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ON 4-TOTAL MEAN CORDIAL GRAPHS

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ABSTRACT. Let G be a graph. Let $f : V(G) \rightarrow \{0, 1, \ldots, k-1\}$ be a function where $k \in \mathbb{N}$ and $k > 1$. For each edge uv, assign the label $f(uv) = \left\lceil \frac{f(u)+f(v)}{2} \right\rceil$. f is called k-total mean cordial labeling of G if $\left|t_{mf}(i) - t_{mf}(j)\right| \leq 1$, for all $i, j \in \{0, 1, \ldots, k-1\}$, where $t_{mf}(x)$ denotes the total number of vertices and edges labelled with x, $x \in \{0, 1, \ldots, k-1\}$. A graph with admit a k-total mean cordial labeling is called k-total mean cordial graph.

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1. Introduction

Graphs in this paper are finite, simple and undirected. Ponraj et al. [3] have been introduced the concept of k-total mean cordial labeling and investigate the k-total mean cordial labeling of certain graphs and investigate the 4-total mean cordial labeling of square of path, double comb, subdivision of star, subdivision of bistar in [4, 5]. In this paper we investigate the 4-total mean cordial labeling behaviour of fan, wheel, jelly fish, jewel graph, ladder, triangular snake. Let x be any real number. Then $\lceil x \rceil$ stands for the smallest integer greater than or equal to x. Terms are not defined here follow from Harary[2] and Gallian[1].

2. Preliminaries

Definition 2.1. The graph $F_n = P_n + K_1$ is called a *Fan graph* where P_n is a path.

Definition 2.2. The graph $W_n = C_n + K_1$ is called a *wheel.*

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Definition 2.3. The *jelly fish graph* $J(m,n)$ is obtained from a cycle C_4 : $uxvwu$ by joining x and w with an edge and appending m pendent edges to u and n pendent edges to v.

Definition 2.4. The *jewel graph* J_n is the graph with the vertex set $V(J_n) = \{u, v, x, y, u_i : 1 \leq i \leq n\}$ and the edge set
$E(J_n) = \{ux, uy, xy, xv, yv, uu_i, vu_i : 1 \leq i \leq n\}$.

Definition 2.5. The *shadow graph* $D_2(G)$ of a connected graph G is obtained by taking two copies of G, say G_1 and G_2. Join each vertex u_1 in G_1 to the the neighbours of corresponding vertex u_2 in G_2.

Definition 2.6. The graph $L_n = P_n + K_2$ is called a *ladder*.

Definition 2.7. The *triangular snake* T_n is obtained from the path P_n : u_1 u_2 ...u_n with $V(T_n) = V(P_n) \cup \{v_i : 1 \leq i \leq n-1\}$ and edge set $E(T_n) = E(P_n) \cup \{u_i v_i, u_{i+1} v_i : 1 \leq i \leq n-1\}$.

3. Main results

Theorem 3.1. The fan graph F_n is 4-total mean cordial for all n.

Proof. Let P_n be the path u_1 u_2...u_n. Let $V(F_n) = \{u\} \cup V(P_n)$ and $E(F_n) = \{uu_i : 1 \leq i \leq n\} \cup E(P_n)$. Clearly $|V(F_n)| + |E(F_n)| = 3n$.
Assign the label 2 to the vertex u.
Case 1. $n \equiv 0 \pmod 4$.
Consider the path vertices u_1 u_2...u_n. Assign the labels 0, 0, 2, 3 respectively to the vertices u_1, u_2, u_3, u_4. Next assign the labels 0, 0, 2, 3 to the vertices u_5, u_6, u_7, u_8 respectively. We now assign the labels 0, 0, 2, 3 respectively to the vertices u_9, u_{10}, u_{11}, u_{12}. Proceeding like this until reach the vertex u_n. Obviously the vertex u_n receive the label 3.
Case 2. $n \equiv 1 \pmod 4$.
As in Case 1 assign the label to the vertices u_i $(1 \leq i \leq n-1)$. Finally assign the label 0 to the vertex u_n.
Case 3. $n \equiv 2 \pmod 4$.
Label the vertices u_i $(1 \leq i \leq n-1)$ as in Case 2. Next assign the label 3 to the vertex u_n.
Case 4. $n \equiv 3 \pmod 4$.
In this case assign the label for the vertices u_i $(1 \leq i \leq n-1)$ as in Case 3. We now assign the label 0 to the vertex u_n.
This vertex labeling f is a 4-total mean cordial labeling of F_n follows from the Tabel 1

