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GENERIC SUBMANIFOLDS OF TRANS-SASAKIAN MANIFOLDS WITH CERTAIN VECTOR FIELDS

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Abstract. The object of the present paper is to deduce some important results on generic submanifolds and generic product of trans-Sasakian manifolds with concurrent vector fields.

1. Introduction

Nowadays submanifold theory is an interesting topic of research in differential geometry and plays an important role in the development of the subject. The research outcomes of this area are mainly used in applied mathematics and theoretical physics [13], [14], [19]. For instance, the method of invariant submanifolds is used in the study of non-linear autonomous systems [14]. Many authors worked on invariant submanifolds [6], [12], [21] and deduced a large number of significant results. Some of them worked on semi-invariant submanifolds which are generalizations of invariant and anti-invariant submanifolds both. The first study on semi-invariant submanifolds of Sasakian manifolds was done by Bejancu and Papaghiuc in [3]. Semi-invariant submanifolds have been studied by several authors [1], [2], [15]. This type of submanifolds help us to understand the beauty of the subject. Generic semi-invariant submanifolds are a special type of semi-invariant submanifolds which give us more attractive and impressive results. For generic submanifolds we refer [26], [27].

In 1985, Oubina [20] introduced a new class of almost contact manifolds namely trans-Sasakian manifolds of type (α, β), which can be considered as a generalization of Sasakian manifolds and Kenmotsu manifolds. Trans-Sasakian structures of type $(0, 0)$, $(0, \beta)$ and $(\alpha, 0)$ are cosymplectic [4], β-Kenmotsu and α-Sasakian respectively. In 2012, the first author and M. Sen worked on trans-Sasakian manifolds [21]. For details about trans-Sasakian manifolds we refer [8], [9], [10], [11], [18], [24].

There are several papers on Riemannian manifolds which admit concircular vector fields and also concurrent vector fields. Recently B. Y. Chen and S. W. Wei studied Riemannian submanifolds with concircular canonical vector fields in [5]. Many papers have been published on related topics [7], [16], [17], [22], [25]. In the paper [27], the authors discussed on generic submanifolds of Sasakian manifolds with concurrent vector fields. Keeping the works in mind we establish some interesting results on generic

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semi-invariant submanifolds of trans-Sasakian manifolds with concurrent vector fields. Also, most of the results of the paper [27] will be shown as corollaries of the present paper.

After introduction in Section 1, we give some basic definitions, notations and formulas of almost contact metric manifolds in Section 2. In Section 3, we deal with the generic submanifolds of trans-Sasakian manifolds with concurrent vector fields.

2. preliminaries

A vector field V on a Riemannian or pseudo-Riemannian manifold $\widetilde{M}$ is called concircular vector field if it satisfies

$$\widetilde{\nabla}_X V = fX$$

for any X tangent to $\widetilde{M}$, where $\widetilde{\nabla}$ is the Levi-Civita connection of $\widetilde{M}$ and f is a real valued function on $\widetilde{M}$. In particular, if $f = 1$, then the concircular vector field V is called a concurrent vector field and also if $f = 0$, then the concircular vector field V is called a parallel vector field.
Throughout this paper we consider the vector field V is concurrent and from definition it follows

$$\widetilde{\nabla}_X V = X \tag{1}$$

for any X tangent to $\widetilde{M}$.

Let, $\widetilde{M}$ be an almost contact metric manifold of dimension m that is a differentiable manifold endowed with an almost contact metric structure (ϕ, ξ, η, g). By definition, ϕ, ξ, η are tensor fields of type $(1,1)$, $(1,0)$, $(0,1)$ respectively and g be a Riemannian metric such that

$$\phi^2 X = -X + \eta(X)\xi, \qquad \eta(\xi) = 1 \tag{2}$$

$$g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y) \tag{3}$$

for all differentiable vector fields X, Y on $\widetilde{M}$. Then also we have

$$\phi\xi = 0, \qquad \eta(\phi X) = 0, \qquad \eta(X) = g(X, \xi). \tag{4}$$

Let, Φ be the fundamental 2-form defined by, $\Phi(X, Y) = g(X, \phi Y)$ for all differentiable vector fields X, Y on $\widetilde{M}$. An almost contact metric structure (ϕ, ξ, η, g) on $\widetilde{M}$ is called trans-Sasakian structure [20] if $(\widetilde{M} \otimes \mathbb{R}, J, G)$ belongs to the class W_4, where J is the almost complex structure on $\widetilde{M} \otimes \mathbb{R}$ defined by

