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On Calculation of Total Power and Allocation for Achieving Near 1+1 Capacity Region of 2PAM NOMA in 5G Networks

Kyuhyuk Chung
Professor, Division of Software Science, Dankook University

5G 네트워크에서 비직교 다중 접속 2PAM의 근접 1+1 용량 영역 달성을 위한 총 전력과 할당의 계산

정규혁
단국대학교 소프트웨어학과 교수

Abstract In binary-modulation non-orthogonal multiple access (NOMA), there has been rare researches for the 1+1 capacity region to be achieved; how much total power is required and what power allocation is assigned for this total power. In this paper, the average total transmitted power to achieve 1+1 capacity region of binary pulse amplitude modulation (2PAM) NOMA is investigated, with a tolerable loss. Then, based on the sufficient average total transmitted power, we calculate the power allocation coefficient to achieve 1+1 capacity region. Furthermore, it is shown by numerical results that with the tolerable loss less than 0.008, near 1+1 capacity region is achieved. We also calculate numerically the power allocation coefficient for both users to achieve near 1+1 capacity region. As a result, for 2PAM NOMA to operate near 1+1 capacity region, proper total power with appropriate power allocation could be calculated in design of NOMA systems.

Key Words : NOMA, 5G, Superposition coding, Successive interference cancellation, Power allocation

요 약 이진 변조 비직교 다중접속에서 1+1 용량 영역을 달성하는 주제에 대한, 다시 말해, 총 전력은 얼마나 필요한가와 이때 전력은 어떻게 할당해야 되는가에 대한 연구가 다소 미흡하다. 본 논문에서는, 허용 가능한 손실 범위 안에서, 2PAM 비직교 다중 접속의 1+1 용량 영역을 달성할 수 있는 평균 총 전송 전력을 고찰한다. 다음으로, 충분한 평균 총 전력을 기반으로 1+1 용량 영역을 달성할 수 있는 전력 할당 계수를 계산한다. 그리고, 수치적 결과를 통해서 0.008 미만의 허용 가능한 손실 범위 안에서, 근접 1+1 용량 영역이 달성됨을 보여준다. 또한, 수치상으로 근접 1+1 용량 영역을 달성하는 양 사용자의 전력 할당 계수를 계산한다. 결론적으로 2PAM 비직교 다중 접속이 근접 1+1 용량 영역에서 동작하기 위해, 적절한 전력 할당과 함께, 적당한 총 전력이 비직교 다중접속 설계에서 계산될 수 있다.

주제어 : 비직교 다중 접속, 5G, 중첩 코딩, 연속 간섭 제거, 전력 할당

1. Introduction

The non-orthogonal multiple access (NOMA) scheme in the fifth-generation (5G) network is considered as a promising candidate multiple access (MA), owing to its larger spectral efficiency and low latency[1,2]. NOMA is based on superposition coding (SC) and successive

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*Corresponding Author : Kyuhyuk Chung(khchug@dankook.ac.kr)

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interference cancellation (SIC)[3,4]. Also NOMA can improve the spectral efficiency by sharing the channel resources[5]. In addition, the bit-error rate (BER) of NOMA networks was analyzed in [6]. The effect of local oscillator imperfection for NOMA was investigated [7]. In [8], the BER with randomly generated signals was analyzed. In [9], the exact BER expression was derived for the two or three-user cases. The average symbol error rate (SER) expression was presented in [10]. The importance of SIC in NOMA was reported [11]. The secure NOMA-enabled mobile network is studied in [12]. In [13], the physical layer security in NOMA was considered. The intelligent reflecting surface (IRS) NOMA was studied in [14]. In [15], the mutual-aid NOMA scheme was studied. The higher order modulation schemes in NOMA-based visible light communication (VLC) systems was investigated in [16]. Various receivers' structures of NOMA were studied in term of BER performance measure [17]. Impacts of channel estimation errors on BER performance was investigated in [18]. Asymmetric binary pulse amplitude modulation (2PAM) NOMA was designed and BER performance of this NOMA scheme was analyzed in [19]. Correlated information sources have been investigated in term of achievable data rates in [20].

In this paper, we calculates the average total transmitted power and allocation for achieving near 1+1 capacity region of 2PAM NOMA. To this end, first, the average total transmitted power is investigated, with a tolerable loss. Then, the power allocation coefficient to achieve 1+1 capacity region is calculated. It is shown that with the tolerable loss less than 0.008, near 1+1 capacity region is achieved. We also calculate numerically the power allocation coefficients for both users to achieve near 1+1 capacity region.