Order of F_n	$t_{mf}(0)$	$t_{mf}(1)$	$t_{mf}(2)$	$t_{mf}(3)$
$n \equiv 0 \pmod 4$	$\frac{3n}{4}$	$\frac{3n}{4}$	$\frac{3n}{4}$	$\frac{3n}{4}$
$n \equiv 1 \pmod 4$	$\frac{3n+1}{4}$	$\frac{3n+1}{4}$	$\frac{3n+1}{4}$	$\frac{3n-3}{4}$
$n \equiv 2 \pmod 4$	$\frac{3n-2}{4}$	$\frac{3n-2}{4}$	$\frac{3n+2}{4}$	$\frac{3n+2}{4}$
$n \equiv 3 \pmod 4$	$\frac{3n-1}{4}$	$\frac{3n-1}{4}$	$\frac{3n+3}{4}$	$\frac{3n-1}{4}$

Table 1:

□

Corollary 3.2. The wheel W_n is 4-total mean cordial if $n \equiv 0, 2, 3 \pmod 4$.

Proof. Let $C_n : u_1u_2u_3 \dots u_nu_1$ be the cycle. Let $V(W_n) = V(C_n) \cup \{u\}$ and $E(W_n) = E(C_n) \cup \{uu_i : 1 \leq i \leq n\}$. Obviously $|V(W_n)| + |E(W_n)| = 3n + 1$. Assign the label 2 to the central vertex u.
Case 1. $n \equiv 0 \pmod 4$.
Clearly the vertex labeling in Case 1 of Theorem 3.1 is also a 4-total mean cordial labeling of W_n.
Case 2. $n \equiv 2 \pmod 4$.
Assign the label to the vertices $u_1, u_2, u_3, \dots, u_{n-2}$ as in Case 1 of Theorem 3.1. Finally assign the labels $2, 0$ to the vertices u_{n-1}, u_n.
Case 3. $n \equiv 3 \pmod 4$.
Obviously the vertex labeling in Case 4 of Theorem 3.1 is also a 4-total mean cordial labeling of W_n. □

Theorem 3.3. The Jelly fish $J(n, n)$ is 4-total mean cordial for all values of n.

Proof. Take the vertex set and edge set as in Definition 2.3. It is easy to verify that $|V(J(n,n))| + |E(J(n,n))| = 4n + 9$.
Case 1. n is even.
Assign the labels 0, 2, 1, 3 respectively to the vertices u, v, x, w.
Now we consider the pendent vertices u_1, u_2,..., u_n. Assign the label 0 to the $\frac{n}{2}$ vertices u_1, u_2,..., $u_{\frac{n}{2}}$. Next assign the label 1 to the $\frac{n}{2}$ vertices $u_{\frac{n+2}{2}}$, $u_{\frac{n+4}{2}}$,..., u_n. We now move to the pendent vertices v_1, v_2,..., v_n. Assign the label 3 to the $\frac{n}{2}$ vertices v_1, v_2,..., $v_{\frac{n}{2}}$. We now assign the label 2 to the $\frac{n-2}{2}$ vertices $v_{\frac{n+2}{2}}$, $v_{\frac{n+4}{2}}$,..., v_{n-1} and finally assign the label 0 to the vertex v_n.
Case 2. n is odd.
Now assign the labels 0, 2, 3, 0 respectively to the vertices u, v, x, w.
Assign the label 0 to the $\frac{n-1}{2}$ vertices u_1, u_2,..., $u_{\frac{n-1}{2}}$. We now assign the label 1 to the $\frac{n+1}{2}$ vertices $u_{\frac{n+1}{2}}$, $u_{\frac{n+3}{2}}$,..., u_n. Then we assign the label 2 to the $\frac{n+1}{2}$ vertices v_1, v_2,..., $v_{\frac{n+1}{2}}$ and finally assign the label 3 to the next $\frac{n-1}{2}$ vertices $v_{\frac{n+3}{2}}$, $v_{\frac{n+5}{2}}$,..., v_n.
Thus this vertex labeling f is 4-total mean cordial labeling of jelly fish follows from the Tabel 2

