo symmetric curvature condition equipped with Riemann solitions. Finally, e determine a necessary condition for shrinking, steady and expanding of the soliton.

2. Preliminaries

According to the definition of Blair ([11]), an *almost contact structure* (ϕ, ξ, η) on a $(2n + 1)$-dimensional Riemannian manifold satisfies the following conditions

$$\text{(5)} \qquad\qquad\qquad \phi^2 = -I + \eta \otimes \xi,$$

$$\text{(6)} \qquad\qquad\qquad \eta(\xi) = 1,$$

$$\text{(7)} \qquad\qquad \phi\xi = 0, \ \eta \circ \phi = 0, \ \text{rank } \phi = n - 1.$$

Moreover, if g is a Riemannian metric on M^{2n+1} satisfying

$$\text{(8)} \qquad\qquad g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y),$$

$$\text{(9)} \qquad\qquad\qquad g(X, \xi) = \eta(X),$$

(10) $$g(\phi X, Y) = -g(X, \phi Y),$$

for any vector fields X, Y on M^{2n+1}, then the manifold M^{2n+1} ([11]) is said to admit an *almost contact metric structure* (ϕ, ξ, η, g).

DEFINITION 2.1. [15] If in an almost contact metric structure (ϕ, ξ, η, g) on M^{2n+1}, the Riemann connection ∇ of g satisfies $(\nabla_X \phi)Y = g(\phi X, Y)\xi - \eta(Y)\phi X$, for any vector fields X, Y on M^{2n+1}, then the structure is called Kenmotsu.

PROPOSITION 2.2. [3,9,15] *If* $(M^{2n+1}, \phi, \xi, \eta, g)$ *is a Kenmotsu manifold, then for any vector fields* X, Y, Z *on* M^{2n+1}, *the following relations hold*

(11) $$\nabla_X \xi = X - \eta(X)\xi,$$

(12) $$(\nabla_X \eta)Y = g(X, Y) - \eta(X)\eta(Y),$$

(13) $$S(X, \xi) = -2n\eta(X),$$

(14) $$\eta(R(X, Y)Z) = g(X, Z)\eta(Y) - g(Y, Z)\eta(X),$$

(15) $$R(\xi, X)Y = \eta(Y)X - g(X, Y)\xi,$$

(16) $$R(X, Y)\xi = \eta(X)Y - \eta(Y)X.$$

DEFINITION 2.3. [1] If a contact metric manifold M^{2n+1} with the almost contact metric structure (ϕ, ξ, η, g) is transformed into $(\phi^d, \xi^d, \eta^d, g^d)$, where

(17) $$\phi^d = \phi, \ \ \xi^d = \frac{1}{p}\xi, \ \ \eta^d = p\eta, \ \ g^d = qg + (p^2 - q)\eta \otimes \eta,$$

where p and q are constants such that $p \neq 0$ and $q > 0$, then the transformation is called a generalized D-conformal deformation.

Note that the generalized D-conformal deformation give rise to conformal deformation (for $p^2 = q$) and D-homothetic deformation (for $p = q = $ constant) ([16], [6], [10]). The generalized D-conformal deformation are studied by various authors in ([22], [23], [21], [24]).

The relation between the Levi-Civita connections ∇ of g and ∇^d of g^d is given by ([1])

(18) $$\nabla_X^d Y = \nabla_X Y + \frac{(p^2 - q)}{p^2} g(\phi X, \phi Y)\xi,$$

for any vector fields X, Y on M^{2n+1}.

In view of (17), (18) and definition of Riemannian curvature tensor, Ricci tensor, scalar curvature, we get the following:

PROPOSITION 2.4. [1] *If a Kenmotsu structure* (ϕ, ξ, η, g) *on* M^{2n+1} *is transformed into* $(\phi^d, \xi^d, \eta^d, g^d)$ *under a generalized D-conformal deformation, then* R, R^d, S, S^d, r *and* r^d *are related by*

(19) $$R^d(X, Y)Z = R(X, Y)Z + \frac{(p^2 - q)}{p^2}[g(\phi Y, \phi Z)X - g(\phi X, \phi Z)Y],$$

(20) $$S^d(X, Y) = S(X, Y) + \frac{2n(p^2 - q)}{p^2}g(\phi X, \phi Y),$$

$$(21) \qquad r^d = \frac{r}{q} + \frac{2n(2n+1)(p^2-q)}{p^2},$$

for any vector fields X, Y, Z on M^{2n+1}.