$$J(X, h\frac{d}{dt}) = (\phi X - h\xi, \eta(X)\frac{d}{dt}),$$

for all vector fields X on $\widetilde{M}$ and smooth function h on $\widetilde{M} \otimes \mathbb{R}$ and G is the product metric on $\widetilde{M} \otimes \mathbb{R}$. This may be expressed by the following condition

$$\begin{aligned}(\widetilde{\nabla}_X \phi)Y &= \alpha(g(X, Y)\xi - \eta(Y)X) \\ &+ \beta(g(\phi X, Y)\xi - \eta(Y)\phi X)\end{aligned} \tag{5}$$

for smooth functions α and β on $\widetilde{M}$, where $\widetilde{\nabla}$ is the Levi-Civita connection on $\widetilde{M}$. We say that the trans-Sasakian structure is of type (α, β). From (5) it follows that

$$\widetilde{\nabla}_X \xi = -\alpha\phi X + \beta(X - \eta(X)\xi), \tag{6}$$

$$(\widetilde{\nabla}_X \eta)Y = -\alpha g(\phi X, Y) + \beta g(\phi X, \phi Y). \tag{7}$$

Let M be an n-dimensional submanifold of an m-dimensional trans-Sasakian manifold $\widetilde{M}$. Here, M is also an n-dimensional trans-Sasakian manifold [21]. Then we have [6]

$$\widetilde{\nabla}_X Y = \nabla_X Y + \sigma(X, Y), \tag{8}$$

$$\widetilde{\nabla}_X N = \nabla^{\perp}_X N - A_N X, \tag{9}$$

for all vector fields X, Y tangent to M and normal vector field N on M, where ∇ is the Riemannian connection on M defined by the induced metric g and $\nabla^{\perp}$ is the normal connection on $T^{\perp}M$ of M, σ is the second fundamental form of M and A_N is shape operator.

It is well known that the relation between second fundamental form σ and the shape operator A_N are given by

$$g(\sigma(X, Y), N) = g(A_N X, Y), \tag{10}$$

for any vector field X, Y tangent to M. Here we denote by the same symbol g the Riemannian metric induced by g on $\widetilde{M}$.

Let M be a real n-dimensional submanifold of m-dimensional trans-Sasakian manifold $\widetilde{M}$ such that ξ is tangent to M. Then, M is called a semi-invariant submanifold of $\widetilde{M}$, if there exist two orthogonal differentiable distributions D and $D^{\perp}$ on M satisfying following conditions

a. the distribution D is invariant by ϕ, i.e, $\phi(D_x) = D_x$ for all $x \in M$.

b. the distribution $D^{\perp}$ is anti-invariant by ϕ, i.e., $\phi(D^{\perp}_x) \subset T^{\perp}_x M$ for all $x \in M$.

Suppose we consider dimension of $\widetilde{M} = m$, dimension of $M = n$, dimension of the distribution $D = p$ and dimension of the distribution $D^{\perp} = q$. Now, if $q = m - n$ then the semi-invariant submanifold M is called generic semi-invariant submanifold of $\widetilde{M}$.

Let M be a semi-invariant submanifold of trans-Sasakian manifold $\widetilde{M}$. By using the definition of semi-invariant submanifold, the tangent bundle and normal bundle of a semi-invariant submanifold M have the orthogonal decomposition

$$TM = D \oplus D^{\perp} \oplus <\xi> \quad T^{\perp}M = \phi(D^{\perp}) \oplus \mu \quad \phi(\mu) = \mu. \tag{11}$$

where μ is the complementary subbundle orthogonal to $\phi(D^{\perp})$ in $\Gamma(T^{\perp}M)$ and $<\xi>$ is the 1-dimensional distribution which is spanned by ξ. Also, if the distributions $D \oplus \xi$ and $D^{\perp}$ are totally geodesic in M, then the submanifold M is called a semi-invariant product.