Nature of n	$t_{mf}(0)$	$t_{mf}(1)$	$t_{mf}(2)$	$t_{mf}(3)$
n is even	$n+2$	$n+3$	$n+2$	$n+2$
n is odd	$n+2$	$n+2$	$n+2$	$n+3$

Table 2:

□

Theorem 3.4. The jewel graph J_n is 4-total mean cordial if and only if $n \equiv 0, 1, 2, 3, 4, 6, 7 \pmod 8$.

Proof. Take the vertex set and edge set as in Definition 2.4. It is easy to show that $|V(J_n)| + |E(J_n)| = 3n + 9$.
Case 1. $n \equiv 0 \pmod 8$. Let $n = r$, $r \in N$.
Assign the labels 0, 2, 1, 0 respectively to the vertices u, v, x, y.
Consider the vertices u_1, u_2, ..., u_n. Assign the label 0 to the $3r$ vertices u_1, u_2, ..., u_{3r}. Next assign the label 1 to the $r-1$ vertices u_{3r+1}, u_{3r+2}, ..., u_{4r-1}. We now assign the label 2 to the r vertices u_{4r}, u_{4r+1}, ..., u_{5r-1}. Finally assign the label 3 to the $3r+1$ vertices u_{5r}, u_{5r+1}, ..., u_{8r}.
Case 2. $n \equiv 1 \pmod 8$.
Let $n = 8r + 1$, $r \in N$.
We now assign the labels 0, 2, 2, 3 respectively to the vertices u, v, x, y.
Then we assign the label 0 to the $3r+1$ vertices u_1, u_2,..., u_{3r+1}. We now assign the label 1 to the $r+1$ vertices u_{3r+2}, u_{3r+3}, ..., u_{4r+2}. Now assign the label 2 to the $r-1$ vertices u_{4r+3}, u_{4r+4}, ..., u_{5r+1} and assign the label 3 to the $3r$ vertices u_{5r+2}, u_{5r+3}, ..., u_{8r+1}.
Case 3. $n \equiv 2 \pmod 8$.
Let $n = 8r + 2$, $r \in N$.
Now assign the labels 0, 2, 1, 0 respectively to the vertices u, v, x, y.
Assign the label 0 to the $3r$ vertices u_1, u_2, ..., u_{3r}. Next assign the label 1 to the r vertices u_{3r+1}, u_{3r+2}, ..., u_{4r}. We now assign the label 2 to the r vertices u_{4r+1}, u_{4r+2}, ..., u_{5r} and finally assign the label 3 to the $3r+2$ vertices u_{5r+1}, u_{5r+2},..., u_{8r+2}.
Case 4. $n \equiv 3 \pmod 8$.
Let $n = 8r + 3$, $r \in N$.
In this case, assign the labels 0, 2, 1, 0 respectively to the vertices u, v, x, y.
Next assign the label 0 to the $3r+1$ vertices u_1, u_2,..., u_{3r+1}. Now assign the label 1 to the $r-1$ vertices u_{3r+2}, u_{3r+3},..., u_{4r}. We now assign the label 2 to the $r+1$ vertices u_{4r+1}, u_{4r+2},..., u_{5r+1}. Finally we assign the label 3 to the $3r+2$ vertices u_{5r+2}, u_{5r+3},..., u_{8r+3}.
Case 5. $n \equiv 4 \pmod 8$.
Let $n = 8r + 4$, $r \geq 0$.
Assign the labels 0, 2, 1, 0 respectively to the vertices u, v, x, y.

Then assign the label 0 to the $3r+1$ vertices u_1, u_2, ..., u_{3r+1}. Next assign the label 1 to the r vertices u_{3r+2}, u_{3r+3}, ..., u_{4r+1}. We now assign the label 2 to the r vertices u_{4r+2}, u_{4r+3}, ..., u_{5r+1}. Finally assign the label 3 to the $3r+3$ vertices u_{5r+2}, u_{5r+3},..., u_{8r+4}.