Now we shall bring out some properties of a generalized D-conformally deformed structure $(\phi^d, \xi^d, \eta^d, g^d)$ of a Kenmotsu manifold M^{2n+1} as follows:

PROPOSITION 2.5. *Under a generalized D-conformal deformation of a Kenmotsu structure (ϕ, ξ, η, g) on M^{2n+1} is transformed into $(\phi^d, \xi^d, \eta^d, g^d)$, then for any vector fields X, Y, Z on M^{2n+1}, we have*

$$(22) \qquad \phi^d = -I + \eta^d \otimes \xi^d,$$

$$(23) \qquad \eta^d(\xi^d) = 1,$$

$$(24) \qquad \phi^d \xi^d = 0, \ \eta^d \circ \phi^d = 0,$$

$$(25) \qquad g^d(\phi^d X, \phi^d Y) = g^d(X, Y) - \eta^d(X)\eta^d(Y),$$

$$(26) \qquad g^d(X, \xi^d) = \eta^d(X),$$

$$(27) \qquad \nabla^d_X \xi^d = \frac{1}{p}[X - \eta^d(X)\xi^d],$$

$$(28) \qquad (\nabla^d_X \eta^d)Y = \frac{1}{p}[g^d(X, Y) - \eta^d(X)\eta^d(Y)],$$

$$(29) \qquad S^d(X, \xi^d) = -\frac{2n}{p^2}\eta^d(X),$$

$$(30) \qquad \eta^d(R^d(X, Y)Z) = \frac{1}{p^2}[g^d(X, Z)\eta^d(Y) - g^d(Y, Z)\eta^d(X)],$$

$$(31) \qquad R^d(\xi^d, X)Y = \frac{1}{p^2}[\eta^d(Y)X - g^d(X, Y)\xi^d],$$

$$(32) \qquad R^d(X, Y)\xi^d = \frac{1}{p^2}[\eta^d(X)Y - \eta^d(Y)X].$$

Now using (28) and (29), we obtain

$$(33) \qquad (\nabla^d_X S^d)(Y, \xi^d) = -\frac{1}{p}[\frac{2n}{p^2}g^d(X, Y) + S^d(X, Y)],$$

for any vector fields X, Y and Z on M^{2n+1}.

3. Main results

In the beginning, we shall define a hyper generalized pseudo symmetric space on a generalized D-conformally deformed structure $(\phi^d, \xi^d, \eta^d, g^d)$ of a Kenmotsu manifold M^{2n+1}.

3.1. Hyper generalized pseudo symmetric deformed Kenmotsu manifold.

DEFINITION 3.1. A generalized D-conformally deformed structure $(\phi^d, \xi^d, \eta^d, g^d)$ of a Kenmotsu manifold M^{2n+1} is said to be hyper generalized pseudo symmetric if it satisfies the condition

$$
\begin{aligned}
(\nabla^d_X \bar{R}^d)&(Y, U, V, W) \\
= \ & 2\alpha^d(X)\bar{R}^d(Y, U, V, \ W) + \alpha^d(Y)\bar{R}^d(X, U, V, \ W) \\
& + \alpha^d(U)\bar{R}^d(Y, X, V, \ W) + \alpha^d(V)\bar{R}^d(Y, U, X, \ W) \\
& + \alpha^d(W)\bar{R}^d(Y, U, V, \ X) + 2\beta^d(X)(g^d \wedge S^d)(Y, U, V, W) \\
& + \beta^d(Y)(g^d \wedge S^d)(X, U, V, W) + \beta^d(U) \ (g^d \wedge S^d)(Y, X, V, W) \\
& + \beta^d(V) \ (g^d \wedge S^d)(Y, U, X, W) + \beta^d(W) \ (g^d \wedge S^d)(Y, U, V, X).
\end{aligned}
\tag{34}
$$

where

$$
\begin{aligned}
(g^d \wedge S^d)(Y, U, V, W) \ = \ & g^d(Y, W)S^d(U, V) + g^d(U, V)S^d(Y, W) \\
& - g^d(Y, V)S^d(U, W) - g^d(U, W)S^d(Y, V),
\end{aligned}
\tag{35}
$$

and A^d_i are non-zero 1-forms defined by $A^d_i(X) = g^d(X, \sigma_i)$, for $i = 1, 2$.