The remainder of this paper is organized as follows. In Section 2, the system and channel model are described. The achievable data rates for both users in NOMA are presented in Section 3. The average total transmitted power is calculated in Section 4. The results are presented and discussed in Section 5. Finally, the conclusions are presented in Section 6.

2. System and Channel Model

To introduce the technical principle of NOMA, we start with a historical development: In 5G mobile networks, the number of users served in a cell coverage increases dramatically, and a new paradigm is required to accommodate the number of users. However, channel resources, such as time, frequency, and space, are fully utilized in orthogonal multiple access (OMA), i.e., time division multiple access (TDMA), frequency division multiple access (FDMA), orthogonal frequency division multiplexing (OFDM), code division multiple access (CDMA), and multiple input multiple output (MIMO). Hence, the standard body for 5G mobile communicatioons has considered new techniques. A candidate for such requirement is NOMA, which is the superposition based multi-user access technique, to provide high system capacity and low latency.

In a cellular downlink NOMA transmission system, all channels are assumed to be block fading. A base station and two users are within the cell. The complex channel coefficient between the *m*th user and the base station is denoted by h_m, $m=1,2$. The channels are sorted as $|h_1|>|h_2|$. The base station will transmit the superimposed signal $x=\sqrt{\alpha P}s_1+\sqrt{(1-\alpha)P}s_2$, where s_m is the message for the *m*th user with unit power, $E[|s_1|^2]=E[|s_2|^2]=1$, where $E[u]$

represents the expectation of a random variable (RV) u, α is the power allocation factor, with $0 \le \alpha \le 1$, and P is the average total transmitted power. The observation at the mth user is given by

$$y_m = |h_m| x + n_m, \tag{1}$$

where $n_m \sim N(0, N_0/2)$ is additive white Gaussian noise (AWGN). The notation $N(\mu, \Sigma)$ represents the distribution of Gaussian RV with mean μ and variance Σ, and N_0 is one-sided power spectral density. In this paper, we assume that the standard 2PAM, $s_2 \in \{+1, -1\}$. It is assumed that for the given information bits $b_1, b_2 \in \{0,1\}$, the bit-to-symbol mapping of the standard 2PAM is given by

$$\begin{cases} s_1(b_1=0)=+1 \\ s_1(b_1=1)=-1 \end{cases} \quad \begin{cases} s_2(b_2=0)=+1 \\ s_2(b_2=1)=-1 \end{cases}, \tag{2}$$

3. Achievable Data Rates for Users in NOMA

In this section, we consider the achievable data rates for each user in the standard 2-user 2PAM NOMA.

First, we consider the single user channel capacity with standard 2PAM. It is well-known that the channel capacity $C^{(b)}$ of a binary-input AWGN channel is given as

$$\begin{aligned} &C^{(b)}\left(\frac{P}{N_0/2}\right) = \\ &-\int_{-\infty}^{+\infty} P_{Y|B}(y|b=0)\log_2 P_{Y|B}(y|b=0)\,dy \\ &-\frac{1}{2}\log_2(2\pi e N_0/2). \end{aligned} \tag{3}$$

The above-mentioned channel capacity in equation (3) is depicted in Fig. 1.

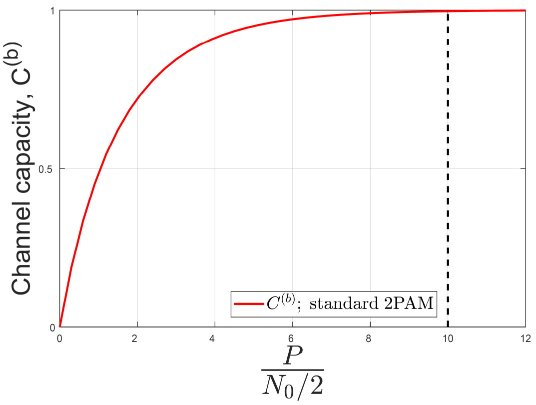

Fig. 1. Channel capacity of binary-input AWGN channel.