Case 6. $n \equiv 6 \pmod 8$.

Let $n = 8r+6$, $r \geq 0$.

Now assign the labels 0, 2, 1, 0 respectively to the vertices u, v, x, y.

Next assign the label 0 to the $3r+2$ vertices u_1, u_2, ..., u_{3r+2}. Then we assign the label 1 to the r vertices u_{3r+3}, u_{3r+4},..., u_{4r+2}. We now assign the label 2 to the $r+1$ vertices u_{4r+3}, u_{4r+4}, ..., u_{5r+3}. Finally we assign the label 3 to the $3r+3$ vertices u_{5r+4}, u_{5r+5}, ..., u_{8r+6}.

Case 7. $n \equiv 7 \pmod 8$.

Let $n = 8r+7$, $r \geq 0$.

We assign the labels 0, 2, 1, 0 respectively to the vertices u, v, x, y.

Assign the label 0 to the $3r+2$ vertices u_1, u_2, ..., u_{3r+2}. Next assign the label 1 to the r vertices u_{3r+3}, u_{3r+4}, ..., u_{4r+2}. We now assign the label 2 to the $r+1$ vertices u_{4r+3}, u_{4r+4}, ..., u_{5r+3}. Finally assign the label 3 to the $3r+4$ vertices u_{5r+4}, u_{5r+5}, ..., u_{8r+7}.

This vertex labeling f is 4-total mean cordial labeling follows from the Tabel 3

Nature of n	$t_{mf}(0)$	$t_{mf}(1)$	$t_{mf}(2)$	$t_{mf}(3)$
$n = 8r$	$6r+3$	$6r+2$	$6r+2$	$6r+2$
$n = 8r+1$	$6r+3$	$6r+3$	$6r+3$	$6r+3$
$n = 8r+2$	$6r+3$	$6r+4$	$6r+4$	$6r+4$
$n = 8r+3$	$6r+5$	$6r+4$	$6r+5$	$6r+4$
$n = 8r+4$	$6r+5$	$6r+5$	$6r+5$	$6r+6$
$n = 8r+6$	$6r+7$	$6r+7$	$6r+7$	$6r+6$
$n = 8t+7$	$6r+7$	$6r+7$	$6r+8$	$6r+8$

Table 3:

Case 8. $n \equiv 5 \pmod 8$.

Let $n = 8r+5$, $r \geq 0$.

Suppose f is a 4-total mean cordial labeling of J_n. $\Rightarrow t_{mf}(0) = t_{mf}(1) = t_{mf}(2) = t_{mf}(3) = 6r+6$.

Subcase (i). $f(u) = f(v) = 0$.

Now $t_{mf}(3) = 6r+6$.

$\Rightarrow$ atleast $\frac{3n+9}{4}$ vertices of u_1, u_2, ..., u_n receive the label 3. In this case, $t_{mf}(2) > 6r+6$, a contradiction.

Subcase (ii). $f(u) = f(v) = 1$.

Then atleast $\frac{3n+9}{4}$ vertices of u_1, u_2, ..., u_n receive the label 0. But $t_{mf}(1) > 6r+6$, a contradiction.

Subcase (iii). $f(u) = f(v) = 2$.