In this section, we consider a Kenmotsu manifold (M^{2n+1}, g) $n \geq 1$ which is hyper generalized pseudo symmetric. Now, making use of (35) in (34), we find

$$
\begin{aligned}
(\nabla_X \bar{R}^d)&(Y, U, V, \ W) \\
= \ & 2\alpha^d(X)\bar{R}^d(Y, U, V, \ W) + \alpha^d(Y)\bar{R}^d(X, U, V, \ W) \\
& + \alpha^d(U)\bar{R}^d(Y, X, V, \ W) + \alpha^d(V)\bar{R}^d(Y, U, X, \ W) \\
& + \alpha^d(W)\bar{R}^d(Y, U, V, \ X) + 2\beta^d(X)[g^d(Y, W)S^d(U, V) \\
& + g^d(U, V)S^d(Y, W) - g^d(Y, V)S^d(U, W) \\
& - g^d(U, W)S^d(Y, V)] + \beta^d(Y)[g^d(X, W)S^d(U, V) \\
& + g^d(U, V)S^d(X, W) - g^d(X, V)S^d(U, W) \\
& - g^d(U, W)S^d(X, V)] + \beta^d(U) \ [g^d(Y, W)S^d(X, V) \\
& + g^d(X, V)S^d(Y, W) - g^d(Y, V)S^d(X, W) \\
& - g^d(X, W)S^d(Y, V)] + \beta^d(V) \ [g^d(Y, W)S^d(U, X) \\
& + g^d(U, X)S^d(Y, W) - g^d(Y, X)S^d(U, W) - g^d(U, W)S^d(Y, X)] \\
& + \beta^d(W)[g^d(Y, X)S^d(U, V) + g^d(U, V)S^d(Y, X) \\
& - g^d(Y, V)S^d(U, X) - g^d(U, X)S^d(Y, V)].
\end{aligned}
\tag{36}
$$

Now, contracting over Y and W in (36), we get

$$
\begin{aligned}
&(\nabla_X^d S^d)(U, V) \\
={}& 2\alpha^d(X)S^d(U, V) + \alpha^d(U)S^d(X, V) \\
&+\alpha^d(R^d(X, U)V) + 2\beta^d(Q^d X)g^d(U, V) \\
&+\alpha^d(R^d(X, V)U) + \alpha^d(V)S^d(X, U) \\
&+2\beta^d(X)[2nS^d(U, V) + r^d g^d(U, V)] \\
&+\beta^d(U)\,[(2n - 2)S^d(X, V) + r^d g^d(X, V)] \\
&+\beta^d(V)\,[(2n - 2)S^d(X, U) + r^d g^d(X, U)] \\
&-\beta^d(Q^d U)g^d(X, V) - \beta^d(Q^d V)g^d(X, U).
\end{aligned}
$$
(37)

Now setting $V = \xi^d$ and using (29), (31), (32) in the foregoing equation, we obtain

$$
\begin{aligned}
&(\nabla_X^d S^d)(U, \xi^d) \\
={}& -\frac{2n}{p^2}[2\alpha^d(X)\eta^d(U) + \alpha^d(U)\eta^d(X)] + \alpha^d(\xi^d)S^d(X, U) \\
&+\frac{1}{p^2}[g^d(X, U)\alpha^d(\xi^d) - 2\eta^d(U)\alpha^d(X) + \eta^d(X)\alpha^d(U)] \\
&+2\beta^d(X)(r^d - \frac{4n^2}{p^2})\eta^d(U) + \beta^d(U)(r^d - \frac{4n(n - 1)}{p^2})\eta^d(X) \\
&+\beta^d(\xi^d)\,[2(n - 1)S^d(U, X) + r^d g^d(U, X)] \\
&+2\beta^d(Q^d X)\eta^d(U) - \beta^d(Q^d U)\eta^d(X) + \frac{2n}{p^2}\beta^d(\xi^d)g^d(U, X)
\end{aligned}
$$
(38)

