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GENERALIZED DERIVATIONS ON PRIME RINGS SATISFYING CERTAIN IDENTITIES

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Abstract. Let R be a ring with characteristic different from 2. An additive mapping $F : R \to R$ is called a generalized derivation on R if there exists a derivation $d : R \to R$ such that $F(xy) = F(x)y + xd(y)$ holds for all $x, y \in R$. In the present paper, we show that if R is a prime ring satisfying certain identities involving a generalized derivation F associated with a derivation d, then R becomes commutative and in some cases d comes out to be zero (i.e., F becomes a left centralizer). We provide some counter examples to justify that the restrictions imposed in the hypotheses of our theorems are not superfluous.

1. Introduction

Throughout the present paper R will denote an associative ring and $Z(R)$ is the center of R. For each $x, y \in R$, the symbol $[x, y]$ will represent the commutator $xy - yx$ and the symbol $x \circ y$ stands for the skew-commutator $xy + yx$. We will make extensive use of the following basic identities:

$$[xy, z] = x[y, z] + [x, z]y,$$
$$[x, yz] = y[x, z] + [x, y]z,$$
$$x \circ (yz) = (x \circ y)z - y[x, z] = y(x \circ z) + [x, y]z,$$
$$(xy) \circ z = x(y \circ z) - [x, z]y = (x \circ z)y + x[y, z].$$

A ring R is said to be a prime ring if $aRb = 0$, then $a = 0$ or $b = 0$ for all $a, b \in R$.

An additive mapping $d : R \to R$ is called a derivation if $d(xy) = d(x)y + xd(y)$ for all $x, y \in R$. In particular, for a fixed $a \in R$, the mapping $I_a : R \to R$ given by $I_a(x) = [a, x]$ is a derivation which is said to be an inner derivation.

An additive function $F : R \to R$ is called a generalized inner derivation if $F(x) = ax + xb$ for fixed $a, b \in R$. For such a mapping F, it is easy to see that

$$F(xy) = F(x)y + x[y, b] = F(x)y + xI_b(y) \text{ for all } x, y \in R.$$

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The above observation leads to the definition that: an additive mapping F: $R \to R$ is called a generalized derivation associated with a derivation d if $F(xy) = F(x)y + xd(y)$ holds for all $x, y \in R$. Familiar examples of generalized derivations are derivations and generalized inner derivations, and the latter includes left multipliers. Since the sum of two generalized derivations is again a generalized derivation, every map of the form $F(x) = cx + d(x)$, where c is a fixed element of R and d a derivation of R, is a generalized derivation. In particular, if R has the multiplicative identity 1, then all generalized derivations have this form. The concept of a generalized derivation includes both the concept of derivation and left centralizer (i.e., an additive mapping F: $R \to R$ satisfying $F(xy) = F(x)y$ for all $x, y \in R$).

In 1978, Herstein [8], proved that if a prime ring R admits a non-zero derivation d on R satisfying the condition $d(x)d(y) = d(y)d(x)$ for all $x, y \in R$, and if $char(R) \neq 2$, then R is a commutative integral domain, and if $char(R) = 2$, then R is commutative or an order in a simple algebra which is 4-dimensional over its center. Following Herstine, several authors studied the commutativity of prime rings or semiprime rings admitting automorphisms, derivations or generalized derivations which are centralizing or commuting on the ring R or on appropriate subsets of R. For details see [5], [6], [7], and [9]. In [4], Ashraf and Rehman proved that if R is a prime ring with a non-zero ideal I, then R is a commutative ring if it admits a derivation d such that $d(xy) \pm xy \in Z(R)$ for all $x, y \in I$.

Again, Ashraf et al. [2], proved that if R is a prime ring, I is a non-zero ideal of R, and R admits a generalized derivation F associated with a non-zero derivation d such that $F(xy) \pm xy \in Z(R)$ or $F(xy) \pm yx \in Z(R)$ or $F(x)F(y) \pm xy \in Z(R)$ for all $x, y \in I$, then R is commutative. Inspired by these identities, several authors considered the different situations to investigate conditions under which a ring becomes commutative, see [12], [13].

Recently, M. K. Abu Nawas and R. M. Al-Omary [1] investigated the commutativity of a prime ring R admitting generalized derivations F and G associated with non-zero derivations d and g, respectively, such that any one of the following conditions is satisfied: (i) $F(x) \circ x \in Z(R)$, (ii) $[F(x), F(y)] - F[x, y] \in Z(R)$, (iii) $F(x) \circ F(y) - F(x \circ y) \in Z(R)$, (iv) $F[x, y] + [F(x), y] - [F(x), F(y)] \in Z(R)$, (v) $F(x \circ y) - [x, y] \in Z(R)$, (vi) $[F(x), F(y)] - x \circ y \in Z(R)$, (vii) $[F(x), G(y)] - [x, y] \in Z(R)$, (viii) $[F(x), x] - [x, G(x)] \in Z(R)$ and (ix) $F(x) \circ x - x \circ G(x) \in Z(R)$ for all $x, y \in I$.