This implies atleast $\frac{3n+9}{4}$ vertices of u_1, u_2, ..., u_n receive the label 0. But $t_{mf}(1) > 6r+6$, a contradiction.
Subcase (iv). $f(u) = f(v) = 3$.
Then atleast $\frac{3n+9}{4}$ vertices of u_1, u_2, ..., u_n receive the label 0. But $t_{mf}(2) > 6r+6$, a contradiction.
Subcase (v). $f(u) = 0$, $f(v) = 1$.
$\Rightarrow$ atleast $\frac{3n+9}{4}$ vertices of u_1, u_2, ..., u_n receive the label 3. In this case, $t_{mf}(2) > 6r+6$, a contradiction.
Subcase (vi). $f(u) = 0$, $f(v) = 2$.
Clearly $t_{mf}(0) < 6r+6$, a contradiction.
Subcase (vii). $f(u) = 0$, $f(v) = 3$.
In this case, $t_{mf}(0) < 6r+6$, a contradiction.
Subcase (viii). $f(u) = 1$, $f(v) = 0$.
Similar to Subcase (v).
Subcase (ix). $f(u) = 1$, $f(v) = 2$.
Then atleast $\frac{3n+9}{4}$ vertices of u_1, u_2, ..., u_n receive the label 0. But $t_{mf}(1) > 6r+6$, a contradiction.
Subcase (x). $f(u) = 1$, $f(v) = 3$.
$\Rightarrow$ atleast $\frac{3n+9}{4}$ vertices of u_1, u_2, ..., u_n receive the label 0. In this case, $t_{mf}(3) < 6r+6$, a contradiction.
Subcase (xi). $f(u) = 2$, $f(v) = 0$.
Similar to Subcase (vi).
Subcase (xii). $f(u) = 2$, $f(v) = 1$.
Similar to Subcase (xi).
Subcase (xiii). $f(u) = 2$, $f(v) = 3$.
Then atleast $\frac{3n+9}{4}$ vertices of u_1, u_2, ..., u_n receive the label 0. But $t_{mf}(3) < 6r+6$, a contradiction.
Subcase (xiv). $f(u) = 3$, $f(v) = 0$.
Similar to Subcase (vii).
Subcase (xv). $f(u) = 3$, $f(v) = 1$.
Similar to Subcase (x).
Subcase (xvi). $f(u) = 3$, $f(v) = 2$.
Similar to Subcase (xiii). □

Theorem 3.5. The graph $D_2(B_{n,n})$ is 4-total mean cordial foa all values of n.

Proof. Let $u_1, u_2, \ldots, u_n, v_1, v_2, \ldots, v_n$ be the pendant vertices and u, v be the central vertices of $B_{n,n}$. Let $V(D_2(B_{n,n})) = \{u, v, x, y\} \cup \{u_i, v_i, x_i, y_i : 1 \leq i \leq n\}$ and $E(D_2(B_{n,n})) = \{uv, xv, uy, xy\} \cup \{uu_i, ux_i, xx_i, xu_i, vv_i, vy_i, yv_i, yy_i : 1 \leq i \leq n\}$. Obviously $|V(D_2(B_{n,n}))| + |E(D_2(B_{n,n}))| = 12n + 8$.
Assign the labels 0, 2, 2, 3 respectively to the vertices u, v, x, y.
Case 1. n is even.
Let $n = 2r$, $r \in N$.
Consider the vertices u_1, u_2, ..., u_{2r}. Assign the label 0 to the $2r$ vertices u_1, u_2, ..., u_{2r}. Now we consider the vertices x_1, x_2, ..., x_{2r}. Assign the label 0 to

the r vertices x_1, x_2, ..., x_r. Next assign the label 1 to the vertex x_{r+1}. Now we assign the label 2 to the $r-1$ vertices x_{r+2}, x_{r+3}, ..., x_{2r}. We now move to the vertices v_1, v_2, ..., v_{2r}. Assign the label 0 to the vertex v_1. Next assign the label 1 to the $2r-1$ vertices v_2, v_3, ..., v_{2r}. Finally consider the vertices y_1, y_2, ..., y_{2r}. Assign the label 3 to the $2r$ vertices y_1, y_2, ..., y_{2r}.
Case 2. n is odd.
Let $n = 2r+1$, $r \in N$.
Assign the label 0 to the $2r+1$ vertices u_1, u_2, ..., u_{2r+1}. Next assign the label 0 to the r vertices x_1, x_2, ..., x_r. Next assign the label 1 to the vertex x_{r+1}. We now assign the label 2 to the r vertices x_{r+2}, x_{r+3}, ..., x_{2r+1}. Assign the label 0 to the vertices v_1, v_2. Next assign the label 1 to the $2r-1$ vertices v_3, v_4, ..., v_{2r+1}. Finally assign the label 3 to the $2r+1$ vertices y_1, y_2, ..., y_{2r+1}.
Note that this vertex labeling f is 4-total mean cordial labeling follows from the Tabel 4

Nature of n	$t_{mf}(0)$	$t_{mf}(1)$	$t_{mf}(2)$	$t_{mf}(3)$
$n = 2r$	$6r+2$	$6r+2$	$6r+2$	$6r+2$
$n = 2r+1$	$6r+5$	$6r+5$	$6r+5$	$6r+5$

Table 4:

□

Theorem 3.6. The ladder L_n is 4-total mean cordial for all n.

