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ON GENERALIZED RIGHT f-DERIVATIONS OF Γ-INCLINE ALGEBRAS

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Abstract. In this paper, we introduce the concept of a generalized right f-derivation associated with a derivation d and a function f in Γ-incline algebras and give some properties of Γ-incline algebras. Also, the concept of d-ideal is introduced in a Γ-incline algebra with respect to right f-derivations.

1. Introduction

Incline algebra is a generalization of both Boolean and fuzzy algebra and it is a special type of semiring. It has both a semiring structure and a poset structure. It can also be used to represent automata and other mathematical systems, to study inequalities for non-negative matrices of polynomials. Z. Q. Cao, K. H. Kim and F. W. Roush [4] introduced the notion of incline algebras in their book and later it was developed by some authors [1, 2, 3, 5]. Ahn et al [1] introduced the notion of quotient incline and obtained the structure of incline algebras. N. O. Alshehri [3] introduced the notion of derivation in incline algebra. Kim[5, 6] studied right derivation and generalized derivation of incline algebras and obtained some results. M. K. Rao etc introduced the concept of generalized right derivation of Γ-incline and obtain some results. In this paper, we introduce the concept of a generalized right f-derivation in Γ-incline algebras and give some properties of Γ-incline algebras. Also, the concept of d-ideal is introduced in a Γ-incline algebra with respect to right f-derivations.

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2. Preliminaries

An *incline algebra* is a set K with two binary operations denoted by " $+$ " and " $*$ " satisfying the following axioms, for all $x, y, z \in K$,

(K1) $x + y = y + x$,
(K2) $x + (y + z) = (x + y) + z$,
(K3) $x * (y * z) = (x * y) * z$,
(K4) $x * (y + z) = (x * y) + (x * z)$,
(K5) $(y + z) * x = (y * x) + (z * x)$,
(K6) $x + x = x$,
(K7) $x + (x * y) = x$,
(K8) $y + (x * y) = y$,
for all $x, y, z \in K$.

Definition 2.1. Let $(K, +)$ and $(\Gamma, +)$ be commutative semigroups. If there exists a mapping $K \times \Gamma \times K \to K((x, \alpha, y) = x\alpha y)$ such that it satisfies the following axioms, for all $x, y \in K$ and $\alpha, \beta \in \Gamma$,

(K9) $x\alpha(y + z) = x\alpha y + x\alpha z$
(K10) $(x + y)\alpha z = x\alpha z + y\alpha z$
(K11) $x(\alpha + \beta)y = x\alpha y + x\beta y$
(K12) $x\alpha(y\beta z) = (x\alpha y)\beta z$
(K13) $x + x = x$
(K14) $x + x\alpha y = x$
(K15) $y + x\alpha y = y$
Then K is called a Γ-*incline algebra*(see[7]).

Example 2.2. Let $K = [0, 1]$ and $\Gamma = N$. Define $+$ by $x + y = \max\{x, y\}$ and ternary operation is defined as $x\alpha y = \min\{x, \alpha, y\}$ for all $x, y \in K$ and $\alpha \in \Gamma$. Then K is a Γ-incline algebra(see [7]).

Note that $x \leq y \Leftrightarrow x + y = y$ for all $x, y \in K$. It is easy to see that " $\leq$ " is a partial order on K and that for any $x, y \in K$, the element $x + y$ is the least upper bound of $\{x, y\}$. We say that $\leq$ is induced by operation $+$.

In a Γ-incline algebra K, the following properties hold.

(K16) $x\alpha y \leq x$ and $y\alpha x \leq x$ for all $x, y \in K$ and $\alpha \in \Gamma$
(K17) $y \leq z$ implies $x\alpha y \leq x\alpha z$ and $y\alpha x \leq z\alpha x$, for all $x, y, z \in K$ and $\alpha \in \Gamma$
(K18) If $x \leq y$ and $a \leq b$, then $x + a \leq y + b$, and $x\alpha \leq y\alpha b$ for all $x, y, a, b \in K$ and $\alpha \in \Gamma$.