Furthermore, on a semi-invariant submanifold M of trans-Sasakian manifold $\widetilde{M}$, the following lemma holds.

Lemma 2.1. [27] *The following properties are equivalent:*
(i) M is semi-invariant product,
(ii) $A_{\phi(Z)}X = 0$,
(iii) the second fundamental form of M satisfies $\sigma(\phi X, Y) = \phi\sigma(X, Y)$, for any $X \in \Gamma(D), Z \in \Gamma(D^{\perp})$ and $Y \in \Gamma(TM)$.

Again, a semi-invariant product is called generic semi-invariant product if $m - n = q$ is satisfied. Then, we have $\mu = \{0\}$ in (11), Therefore, we get the following decomposition

$$TM = D \oplus D^{\perp} \oplus < \xi >, \qquad T^{\perp}M = \phi(D^{\perp}). \tag{12}$$

For a generic semi-invariant product, we can write

$$V = V^T + V^{\perp} + \phi(V^{\perp}) + f\xi, \tag{13}$$

where $V \in \Gamma(T\widetilde{M}), V^T \in \Gamma(D), V^{\perp} \in \Gamma(D^{\perp})$ and here we consider the function f is constant.

Let M be a semi-invariant submanifold of trans-Sasakian manifold $\widetilde{M}$. The semi-invariant submanifold M is called D-geodesic if it satisfies the following

$$\sigma(X, Y) = 0, \tag{14}$$

for all $X, Y \in \Gamma(D)$.

Similarly, for any $X, Y \in \Gamma(D^{\perp})$ if the relation (14) is satisfied on M, then the semi-invariant submanifold is called $D^{\perp}$-geodesic. Furthermore, for any $X \in \Gamma(D), Y \in \Gamma(D^{\perp})$ if the relation (14) is satisfied on M, the semi-invariant submanifold M is called $(D, D^{\perp})$-geodesic or mixed geodesic.
Again, for any $X, Y \in \Gamma(TM)$ if we get $\sigma(X, Y) = 0$, then the semi-invariant submanifold is called totally-geodesic.

Now the distribution D is called parallel with respect to $\widetilde{\nabla}$, if it satisfies $\widetilde{\nabla}_X Y \in \Gamma(D)$, where $\widetilde{\nabla}$ is the Levi-Civita connection of $\widetilde{M}$, for any $X \in \Gamma(T\widetilde{M})$ and $Y \in \Gamma(D)$.

3. Generic submanifolds of trans-Sasakian manifolds with concurrent vector fields

In this section first we discuss some important lemma and proposition then we deduce the main results on this topic.

Lemma 3.1. *Let M be a generic submanifold of a trans-Sasakian manifold $\widetilde{M}$ with concurrent vector field V. Then we get the following*

$$\nabla_X V^T + \nabla_X V^{\perp} - A_{\phi V^{\perp}} X = (1 - f\beta)X + f\alpha\phi X,$$

$$\sigma(X,V^T)+\sigma(X,V^{\perp})+\nabla^{\perp}_X\phi V^{\perp}=0,$$

for $X\in\Gamma(D)$ and others are usual notations discussed in Section 2.

Proof. Since V is a concurrent vector field then from (13) we get

$$\widetilde{\nabla}_X V^T+\widetilde{\nabla}_X V^{\perp}+\widetilde{\nabla}_X\phi V^{\perp}+\widetilde{\nabla}_X f\xi=X.$$

Now from (8) and (9) we get,

$$\nabla_X V^T+\sigma(X,V^T)+\nabla_X V^{\perp}+\sigma(X,V^{\perp})+\nabla^{\perp}_X\phi V^{\perp}-A_{\phi V^{\perp}}X+f\widetilde{\nabla}_X\xi=X.$$

Putting the value of $\widetilde{\nabla}_X\xi$ from (6) and compairing the tangential and normal components we have the required results. □