Proof. Let $V(L_n) = \{u_i, v_i : 1 \le i \le n\}$ and
$E(L_n) = \{u_iu_{i+1}, v_iv_{i+1} : 1 \le i \le n-1\} \cup \{u_iv_i : 1 \le i \le n\}$.
Clearly, $|V(L_n)| + |E(L_n)| = 5n-2$.
Case 1. $n \equiv 0 \pmod 4$.
Let $n = 4r$, $r \ge 2$. Consider the vertices u_1, u_2, $\ldots$, u_n. Assign the label 0 to the r vertices $u_1, u_2, \ldots, u_r$. Next assign the label 1 to the r vertices u_{r+1}, u_{r+2}, $\ldots$, u_{2r}. We now assign the label 2 to the r vertices u_{2r+1}, u_{2r+2}, $\ldots$, u_{3r} and assign the label 3 to the r vertices u_{3r+1}, u_{3r+2}, $\ldots$, u_{4r}. Consider the vertices v_1, v_2, $\ldots$, v_n. Assign the label 0 to the r vertices v_1, v_2, $\ldots$, v_r. Next assign the label 1 to the $r-1$ vertices v_{r+1}, v_{r+2}, $\ldots$, v_{2r-1}. Now we assign the label 0 to the vertex v_{2r}. We now assign the label 2 to the r vertices v_{2r+1}, v_{2r+2}, $\ldots$, v_{3r} and finally we assign the label 3 to the r vertices v_{3r+1}, v_{3r+2}, $\ldots$, v_{4r}.
Case 2. $n \equiv 1 \pmod 4$.
Let $n = 4r+1$, $r \ge 2$. As in Case 1 assign the label to the vertices u_i $(1 \le i \le 4r)$ and v_i $(1 \le i \le 4r)$. Finally assign the labels 0, 2 to the vertices u_{4r+1}, v_{4r+1}.
Case 3. $n \equiv 2 \pmod 4$.
Let $n = 4r+2$, $r \ge 2$. Label the vertices u_i $(1 \le i \le 4r)$ and v_i $(1 \le i \le 4r)$ as in Case 1. Next assign the labels 2, 0 to the vertices u_{4r+1}, u_{4r+2} and finally assign the labels 2, 0 to the vertices v_{4r+1}, v_{4r+2}.
Case 4. $n \equiv 3 \pmod 4$.

Let $n = 4r+3$, $r \geq 2$. In this case assign the label for the vertices u_i $(1 \leq i \leq 4r)$ and v_i $(1 \leq i \leq 4r)$ as in Case 1. We now assign the labels 2, 0, 0 to the vertices u_{4r+1}, u_{4r+2}, u_{4r+3} and assign the labels 0, 2, 3 to the vertices v_{4r+1}, v_{4r+2}, v_{4r+3}.
This vertex labeling f is a 4-total mean cordial labeling of L_n follows from the Tabel 5

Order of L_n	$t_{mf}(0)$	$t_{mf}(1)$	$t_{mf}(2)$	$t_{mf}(3)$
$n = 4r$	$5r-1$	$5r$	$5r-1$	$5r$
$n = 4r+1$	$5r$	$5r+1$	$5r+1$	$5r+1$
$n = 4r+2$	$5r+2$	$5r+2$	$5r+2$	$5r+2$
$n = 4r+3$	$5r+3$	$5r+4$	$5r+3$	$5r+3$

Table 5:

Case 5. $3 \leq n \leq 7$.
A 4-total mean cordial labeling of L_n is given in Tabel 6

Value of n	u_1	u_2	u_3	u_4	u_5	u_6	u_7	v_1	v_2	v_3	v_4	v_5	v_6	v_7
3	0	2	3					0	1	2				
4	0	1	2	3				0	0	2	3			
5	0	1	2	3	0			0	0	2	3	2		
6	0	1	2	3	3	0		0	0	2	3	0	1	
7	0	1	2	3	2	0	0	0	0	2	3	0	2	3

Table 6:

□

Theorem 3.7. The triangular snake T_n is 4-total mean cordial for all n.