Furthermore, a Γ-incline algebra K is said to be *commutative* if $x\alpha y = y\alpha x$ for all $x, y \in K$ for all $\alpha \in \Gamma$.

A Γ-*subincline* of a Γ-incline algebra K is a non-empty subset I of K which is closed under the addition and multiplication. A Γ-subincline I is called an *ideal* if $x \in I$ and $y \leq x$ then $y \in I$. An element "0" in an Γ-incline algebra K is a *zero element* if $x + 0 = x = 0 + x$ and $x\alpha 0 = 0 = 0\alpha x$ for any $x \in K$ and $\alpha \in \Gamma$. An non-zero element "1" is called a *multiplicative identity* if $x\alpha 1 = 1\alpha x = x$ for any $x \in K$ and $\alpha \in \Gamma$. A non-zero element $a \in K$ is said to be a *left* (resp. *right*) *zero divisor* if there exists a non-zero $b \in K$ such hat $a\alpha b = 0$ (resp. $b\alpha a = 0$) for all $\alpha \in \Gamma$. A zero divisor is an element of K which is both a left zero divisor and a right zero divisor. An incline algebra K with multiplicative identity 1 and zero element 0 is called an *integral incline* if it has no zero divisors. By a homomorphism of Γ-incline algebras, we mean a mapping f from a Γ-incline algebra K into a Γ-incline algebra L such that $f(x + y) = f(x) + f(y)$ and $f(x\alpha y) = f(x)\alpha f(y)$ for all $x, y \in K$ for all $\alpha \in \Gamma$.

Definition 2.3. Let K be a Γ-incline algebra. An element $a \in K$ is said to be *idempotent* of K if there exists $\alpha \in \Gamma$ such that $a = a\alpha a$.

Let K be a Γ-incline algebra. If every element of K is idempotent, then K is said to be *idempotent* Γ-*incline algebra*. A Γ-incline algebra K with unity 1 and zero element 0 is called an *integral* Γ-*incline* if it has no zero divisors.

Definition 2.4. Let K be a Γ-incline algebra. By a *right derivation* of K, we mean a self map d of K satisfying the identities

$$d(x + y) = d(x) + d(y) \text{ and } d(x\alpha y) = (d(x)\alpha y) + (d(y)\alpha x)$$

for all $x, y \in K$ and $\alpha \in \Gamma$.

3. Generalized right f-derivations of Γ-incline algebras

In what follows, let K denote a Γ-incline algebra with a zero-element unless otherwise specified.

Definition 3.1. Let K be a Γ-incline algebra. A mapping $D : K \to K$ is called a *generalized right* f-*derivation* of K if there exists a right derivation d and a function f of K such that

$$D(x + y) = D(x) + D(y) \text{ and } D(x\alpha y) = (D(x)\alpha f(y)) + (d(y)\alpha f(x))$$

for all $x, y \in K$ and $\alpha \in \Gamma$.

EXAMPLE 3.2. Let $K = \{0, a, b, 1\}$ be a set in which "+" and "α" is defined by

$+$	0	a	b	1
0	0	a	b	1
a	a	a	b	1
b	b	b	b	1
1	1	1	1	1

α	0	a	b	1
0	0	0	0	0
a	0	a	a	a
b	0	a	b	b
1	0	a	b	1

Then it is easy to check that $(K, +, \alpha)$ is a Γ-incline algebra. Define a map $d : K \to K$ by

$$d(x) = \begin{cases} a & \text{if } x = a, b, 1 \\ 0 & \text{if } x = 0 \end{cases}$$

and define a function $f : K \to K$ by

$$f(x) = \begin{cases} a & \text{if } x = b \\ b & \text{if } x = a \\ 1 & \text{if } x = 1 \\ 0 & \text{if } x = 0. \end{cases}$$

Also, define a map $D : K \to K$ by

$$D(x) = \begin{cases} a & \text{if } x = a, b \\ 1 & \text{if } x = 1 \\ 0 & \text{if } x = 0. \end{cases}$$

Then we can see that D is a generalized right f-derivation associated with a derivation d and a function f of K.