PROPOSITION 3.2. *Let M be a generic submanifold of trans-Sasakian manifold $\widetilde{M}$ with concurrent vector field V. Then we get*

$$\begin{aligned}\nabla_X\phi V^T \;&-\; \nabla_X V^{\perp}-A_{\phi V^{\perp}}X=\alpha g(X,V^T)\xi\\ &+\;\beta g(\phi X,V^T)\xi-\alpha fX+(1+\beta f)\phi X,\end{aligned}$$

$$\sigma(X,\phi V^T)-\sigma(X,V^{\perp})+\nabla^{\perp}_X\phi V^{\perp}=0.$$

Proof. Since $\widetilde{M}$ is a trans-Sasakian manifold then putting $Y=V$ in (5), we get

$$\widetilde{\nabla}_X\phi V-\phi X=\alpha[g(X,V)\xi-\eta(V)X]+\beta[g(\phi X,V)\xi-\eta(V)\phi X].$$

Now from (13), we have

$$\begin{aligned}\widetilde{\nabla}_X\phi V^T \;&+\; \widetilde{\nabla}_X\phi V^{\perp}+\widetilde{\nabla}_X\phi^2V^{\perp}-\phi X\\ &=\; \alpha[g(X,V^T)\xi-fX]+\beta[g(\phi X,V^T)\xi-f\phi X].\end{aligned}$$

Now applying (8), (9) and (2) in the above equation

$$\begin{aligned}\nabla_X\phi V^T \;&+\; \sigma(X,\phi V^T)+\nabla^{\perp}_X\phi V^{\perp}\\ &-\; A_{\phi V^{\perp}}X-\nabla_X V^{\perp}-\sigma(X,V^{\perp})-\phi X\\ &=\; \alpha[g(X,V^T)\xi-fX]+\beta[g(\phi X,V^T)\xi-f\phi X].\end{aligned}$$

From the tangential and normal components of above equation, we have the required results. Hence the proposition is proved. □

PROPOSITION 3.3. *Let M be a generic submanifold of trans-Sasakian manifold $\widetilde{M}$ with concurrent vector field V and M be D-geodesic. Then we get*

$$\nabla_X\phi V^T-\phi(\nabla_X V^T)=\alpha g(X,V^T)\xi+\beta g(\phi X,V^T)\xi.$$

Proof. Since $\widetilde{M}$ is a trans-Sasakian manifold then putting $Y=V^T$ in (5), we get

$$\begin{aligned}\widetilde{\nabla}_X\phi V^T-\phi(\widetilde{\nabla}_X V^T) \;&=\; \alpha[g(X,V^T)\xi-\eta(V^T)X]\\ &+\; \beta[g(\phi X,V^T)\xi-\eta(V^T)\phi X].\end{aligned}$$

Now applying (8) in the above equation

$$\begin{aligned}\nabla_X \phi V^T + \sigma(X, \phi V^T) &- \phi(\nabla_X V^T + \sigma(X, V^T)) \\ &= \alpha[g(X, V^T)\xi] + \beta[g(\phi X, V^T)\xi].\end{aligned}$$

Since, M is D-geodesic, then we get

$$\nabla_X \phi V^T - \phi(\nabla_X V^T) = \alpha g(X, V^T)\xi + \beta g(\phi X, V^T)\xi. \tag{15}$$

Hence the proposition is proved.

□

Proposition 3.4. *Let M be a generic submanifold of trans-Sasakian manifold $\widetilde{M}$ with concurrent vector field V. And also M is mixed-geodesic. Then we get*

$$\phi(\nabla_X V^\perp) = -A_{\phi V^\perp} X,$$
$$\nabla_X^\perp \phi V^\perp = 0.$$

Proof. Since $\widetilde{M}$ is a trans-Sasakian manifold then putting $Y = V^\perp$ in (5), we get

$$\begin{aligned}\widetilde{\nabla}_X \phi V^\perp - \phi(\widetilde{\nabla}_X V^\perp) &= \alpha[g(X, V^\perp)\xi - \eta(V^\perp)X] \\ &+ \beta[g(\phi X, V^\perp)\xi - \eta(V^\perp)\phi X].\end{aligned}$$