As shown in Fig. 1, the binary input channel capacity $C^{(b)}$ in equation (3) asymptotically approaches the value of one, as the SNR $P/(N_0/2)$ increase. We note that the capacity of equiprobable M-ary constellations cannot be larger than $\log_2(M)$. In our case of 2PAM, the maximum capacity is one, even though the SNR $P/(N_0/2) \to \infty$. Therefore, in this paper, based on Fig. 1, we assume that

$$C^{(b)}\left(\frac{P}{N_0/2} = 10\right) \simeq 1. \tag{4}$$

In the standard 2-user 2PAM NOMA, for the first user, the achievable data rate is given as

$$\begin{aligned} &R_1 = h(y_1 | b_2) - h(y_1 | b_1, b_2) \\ &-\int_{-\infty}^{+\infty} P_{Y_1|B_2}(y_1 | b_2=0)\log_2 P_{Y_1|B_2}(y_1 | b_2=0)\,dy_2 \\ &-E[-N(0, N_0/2)], \end{aligned} \tag{5}$$

where $E[-N(0,N_0/2)] = \frac{1}{2}\log_2(2\pi e N_0/2)$ and

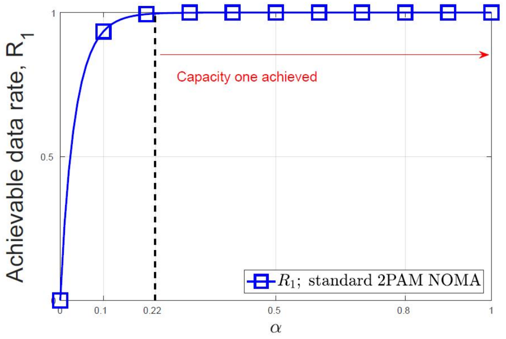

Fig. 2. Achievable data rate of first user for standard 2PAM NOMA.

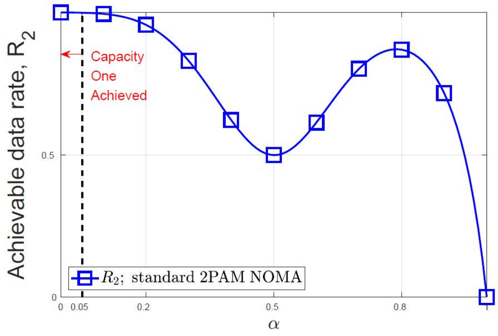

Fig. 3. Achievable data rate of second user for standard 2PAM NOMA.

$$P_{Y_1|B_2}(y_1 \mid b_2=0)= \tag{6}$$
$$+\frac{1}{2}\frac{1}{\sqrt{2\pi N_0/2}}e^{-\frac{(y_1-|h_1|\sqrt{P\alpha}\,s_1(b_1=0\,|\,b_2=0))^2}{2N_0/2}}$$
$$+\frac{1}{2}\frac{1}{\sqrt{2\pi N_0/2}}e^{-\frac{(y_1-|h_1|\sqrt{P\alpha}\,s_1(b_1=1\,|\,b_2=0))^2}{2N_0/2}}.$$

For the second user, the achievable data rate is given as

$$R_2 = h(y_2)-h(y_2 \mid b_2)$$
$$=\int_{-\infty}^{+\infty} P_{Y_2|B_2}(y_2 \mid b_2=0)\log_2\frac{P_{Y_2|B_2}(y_2 \mid b_2=0)}{P_{Y_2}(y_2)}dy_2, \tag{7}$$

where

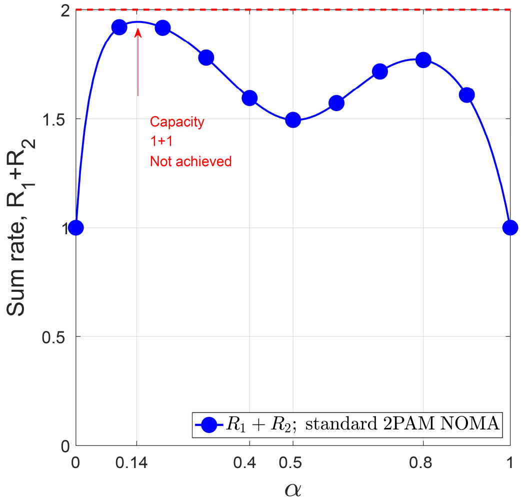

Fig. 4 Achievable sum rate of first and second users for standard 2PAM NOMA.

$$P_{Y_2}(y_2)= \tag{8}$$
$$+\frac{1}{4}\frac{1}{\sqrt{2\pi N_0/2}}e^{-\frac{(y_2-|h_2|\sqrt{P}(+\sqrt{(1-\alpha)}+\sqrt{\alpha}))^2}{2N_0/2}}$$
$$+\frac{1}{4}\frac{1}{\sqrt{2\pi N_0/2}}e^{-\frac{(y_2-|h_2|\sqrt{P}(+\sqrt{(1-\alpha)}-\sqrt{\alpha}))^2}{2N_0/2}}$$
$$+\frac{1}{4}\frac{1}{\sqrt{2\pi N_0/2}}e^{-\frac{(y_2-|h_2|\sqrt{P}(-\sqrt{(1-\alpha)}+\sqrt{\alpha}))^2}{2N_0/2}}$$
$$+\frac{1}{4}\frac{1}{\sqrt{2\pi N_0/2}}e^{-\frac{(y_2-|h_2|\sqrt{P}(-\sqrt{(1-\alpha)}-\sqrt{\alpha}))^2}{2N_0/2}},$$