PROPOSITION 3.3. *Let K be a Γ-incline algebra and let D be a generalized f derivation associated with a derivation d and a function f of K. Then the following conditions hold for all $x, y \in K$ and $\alpha \in \Gamma$.*

(1) *If K is idempotent, then $d(x) \leq D(x) \leq x$ for all $x \in K$*
(2) *$D(x\alpha y) \leq D(x) + d(y)$*
(3) *If $x \leq y$ and f is an order preserving mapping, then $D(x\alpha y) \leq f(y)$.*

Proof. (1) Let D be a generalized right f derivation associated with a derivation d and a function f of K. If K is idempotent, then we get $D(x) = D(x\alpha x) = (D(x)\alpha x) + (d(x)\alpha x) \leq d(x)\alpha f(x) \leq d(x)$ for all $x, y \in K$ and $\alpha \in \Gamma$. (2) Let $x, y \in K$ and $\alpha \in \Gamma$. Then we have

$D(x)\alpha(y) \leq D(x)$ and $d(y)\alpha f(x) \leq d(y)$. Hence $D(x\alpha y) = D(x)\alpha f(y) + d(y)\alpha f(x) \leq D(x) + d(y)$. So, we find $D(x\alpha y) \leq D(x) + d(y)$. Also, since $D(x) = D(x\alpha x) = D(x)\alpha x + d(x)\alpha x$, we get from (K15) and (K16),

$$\begin{aligned} D(x) + x &= ((D(x)\alpha x) + (d(x)\alpha x)) + x \\ &= (D(x)\alpha x) + (x + d(x)\alpha x) \\ &= (D(x)\alpha x) + x = x + (D(x)\alpha x) \\ &= x \end{aligned}$$

which implies $D(x) \leq x$ for all $x \in K$ and $\alpha \in \Gamma$. (3) Let $x \leq y$ and let f be an order preserving mapping. Then we by using (K16) and (K18), we have $d(y)\alpha f(x) \leq d(y)\alpha f(y) \leq f(y)$. Similarly, we get $D(x)\alpha f(y) \leq f(y)$. Then we obtain $D(x\alpha y) = (D(x)\alpha f(y)) + (d(y)\alpha f(x)) \leq f(y) + f(y) = f(y)$ for all $\alpha \in \Gamma$. Hence we have $D(x\alpha y) \leq f(y)$.

□

Proposition 3.4. *Let D be a generalized right f-derivation associated with a derivation d and a function f of K. If $f(0) = 0$, then we have $D(0) = 0$.*

Proof. Let D be a generalized right f-derivation associated with a derivation d and a function f of K Then we have for all $\alpha \in \Gamma$,

$$\begin{aligned} D(0) &= D(0\alpha 0) = D(0)\alpha f(0) + d(0)\alpha f(0) \\ &= D(0)\alpha 0 + d(0)\alpha 0 = 0 + 0 \\ &= 0. \end{aligned}$$

□

Proposition 3.5. *Let D be a generalized right f-derivation associated with a derivation d and a function f of K. If K is idempotent, then $D(x) \leq f(x)$ for all $x \in K$.*

Proof. Let D be a generalized right f-derivation associated with a derivation d and a function f of K. If K is idempotent, then

$$\begin{aligned} D(x) &= D(x\alpha x) = D(x)\alpha f(x) + d(x)\alpha f(x) \\ &\leq f(x) + f(x) = f(x) \end{aligned}$$

from (K9) for all $x \in K$ and $\alpha \in \Gamma$. □

Proposition 3.6. *Let K be an incline algebra and let D be a generalized right f-derivation associated with a derivation d and a function f of K. Then for all $x, y \in K$ and $\alpha \in \Gamma$, $D(x\alpha y) \leq D(x)$ and $D(x\alpha y) \leq D(y)$.*

Proof. Let $x, y \in K$ and $\alpha \in \Gamma$, Then by using (K14), we obtain

$$D(x) = D(x + x\alpha y) = D(x) + D(x\alpha y).$$

Hence we get $D(x\alpha y) \leq D(x)$. Also, $D(y) = D(y + (x\alpha y)) = D(y) + D(x\alpha y)$, and so $D(x\alpha y) \leq D(y)$. □