which yields by using (33)

$$
\begin{aligned}
&-\frac{1}{p}[\frac{2n}{p^2}g^d(X, U) + S^d(X, U)] \\
={}& [-\frac{2(2n + 1)}{p^2}\alpha^d(X) + 2\beta^d(X)(r^d - \frac{4n^2}{p^2}) + 2\beta^d(Q^d X)]\eta^d(U) \\
&+[-\frac{(2n - 1)}{p^2}\alpha^d(U) + (r^d - \frac{4n(n - 1)}{p^2})\beta^d(U) - \beta^d(Q^d U)]\eta^d(X) \\
&+\frac{1}{p^2}\alpha^d(\xi^d)g^d(X, U) + \alpha^d(\xi^d)S^d(X, U) + \frac{2n}{p^2}\beta^d(\xi^d)g^d(U, X) \\
&+\beta^d(\xi^d)\,[2(n - 1)S^d(U, X) + r^d g^d(U, X)].
\end{aligned}
$$
(39)

Putting $X = U = \xi^d$ in succession, we obtain from (39) that

$$
[r^d - \frac{2n(2n - 1)}{p^2}]\beta^d(\xi^d) = \frac{2n}{p^2}\alpha^d(\xi^d).
$$
(40)

$$
\begin{aligned}
&-\frac{2(2n + 1)}{p^2}\alpha^d(X) + 2\beta^d(X)(r^d - \frac{4n^2}{p^2}) + 2\beta^d(Q^d X) \\
={}& [\frac{2(2n - 1)}{p^2}\alpha^d(\xi^d) - 2\beta^d(\xi^d)(r^d - \frac{4n(n - 1)}{p^2}) + 2\beta^d(Q^d \xi^d)]\eta^d(X).
\end{aligned}
$$
(41)

and

$$-\frac{(2n-1)}{p^2}\alpha^d(U) + (r^d - \frac{4n(n-1)}{p^2})\beta^d(U) - \beta^d(Q^dU)$$

(42)
$$= [\frac{(6n+1)}{p^2}\alpha^d(\xi^d) - (3r^d - \frac{12n^2-2n}{p^2})\beta^d(\xi^d)]\eta^d(U).$$

By virtue of (40), (41) and (42), the equation (39) yields

$$S^d(X,U)$$

$$= -\left(\frac{\frac{1}{p^2}\alpha^d(\xi^d) + (r^d + \frac{2n}{p^2})\beta^d(\xi^d) + \frac{2n}{p^3}}{\frac{1}{p} + \alpha^d(\xi^d) + 2(n-1)\beta^d(\xi^d)}\right) g^d(U,X)$$

(43)
$$-\left(\frac{\frac{(10n-1)}{p^2}\alpha^d(\xi^d) - (5r^d - \frac{20n^2-14n}{p^2})\beta^d(\xi^d)}{\frac{1}{p} + \alpha^d(\xi^d) + 2(n-1)\beta^d(\xi^d)}\right) \eta^d(U)\eta^d(X).$$

and (40) gives

(44)
$$r^d = \frac{2n}{p^2}[\frac{\alpha^d(\xi^d)}{\beta^d(\xi^d)} + (2n-1)].$$

Next using (44) in (43) we have

$$S^d(X,U)$$

$$= -\left(\frac{\frac{(2n+1)}{p^2}\alpha^d(\xi^d) + \frac{4n^2}{p^2}\beta^d(\xi^d) + \frac{2n}{p^3}}{\frac{1}{p} + \alpha^d(\xi^d) + 2(n-1)\beta^d(\xi^d)}\right) g^d(U,X)$$

(45)
$$+\left(\frac{\frac{1}{p^2}\alpha^d(\xi^d) + \frac{4n}{p^2}\beta^d(\xi^d)}{\frac{1}{p} + \alpha^d(\xi^d) + 2(n-1)\beta^d(\xi^d)}\right) \eta^d(U)\eta^d(X).$$

This motivate us to state:

THEOREM 3.2. Let $(\phi^d, \xi^d, \eta^d, g^d)$ be a generalized D-conformally deformed hyper generalized pseudo symmetric Kenmotsu manifold M^{2n+1}. Then such a space is conformally flat provided $\frac{1}{p} + \alpha^d(\xi^d) + 2(n-1)\beta^d(\xi^d) \neq 0$.