and

$$P_{Y_2|B_2}(y_2 \mid b_2=0)= \tag{9}$$
$$+\frac{1}{2}\frac{1}{\sqrt{2\pi N_0/2}}e^{-\frac{(y_2-|h_2|\sqrt{P}(\sqrt{(1-\alpha)}+\sqrt{\alpha}))^2}{2N_0/2}}$$
$$+\frac{1}{2}\frac{1}{\sqrt{2\pi N_0/2}}e^{-\frac{(y_2-|h_2|\sqrt{P}(\sqrt{(1-\alpha)}-\sqrt{\alpha}))^2}{2N_0/2}}.$$

4. Calculation of Average Total Transmitted Power for 1+1 Capacity Region

It is assumed that $|h_1| = \sqrt{1.5}$ and $|h_2| = \sqrt{0.5}$.

Table 1. Maximum sum rate for varying total power

$P/(N_0/2)$	Max Sum Rate
30	1.945
40	1.979
50	1.992
60	1.997

We consider the average total transmitted signal power to noise power ratio (SNR) $P/(N_0/2)=30$. Now, based on the assumption of equation (4), we calculate the average total transmitted power to be required for achieving 1+1 capacity region of 2-user 2PAM NOMA. The SNR of the first user is given by

$$\frac{|h_1|^2 P\alpha}{N_0/2}. \qquad (10)$$

Thus, for $\frac{|h_1|^2 P\alpha}{N_0/2} \simeq 10$, based on equation (10), the power allocation $\alpha=0.22$, as shown in Fig. 2. Similary, for the second user, the power allocation $\alpha=0.05$, as shown in Fig. 3. . However, as shown in Fig. 2 and Fig. 3, the power allocation ranges for the capacity one to be achieved does not overlap each other, i.e., the 1+1 capacity region cannot be achieved with the given $P/(N_0/2)=30$. We also depict the sum rate in Fig. 4. As shown in Fig. 4, we observe the similar results, as those in Fig. 2 and Fig. 3, i.e., the 1+1 capacity region cannot be achieved with the given $P/(N_0/2)=30$. Thus, we need more power, i.e., $P/(N_0/2)>30$. Based on Table 1, when $P/(N_0/2)\geq 50$, there is no significant gain of the maximum sum rate. Hence, we choose $P/(N_0/2)=50$ for achieving near 1+1 capacity. In the next section. we calculate the power allocation for this near 1+1 capacity.

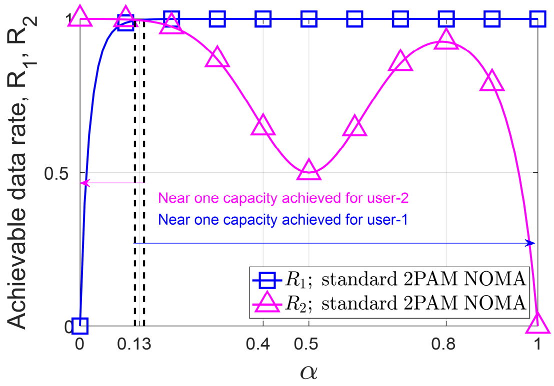

Fig. 5. Achievable data rates of first and second users for standard 2PAM NOMA.

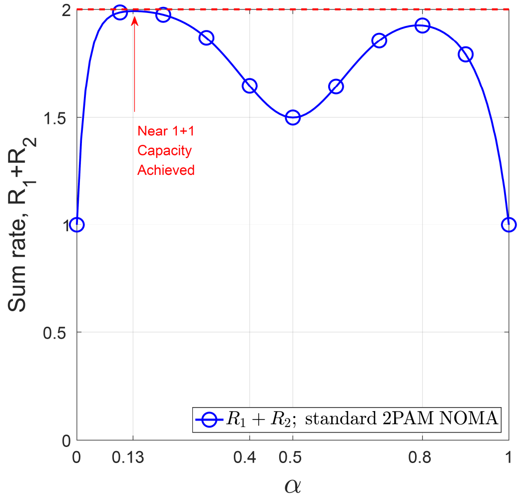

Fig. 6. Achievable sum rate of first and second users for standard 2PAM NOMA.