Now applying (8) and (9) in the above equation

$$\nabla_X^\perp \phi V^\perp - A_{\phi V^\perp} X - \phi(\nabla_X V^\perp + \sigma(X, V^\perp)) = 0.$$

Since, M is mixed-geodesic, then we get

$$\nabla_X^\perp \phi V^\perp - A_{\phi V^\perp} X - \phi(\nabla_X V^\perp) = 0.$$

Now, comparing the tangential and normal components, we have

$$\begin{aligned}\phi(\nabla_X V^\perp) &= -A_{\phi V^\perp} X, \\ \nabla_X^\perp \phi V^\perp &= 0.\end{aligned} \tag{16}$$

Hence the proposition is proved.

□

Theorem 3.5. *Let M be a generic submanifold of trans-Sasakian manifold $\widetilde{M}$ with concurrent vector field V. If the submanifold M is D-geodesic then V^T on D is never concurrent.*

Proof. From Proposition 3.2 we get

$$\begin{aligned}\nabla_X \phi V^T - \nabla_X V^\perp - A_{\phi V^\perp} X = \alpha g(X, V^T)\xi &+ \beta g(\phi X, V^T)\xi \\ &- \alpha f X + (1 + \beta f)\phi X.\end{aligned} \tag{17}$$

Since, M is D-geodesic, then from (15) we get

$$\nabla_X \phi V^T = \phi(\nabla_X V^T) + \alpha g(X, V^T)\xi + \beta g(\phi X, V^T)\xi. \tag{18}$$

Combining (17) and (18)

$$\phi(\nabla_X V^T) - \nabla_X V^\perp - A_{\phi V^\perp} X = -\alpha f X + (1+\beta f)\phi X. \tag{19}$$

From Lemma 3.1 we have the following

$$\nabla_X V^T + \nabla_X V^\perp - A_{\phi V^\perp} X = (1 - f\beta)X + f\alpha\phi X. \tag{20}$$

Adding (19) and (20),

$$\begin{aligned} \phi(\nabla_X V^T) + \nabla_X V^T - 2A_{\phi V^\perp} X &= (1 - f\beta - f\alpha)X \\ &+ (1 + f\beta + f\alpha)\phi X. \end{aligned} \tag{21}$$

If possible let V^T on D is concurrent then from the definition of concurrent vector field $\nabla_X V^T = X$ and then from the equation (21)

$$A_{\phi V^\perp} X = \frac{1}{2}(f\beta + f\alpha)X - \frac{1}{2}(f\beta + f\alpha)\phi X. \tag{22}$$

From (10), we have

$$g(\sigma(X,Y), \phi V^\perp) = g(A_{\phi V^\perp} X, Y), \tag{23}$$

for any $Y \in \Gamma(D)$.
Combining (22) and (23),

$$\begin{aligned} g(\sigma(X,Y), \phi V^\perp) &= \frac{1}{2}(f\beta + f\alpha)g(X,Y) \\ &- \frac{1}{2}(f\beta + f\alpha)g(\phi X, Y). \end{aligned} \tag{24}$$

Interchanging X and Y in (24), we have

$$\begin{aligned} g(\sigma(Y,X), \phi V^\perp) &= \frac{1}{2}(f\beta + f\alpha)g(Y,X) \\ &- \frac{1}{2}(f\beta + f\alpha)g(\phi Y, X). \end{aligned} \tag{25}$$

Now from (24) and (25), we get the following equality

$$g(\sigma(X,Y), \phi V^\perp) = \frac{1}{2}(f\beta + f\alpha)g(X,Y)$$

for $X, Y \in \Gamma(D)$.
Since M is D-geodesic, so it is a contradiction. Therefore our assumption is wrong. Then, V^T on D is never concurrent.

□

Now the following question arises:
When do V^T on D will be concurrent ? The next theorem gives the answer.