PROPOSITION 3.7. *Let K be an incline algebra. A mapping $D : K \to K$ is isotone if $x \leq y$ implies $D(x) \leq D(y)$ for all $x, y \in K$.*

PROPOSITION 3.8. *Let D be a generalized right f-derivation associated with a derivation d and a function f of K. Then D is isotone.*

Proof. Let $x, y \in K$ be such that $x \leq y$. Then $x + y = y$. Hence we have $D(y) = D(x+y) = D(x)+D(y)$, which implies $D(x) \leq D(y)$. This completes the proof. □

PROPOSITION 3.9. *Let K is an Γ-incline algebra. Then a sum of two generalized f-right derivations associated with a function f of K is again a generalized right f-derivation associated with a function f of K.*

Proof. Let D_1 and D_2 be two generalized right f-derivations associated with derivations d_1 and d_2, respectively. Then we have for all $a, b \in K$ and $\alpha \in \Gamma$,

$$\begin{aligned}(D_1 + D_2)(a\alpha b) &= D_1(a\alpha b) + D_2(a\alpha b)\\ &= D_1(a)\alpha f(b) + d_1(b)\alpha f(a) + D_2(a)\alpha f(b) + d_2(b)\alpha f(a)\\ &= D_1(a)\alpha f(b) + D_2(a)\alpha f(b) + d_1(b)\alpha f(a) + d_2(b)\alpha f(a)\\ &= (D_1 + D_2)(a)\alpha f(b) + (d_1 + d_2)(b)\alpha f(a).\end{aligned}$$

Clearly, $(D_1 + D_2)(a+b) = (D_1 + D_2)(a) + (D_1 + D_2)(b)$ for all $a, b \in K$ and $\alpha \in \Gamma$. This completes the proof. □

PROPOSITION 3.10. *Let K be an integral Γ-incline and let D be a generalized right f-derivation associated with a derivation d and a function f of K. If $f(1) = 1$ and $a \in K$, then $a\alpha D(x) = 0$ implies $a = 0$ or $d = 0$ for all $a \in K$ and $\alpha \in \Gamma$.*

Proof. Let $a\alpha D(x) = 0$ for all $x \in K$ and $\alpha \in \Gamma$. Putting x on $x\alpha y$, for all $y \in K$, we get

$$\begin{aligned}0 &= a\alpha D(x) = a\alpha D(x\alpha y)\\ &= a\alpha[(D(x)\alpha f(y)) + (d(y)\alpha f(x))]\\ &= (a\alpha(D(x)\alpha f(y)) + (a\alpha(d(y)\alpha f(x)))\\ &= a\alpha(d(y)\alpha f(x)).\end{aligned}$$

In this equation, by taking $x = 1$, we have $a\alpha d(y) = 0$ for any $y \in K$. Since K is an integral Γ-incline algebra, we have $a = 0$ or $d = 0$.

□

THEOREM 3.11. *Let I be a nonzero ideal of integral Γ-incline K and let D be a nonzero generalized right f-derivation associated with a nonzero derivation d and a function f of K. Then D is nonzero on I.*

Proof. Suppose that D is a nonzero generalized right f-derivation of K associated with a nonzero derivation d and a function f of K but D is zero generalized right f-derivation on I. Let $x \in I$. Then we have $D(x) = 0$. Let $y \in K$. Then by (K16), we get $x\alpha y \leq x$ for all $\alpha \in \Gamma$. Since I is an ideal of K, we have $x\alpha y \in L$. Hence $D(x\alpha y) = 0$, which implies

$$0 = D(x\alpha y) = (D(x)\alpha f(y)) + (d(y)\alpha f(x)) = d(y)\alpha f(x).$$

By hypothesis, K has no zero divisors. Hence $f(x) = 0$ for all $x \in I$ or $d(y) = 0$ for all $y \in K$. Since f is a nonzero function of K, we get $d(y) = 0$ for all $y \in K$. This contradicts with our assumption that is d is a nonzero derivation on K. Hence D is nonzero on I.