Proof. Take the vertex set and edge set as in Definition 2.7. In this graph, $|V(T_n)| + |E(T_n)| = 5n - 4$.
Case 1. $n \equiv 0 \pmod 4$.
Let $n = 4r$, $r \geq 1$. Consider the vertices $u_1, u_2, \ldots, u_n$. Assign the label 0 to the r vertices $u_1, u_2, \ldots, u_r$. Next assign the label 1 to the r vertices $u_{r+1}, u_{r+2}, \ldots, u_{2r}$. We now assign the label 2 to the r vertices $u_{2r+1}, u_{2r+2}, \ldots, u_{3r}$ and assign the label 3 to the r vertices $u_{3r+1}, u_{3r+2}, \ldots, u_{4r}$. Consider the vertices $v_1, v_2, \ldots, v_n$. Assign the label 0 to the r vertices $v_1, v_2, \ldots, v_r$. Next assign the label 1 to the $r-1$ vertices $v_{r+1}, v_{r+2}, \ldots, v_{2r-1}$. Now assign the label 3 to the vertex v_{2r}. We now assign the label 2 to the $r-1$ vertices $v_{2r+1}, v_{2r+2}, \ldots, v_{3r-1}$. Now assign the label 0 to the vertex v_{3r} and finally assign the label 3 to the $r-1$ vertices $v_{3r+1}, v_{3r+2}, \ldots, v_{4r-1}$.
Case 2. $n \equiv 1 \pmod 4$.
Let $n = 4r+1$, $r \geq 1$. Label the vertices u_i $(1 \leq i \leq 4r)$ and v_i $(1 \leq i \leq 4r-1)$ as in Case 1. Finally assign the labels 0, 2 to the vertices u_{4r+1}, v_{4r}.

Case 3. $n \equiv 2 \pmod 4$.
Let $n = 4r+2$, $r \geq 0$. As in Case 1 assign the label to the vertices u_i $(1 \leq i \leq 4r)$ and v_i $(1 \leq i \leq 4r-1)$. Next assign the labels 2, 3 to the vertices u_{4r+1}, u_{4r+2} and finally assign the labels 0, 0 to the vertices v_{4r}, v_{4r+1}.
Case 4. $n \equiv 3 \pmod 4$.
Let $n = 4r+3$, $r \geq 1$. In this case, assign the label for the vertices u_i $(1 \leq i \leq 4r)$ and v_i $(1 \leq i \leq 4r-1)$ as in Case 1. We now assign the labels 2, 1, 0 to the vertices u_{4r+1}, u_{4r+2}, u_{4r+3} and assign the labels 0, 3, 0 to the vertices v_{4r}, v_{4r+1}, v_{4r+2}.
This vertex labeling f is a 4-total mean cordial labeling of T_n follows from the Tabel 7

Order of T_n	t_{mf} (0)	t_{mf} (1)	t_{mf} (2)	t_{mf} (3)
$n = 4t$	$5t-1$	$5t-1$	$5t-1$	$5t-1$
$n = 4t+1$	$5t$	$5t$	$5t+1$	$5t$
$n = 4t+2$	$5t+1$	$5t+1$	$5t+2$	$5t+2$
$n = 4t+3$	$5t+3$	$5t+3$	$5t+3$	$5t+2$

Table 7:

Case 5. $n = 3$.
A 4-total mean cordial labeling of T_n is given in Tabel 8

Vertex	u_1	u_2	u_3	v_1	v_2
Label	0	0	2	2	3

Table 8:

□

4. conclusion

In this paper we have studied about the 4-total mean cordial labeling of fan, wheel, jellyfish, jewel graph, ladder, triangular snake. Investigation of 4-total mean cordinality of some graphs using graph operations is in open problems.

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