THEOREM 3.3. The scalar curvature of a hyper generalized pseudo symmetric generalized D-conformally deformed Kenmotsu manifold is constant.

COROLLARY 3.4. Let $(\phi^d, \xi^d, \eta^d, g^d)$ be a generalized D-conformally deformed pseudo symmetric Kenmotsu manifold M^{2n+1}. Then such a space is conformally flat provided $p\alpha^d(\xi^d) \neq -1$.

4. Generalized D-conformally deformed hyper generalized pseudo symmetric Kenmotsu manifold and the case of Riemann soliton

In this section, we consider a generalized D-conformally deformed Kenmotsu manifold $(\phi^d, \xi^d, \eta^d, g^d)$ admitting a Riemann soliton. Then with the aid of (3) and (4),

we obtain

$$2R^d\left(Y,U,V,Z\right) + g^d\left(Y,Z\right)\left(\pounds_{\xi^d}g^d\right)\left(U,V\right)$$
$$+g^d\left(U,V\right)\left(\pounds_{\xi^d}g^d\right)\left(Y,Z\right)$$
$$-g^d\left(Y,V\right)\left(\pounds_{\xi^d}g^d\right)\left(U,Z\right) - g^d\left(U,Z\right)\left(\pounds_{\xi^d}g^d\right)\left(Y,V\right)$$
$$(46) \qquad = \ 2\kappa\left[g^d\left(Y,Z\right)g^d\left(U,V\right) - g^d\left(Y,V\right)g^d\left(U,Z\right)\right].$$

Now by contraction over Y and Z we get

$$(47) \qquad \frac{1}{2}\left(\pounds_{\xi^d}g^d\right)\left(U,V\right) + \frac{1}{2n-1}S^d(U,V) = \frac{2n\kappa - div(\xi^d)}{2n-1}g^d(U,V).$$

and then using (47) in (46), we get

$$(48) \qquad r^d = 2n[(2n+1)\kappa - \frac{4n}{p}].$$

Comparing (44) with (48) we have

$$(49) \qquad (2n+1)p^2\kappa = 4np + (2n-1) + \frac{\alpha^d(\xi^d)}{\beta^d(\xi^d)}.$$

This leads to the following:

THEOREM 4.1. *Assume that a Kenmotsu structure (ϕ, ξ, η, g) on M^{2n+1} is transformed into $(\phi^d, \xi^d, \eta^d, g^d)$ under a generalized D-conformally deformation which is a hyper generalized pseudo symmetric space. Then the Riemann soliton is expanding, steady and shrinking as $\frac{\alpha^d(\xi^d)}{\beta^d(\xi^d)} + 2n(2p+1) <=> 1$.*

References

[1] P. Alegre, D. E. Blair, and A. Carriazo, *Generalized Sasakian-space-forms*, Israel J. Math. **141** (1) (2004), 157–183.

[2] K K Baishya, F. Ozen Zengin† and J Mike, *On hyper generalised weakly symmetric manifolds*, Nineteenth International Conference on Geometry, Integrability and Quantization June 02–07, 2017, Varna, Bulgaria Ivaïlo M. Mladenov and Akira Yoshioka, Editors Avangard Prima, Sofia 2018, pp 1–10 doi:10.7546/giq-19-2018-1-10

[3] K. K. Baishya, P. Peska, and P. R. Chowdhury, *On almost generalized weakly symmetric Kenmotsu manifolds*, Acta Univ. Palacki. Olomuc., Fac. rer. nat. Mathematica **55** (2) (2016), 5–15.

[4] M. R. Bakshi and K. K. Baishya, *Certain types of $(LCS)_n$ -manifolds and the case of Riemann solitons*, Differential Geometry-Dynamical Systems **22** (2020),11–25.