□

THEOREM 3.12. *Let I be a nonzero ideal of integral Γ-incline K and let D be a nonzero generalized right f-derivation associated with a nonzero derivation d and a function f of K. Then D is nonzero on I. If $a\alpha D(x) = 0$ for $a \in K$ and $\alpha \in \Gamma$, then $a = 0$.*

Proof. By Theorem 3.11, we know that there exists $m \in I$ such that $D(m) \neq 0$. Let I be a Let $a\alpha D(I) = 0$ $a \in K$ and $\alpha \in \Gamma$. Then for $m, n \in I$ we can write

$$\begin{aligned} 0 &= a\alpha D(m\alpha n) \\ &= a\alpha(D(m)\alpha f(n) + d(n)\alpha f(m)) \\ &= a\alpha D(m)\alpha f(n) + a\alpha d(n)\alpha f(m) \\ &= a\alpha d(n)\alpha f(m). \end{aligned}$$

Since K is an integral Γ-incline algebra and let d is a nonzero right f-derivation of K and f is a nonzero function on I, we get $a = 0$.

□

THEOREM 3.13. *Let K be a Γ-incline with a multiplicative identity element and let D be a generalized right f-derivation associated with a derivation d and a function f of K. If $f(1) = 1$, then we have $D(x) = D(1)\alpha f(x)) + d(x)$ for all $x \in K$.*

Proof. Since $D(x) = D(1\alpha x) = (D(1)\alpha f(x)) + (d(x)\alpha f(1)$ for all $x \in K$ and by using $f(1) = 1$, we have $D(x) = (D(1)\alpha f(x)) + d(x)$. □

DEFINITION 3.14. Let K be a Γ-incline algebra and let d be a non-trivial generalized right f-derivation associated with a derivation d and a function f of K. An ideal I of K is called a *d-ideal* if $d(I) = I$.

Since $d(0) = 0$, it can be easily observed that the zero ideal $\{0\}$ is a d-ideal of K. If d is onto, then $d(K) = K$, which implies K is a d-ideal of K.

EXAMPLE 3.15. In Example 3.2, let $I = \{0, a\}$. Then I is an ideal of K. It can be verified that $d(I) = I$. Therefore, I is a d-ideal of K.

LEMMA 3.16. *Let d be a generalized right f-derivation associated with a derivation d and a function f of K and let I, J be any two d-ideals of K. Then we have $I \subseteq J$ implies $d(I) \subseteq d(J)$.*

Proof. Let $I \subseteq J$ and $x \in d(I)$. Then we have $x = d(y)$ for some $y \in I \subseteq J$. Hence we get $x = d(y) \in d(J)$. Therefore, $d(I) \subseteq d(J)$. □

PROPOSITION 3.17. *Let K be a Γ-incline algebra. Then, a sum of any two d-ideals is also a d-ideal of K.*

Proof. Let I and J be d-ideals of K. Then $I + J = d(I) + d(J) = d(I + J)$. Hence $I + J$ is a d-ideal of K. □

Let d be a generalized right f-derivation associated with a derivation d and a function f of K. Define a set $Kerd$ by

$$Kerd := \{x \in K \mid d(x) = 0\}$$

for all $x \in K$.

PROPOSITION 3.18. *Let d be a generalized right f-derivation associated with a derivation d and a function f of K. Then $Kerd$ is a subincline of K.*

Proof. Let $x, y \in Kerd$. Then $d(x) = 0, d(y) = 0$ and

$$\begin{aligned} d(x\alpha y) &= (d(x)\alpha f(y)) + (d(y)\alpha f(x)) \\ &= (0\alpha f(y)) + (0\alpha f(x)) \\ &= 0 + 0 = 0, \end{aligned}$$

and

$$\begin{aligned} d(x+y) &= d(x) + d(y) \\ &= 0 + 0 = 0. \end{aligned}$$

Therefore, $x\alpha y, x+y \in Kerd$. This completes the proof. □

PROPOSITION 3.19. *Let d be a generalized right f-derivation associated with a derivation d and a function f of an integral Γ-incline algebra K. If f is an one to one function, then $Kerd$ is an ideal of K.*