Theorem 3.6. *Let M be a generic semi-invariant product of a trans-Sasakian manifold $\widetilde{M}$ with concurrent vector field V and also let $\nabla_X V^T \in \Gamma(D)$ for any $X \in \Gamma(D)$. Then V^T on D is concurrent if the function f vanishes identically or the trans-Sasakian structure is of type $(0,0)$.*

Proof. From Proposition 3.2 we get

$$\begin{aligned} \nabla_X \phi V^T - \nabla_X V^\perp - A_{\phi V^\perp} X &= \alpha g(X, V^T)\xi + \beta g(\phi X, V^T)\xi \\ &- \alpha f X + (1+\beta f)\phi X. \end{aligned} \tag{26}$$

Since, M is semi-invariant product, then from (15) we get

$$\nabla_X \phi V^T = \phi(\nabla_X V^T) + \alpha g(X, V^T)\xi + \beta g(\phi X, V^T)\xi. \tag{27}$$

Combining (26) and (27) and using Lemma 2.1, we have

$$\phi(\nabla_X V^T) - \nabla_X V^\perp = -\alpha f X + (1+\beta f)\phi X. \tag{28}$$

Using Lemma 2.1 in Lemma 3.1, we have the following

$$\nabla_X V^T + \nabla_X V^\perp = (1 - f\beta)X + f\alpha\phi X. \tag{29}$$

Adding (28) and (29),

$$\begin{aligned} \phi(\nabla_X V^T) + \nabla_X V^T &= (1 - f\beta - f\alpha)X \\ &+ (1 + f\beta + f\alpha)\phi X. \end{aligned} \tag{30}$$

Applying ϕ on both sides of (30), we have the following

$$\begin{aligned} \phi(\nabla_X V^T) - \nabla_X V^T + \eta(\nabla_X V^T)\xi &= (1 - f\beta - f\alpha)\phi X \\ &- (1 + f\beta + f\alpha)X. \end{aligned} \tag{31}$$

Since, $\nabla_X V^T \in \Gamma(D)$ for any $X \in \Gamma(D)$ so, we have from(31)

$$\begin{aligned} \phi(\nabla_X V^T) - \nabla_X V^T &= (1 - f\beta - f\alpha)\phi X \\ &- (1 + f\beta + f\alpha)X. \end{aligned} \tag{32}$$

Now subtracting (32) from (30) we get

$$\nabla_X V^T = X + (f\beta + f\alpha)\phi X. \tag{33}$$

Now if f is identically 0 or the trans-Sasakian structure $\widetilde{M}$ is of type $(0,0)$, then clearly we have

$$\nabla_X V^T = X.$$

Therefore, the vector field V^T is concurrent. □

In the next theorem we study $V^\perp$ on M.

Theorem 3.7. *Let M be a generic semi-invariant product of a trans-Sasakian manifold $\widetilde{M}$ with concurrent vector field V. If the submanifold M is mixed-geodesic and $\nabla_X V^\perp \in \Gamma(D^\perp)$ for any $X \in \Gamma(D)$ then $V^\perp$ is a parallel vector field.*

Proof. From Proposition 3.4 we get
$$\phi(\nabla_X V^\perp) = -A_{\phi V^\perp} X.$$
Using Lemma 2.1,
$$\phi(\nabla_X V^\perp) = 0.$$
Applying ϕ on both sides of this we have
$$-\nabla_X V^\perp + \eta(\nabla_X V^\perp)\xi = 0.$$
Now from the given condition we get
$$\nabla_X V^\perp = 0.$$
Therefore $V^\perp$ is a parallel vector field. □

Corollary 3.8. *Let M be a generic semi-invariant product of Sasakian manifold $\widetilde{M}$ with concurrent vector field V and also let $\nabla_X V^T \in \Gamma(D)$ for any $X \in \Gamma(D)$. Then V^T on D is concurrent if the function f vanishes identically.*

Corollary 3.9. *Let M be a generic semi-invariant product of Kenmotsu manifold $\widetilde{M}$ with concurrent vector field V and also let $\nabla_X V^T \in \Gamma(D)$ for any $X \in \Gamma(D)$. Then V^T on D is concurrent if the function f vanishes identically.*

Corollary 3.10. *Let M be a generic semi-invariant product of a Cosymplectic manifold $\widetilde{M}$ with concurrent vector field V and also let $\nabla_X V^T \in \Gamma(D)$ for any $X \in \Gamma(D)$. Then V^T on D is concurrent.*

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