[5] M. R. Bakshi and K. K. Baishya, *Four classes of Riemann solitons on alpa-cosymplectic manifolds*, Afrika Matematika, https://doi.org/10.1007/s13370-020-00846-6

[6] M. R. Bakshi, K. K. Baishya, D. G. Prakasha and P. Veeresha, *Ricci solitons in a hyper generalized pseudo symmetric D-homothetically deformed Kenmotsu manifold*, submitted

[7] A. Biswas, A. Das, K. K. Baishya and M. R. Bakshi, *η -Ricci solitons on Kenmotsu manifolds admitting General connection*, Korean J. Math. **28** (2020) (4), 803–817, http://dx.doi.org/10.11568/kjm.2020.28.4.803

[8] A. M. Blaga, M. R. Bakshi, and K. K. Baishya, *Hyper generalized pseudo Q-symmetric semi-Riemanian manifold*, Cubo, A Mathematical Journal,Vol. **23** (1) (2021), 87–96,

[9] K. K. Baishya and P. R. Chowdhury, *On Generalized Weakly Symmetric Kenmotsu Manifolds*, Bol. Soc. Paran. Mat, (3s.) v. 39 6 (2021): 211–222.

[10] A. M. Blaga, K. K. Baishya and N. Sarkar, *Ricci solitons in a generalized weakly (Ricci) symmetric D-homothetically deformed Kenmotsu manifold*, Ann. Univ. Paedagog. Crac. Stud. Math. **18** (2019), 123–136

[11] D. E. Blair, *Contact Manifolds in Riemannian Geometry*, Lect. Notes in Math. 509, Springer-Verlag, New York (1976).

[12] M. C. Chaki, *On pseudo symmetric manifolds*, Analele Stiintifice ale Universitatii "Al I. Cuza" din Iasi **33** (1987), 53–58.

[13] R. S. Hamilton, *The Ricci flow on surfaces* , Contemp. Math. **71** (1988), 237–261.

[14] R. S. Hamilton, *Three-manifolds with positive Ricci curvature*, J. Diff. Geom. **17** (1982), 255–306.

[15] K. Kenmotsu, *A class of almost contact Riemannian manifolds*, Tohoku Math. J. **24** (1972), 93–103

[16] Tanno, S., *The topology of contact Riemannian manifolds* , Illinois J. Math. **12** (1968), 700–717.

[17] R. S. Hamilton, *The Ricci flow on surfaces*, Mathematics and general relativity, Contemp. Math. **71**, American Math. Soc. (1988), 237–262.

[18] I.E. Hirică, C. Udriste, *Ricci and Riemann solitons* , Balkan J. Geom. Applications. **21** (2) (2016), 35–44.

[19] C. Udrişte, *Riemann flow and Riemann wave via bialternate product Riemannian metric.* preprint, arXiv.org/math.DG/1112.4279v4 (2012).

[20] C. Udriste, *Riemann flow and Riemann wave*, Ann. Univ. Vest, Timisoara. Ser. Mat. Inf. **48** (1-2) (2010), 265–274.

[21] Nülifer Özdemir, Sirin Aktay, Mehmet Solgun, *On generalized D-conformal deformations of certain almost contact metric manifolds*, Mathematics 2019, 0**7**00168.

[22] Nagaraja, H.G., Kiran Kumar, D.L., *Ricci Solitons in Kenmotsu Manifold under Generalized D-Conformal Deformation.* Lobachevskii J Math **40** (2019), 195–200. https://doi.org/10.1134/S1995080219020112.

[23] HG Nagaraja, DL Kiran Kumar, VS Prasad, *Ricci solitons on Kenmotsu manifolds under D-homothetic deformation*, Khayyam J. Math. **4** (1) (2018), 102–109.

[24] T. Suguri and S. Nakayama, *D-conformal deformations on almost contact metric structure*, Tensor (N.S.) **28** (1974), 125–129.

Manoj Ray Bakshi
Department of Mathematics, Raiganj University, Uttar Dinajpur,
West Bengal, India
E-mail: raybakshimanoj@gmail.com

Kanak Kanti Baishya
Department of Mathematics, Kurseong College, Kurseong, Darjeeling,
West Bengal, India
E-mail: kanakkanti.kc@gmail.com

Ashoke Das
Department of Mathematics, Raiganj University, Uttar Dinajpur,
West Bengal, India
E-mail: ashoke.avik@gmail.com