Proof. By Proposition 3.18, $Kerd$ is a subincline of K. Now let $x \in K$ and $y \in Kerd$ such that $x \leq y$. Then $d(y) = 0$ and

$$0 = d(y) = d(y + x\alpha y) = d(y) + d(x\alpha y) = 0 + d(x\alpha y),$$

which $d(x\alpha y) = 0$. Hence we have

$$0 = d(x\alpha y) = (d(x)\alpha f(y)) + (d(y)\alpha f(x)) = d(x)\alpha f(y).$$

Since K has no zero divisors, either $d(x) = 0$ or $f(y) = 0$. If $d(x) = 0$, then $x \in Kerd$. If $f(y) = 0$, then $y = 0$ and so $x \leq y = 0$, i.e., $x = 0$, which implies $x \in Kerd$. □

Let d be a generalized right f-derivation associated with a derivation d and a function f of K. Define a set $Fix_d(K)$ by

$$Fix_d(K) := \{x \in K \mid d(x) = f(x)\}$$

for all $x \in K$.

PROPOSITION 3.20. *Let K be a commutative Γ-incline algebra and let d be a generalized right f-derivation associated with a derivation d and a function f of K. Then $Fix_d(K)$ is a subincline of K.*

Proof. Let $x, y \in Fix_d(K)$. Then we have $d(x) = f(x)$ and $d(y) = f(y)$, and so

$$\begin{aligned} d(x\alpha y) &= d(x)\alpha f(y) + d(y)\alpha f(x) = f(x)\alpha f(y) + f(y)\alpha f(x) \\ &= f(x)\alpha f(y) + f(x)\alpha f(y) = f(x)\alpha f(y) = f(x\alpha y). \end{aligned}$$

Now

$$d(x+y) = d(x) + d(y) = f(x) + f(y) = f(x+y),$$

which implies $x+y, x\alpha y \in Fix_d(K)$. This completes the proof. □

DEFINITION 3.21. Let K be an Γ-incline algebra. An element $a \in K$ is said to be *additively left cancellative* if for all $a, b \in K$, $a+b = a+c \Rightarrow b = c$. An element $a \in K$ is said to be *additively right cancellative* if for all $a, b \in K$, $b+a = c+a \Rightarrow b = c$. It is said to be *additively cancellative* if it is both left and right cancellative. If every element of K is additively left cancellative, it is said to be *additively left cancellative*. If every element of K is additively right cancellative, it is said to be *additively right cancellative*.

DEFINITION 3.22. A subincline I of an Γ-incline algebra K is called a *k-ideal* if $x+y \in I$ and $y \in I$, then $x \in I$.

EXAMPLE 3.23. In Example 3.2, $I = \{0, a, b\}$ is an k-ideal of K.

THEOREM 3.24. *Let K be a commutative Γ-incline algebra and additively right cancellative. If d is a generalized right f-derivation associated with a derivation d and a function f of K, then $Fix_d(K)$ is a k-ideal of K.*

Proof. By Proposition 3.20, $Fix_d(K)$ is a subincline of K. Let $x+y, y \in Fix_d(K)$. Then $d(y) = f(y)$ and $f(x+y) = d(x+y)$. Hence $f(x) + f(y) = d(x+y) = d(x) + d(y) = d(x) + f(y)$, which implies $x \in Fix_d(K)$. Hence $Fix_d(K)$ is a k-ideal of K. □

PROPOSITION 3.25. *Let K be an Γ-incline algebra and let d be a generalized right f-derivation associated with a derivation d and a function f of K. Then $Kerd$ is a k-ideal of K.*

Proof. From Proposition 3.18, $Kerd$ is a subincline of K. Let $x+y \in K$ and $y \in Kerd$. Then we have $d(x+y) = 0$ and $d(y) = 0$, and so

$$0 = d(x+y) = d(x) + d(y) = d(x) + 0 = d(x).$$

This implies $x \in Kerd$. □

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