5. Numerical Results and Discussions

In this section, with the sufficient total transmitted power $P/(N_0/2)=50$, we calculate the power allocation α for achieving near 1+1 capacity region of 2-user 2PAM NOMA. First, we depict the achievable data rates R_1 and R_2, especially as a single figure, in Fig. 5. As shown in Fig. 5, the power allocation range to be achieved by both users is calculated as

$$0.12<\alpha<0.14. \qquad (11)$$

Therefore, we choose a reasonable power allocation $\alpha = 0.13$. With such power allocation, we depict the sum rate $R_1 + R_2$, in Fig. 6. As shown in Fig. 6, the near 1+1 capacity region is achieved with the power allocation $\alpha = 0.13$ and $P/(N_0/2) = 50$. It should be noted that user-fairness in the conventional NOMA schemes is established with $0 \le \alpha \ll 0.5$. Typical values in the literature of NOMA can be found as $0 \le \alpha < 0.25$. This user-fairness can be stated as follows; the stronger the channel gain is, the less power is allocated. Hence, our calculated value of the power allocation coefficient could be reasonable in the practical NOMA systems.

In addition, for a better comparison, we present the results of experiments for different channel environments, i.e., equal channel gains $|h_1| = |h_2| = 1$. It is observed in Fig. 7 that the power allocation range to be achieved by both users is calculated as

$$0.20 < \alpha < 0.22. \qquad (12)$$

Therefore, we choose a reasonable power allocation $\alpha = 0.21$. With such power allocation, we depict the sum rate $R_1 + R_2$, in Fig. 8. As shown in Fig. 8, the similar results as those in Fig. 6 are observed, i.e., the near 1+1 capacity region is achieved with the power allocation $\alpha = 0.21$ and $P/(N_0/2) = 50$.

6. Conclusion

In this paper, we calculated the average total transmitted power and allocation for achieving near 1+1 capacity region of 2PAM NOMA. The average total transmitted power to achieve 1+1 capacity region was studied, with a tolerable loss.

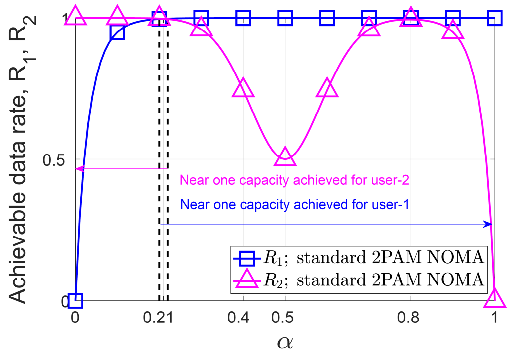

Fig. 7. Achievable data rates of first and second users for standard 2PAM NOMA.

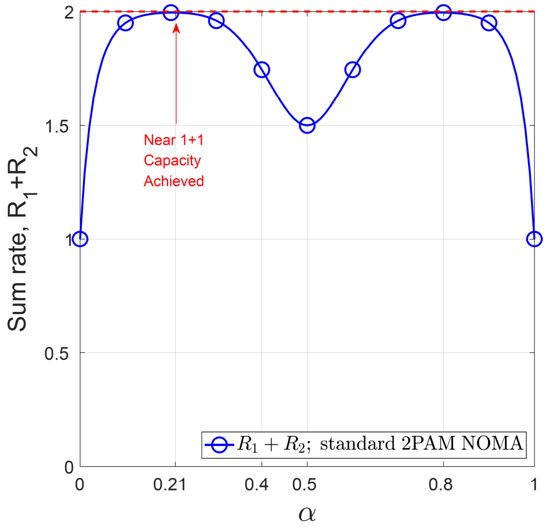

Fig. 8. Achievable sum rate of first and second users for standard 2PAM NOMA.

The power allocation coefficient to achieve 1+1 capacity region was calculated. Then, it was shown that with the tolerable loss less than 0.008, near 1+1 capacity region is achieved. We also calculated numerically the power allocation coefficient to achieve near 1+1 capacity region.

As a result, for 2PAM NOMA to operate near 1+1 capacity region, proper total power with appropriate power allocation could be calculated in design of NOMA systems in 5G mobile networks.

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정 규 혁(kyuhyuk Chung) [정회원]

· 1997년 2월 : 성균관대학교 전자공학과 졸업(공학사)
· 1998년 12월 : Univ. of Southern California 전기공학과 졸업(공학석사)
· 2003년 12월 : Univ. of Southern California 전기공학과 졸업(공학박사)
· 2005년 9월 ~ 현재 : 단국대학교 SW융합대학 소프트웨어학과 교수
· 관심분야 : 5G이동통신시스템, 비직교다중접속
· E-Mail : khchung@dakook.ac.kr