More recently, Ashraf [3] explored the condition under which generalized derivation F becomes a left centralizer, i.e., associated derivation d becomes a zero map on R.

In the present paper, we shall discuss the situation when derivation d is a trivial map (i.e., F is a left centralizer) such that R is a non-zero prime ring with generalized derivation F associated with d satisfying any one of the following properties: (i) $d(x) \circ y = d(xy)$, (ii) $F(x \circ y) = F(x) \circ y - F(y) \circ x$, (iii) $F[x, y] = F(x) \circ y - F(y) \circ x$, (iv) $F(x) \circ x = -d(x^2)$, (v) $[F(x), x] = d(x^2)$ for all

$x, y \in R$. We have also obtained the commutativity of prime rings admitting generalized derivation F associated with non-zero derivation d satisfying the condition $F[x, y] = [F(x), y] + [F(y), x]$ for all $x, y \in R$. Further, examples are given to show that the hypotheses in our theorems are not superfluous.

2. Results

Theorem 2.1. *Let R be a non-zero prime ring. If R admits a derivation $d : R \to R$ such that $d(x) \circ y = d(xy)$ for all $x, y \in R$, then $d = 0$.*

Proof. In the sequel, following the previous work, first we will prove that R is commutative.

We have

$$d(x) \circ y = d(xy) \text{ for all } x, y \in R.$$

This can be rewritten as $d(x)y + yd(x) = d(x)y + xd(y)$ for all $x, y \in R$, which gives

$$yd(x) = xd(y) \text{ for all } x, y \in R. \tag{2.1}$$

Replacing y by zy in (2.1), we get

$$zyd(x) = xd(z)y + xzd(y) \text{ for all } x, y, z \in R. \tag{2.2}$$

Multiply (2.1) from left by z and compare with (2.2), we get

$$zxd(y) = xd(z)y + xzd(y) \text{ for all } x, y, z \in R. \tag{2.3}$$

This implies that

$$[z, x]d(y) = xd(z)y \text{ for all } x, y, z \in R. \tag{2.4}$$

For any $m \in R$ replacing x by mx in the last equation, we get

$$m[z, x]d(y) + [z, m]xd(y) = mxd(z)y \text{ for all } x, y, z, m \in R. \tag{2.5}$$

Finally, multiply (2.4) from left by m and compare with (2.5), we get

$$[z, m]Rd(y) = 0 \text{ for all } z, m, y \in R. \tag{2.6}$$

Since R is prime, so either $[z, m] = 0$ for all $z, m \in R$ or $d(y) = 0$ for all $y \in R$.

If $d \neq 0$, then $[z, m] = 0$ for all $z, m \in R$, and so R is commutative.

But then using the commutativity of R and by using Equation (2.3) we get

$$xd(z)y = xyd(z) = 0 \text{ for all } x, y, z \in R$$

which implies that $x = 0$, and so $R = 0$. This contradicts our assumption about R, hence $d = 0$.

Following example demonstrates that the primeness of R in the hypothesis in above theorem is necessary. But before we explore the example we need to define the following ring (see details in [10], [11]).

Definition 2.1. We define

$$K_{2^n} := \langle x_1, \ldots, x_n \,|\, x_i x_j = x_i,\ 2x_i = 0,\ \forall i = 1, 2, \ldots, n \rangle .$$

Remark 2.1. K_{2^n} is an algebra over $\mathbb{Z}_2$ with n-generators. This is an n-dimensional vector algebra over the field $\mathbb{Z}_2$. This algebra is a noncommutative ring without 1, its characteristic is 2, and every element in it is a zero divisor. Moreover, it is not a prime ring.

Remark 2.2. The opposite ring of K_{2^n} is defined by

$$(K_{2^n})^{op} := \langle x_1, \dots, x_n \,|\, x_i x_j = x_j,\ 2x_i = 0,\ \forall i = 1, 2, \dots, n \rangle .$$

This ring has the same properties as that of K_{2^n}.

Example 2.1. For example, for $n = 2$, we can rewrite $K_{2^2} = \langle a, b \rangle = \{0, a, b, c\}$ with the following relations:

$$2a = 2b = 0,\ c = a + b,\ a^2 = ab = a,\ b^2 = ba = b.$$

The additive and multiplicative tables of this ring are given by

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

;

.	0	a	b	c
0	0	0	0	0
a	0	a	a	0
b	0	b	b	0
c	0	c	c	0

.

Note that concrete forms of K_{2^2} and $(K_{2^2})^{op}$ in matrix notation are

$$K_{2^2} = \left\{ 0 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, a = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, b = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, c = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix},\ 0, 1 \in \mathbb{Z}_2 \right\}$$

and

$$(K_{2^2})^{op} = \left\{ 0 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, a = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, b = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, c = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, 0, 1 \in \mathbb{Z}_2 \right\},$$

respectively.

We give here full proof of the following proposition for K_{2^n}.

Proposition 2.1. *Let $X = K_{2^n}$ be as above where n is an even integer. Let for $1 \le \lambda \le n$, $d_\lambda : X \to X$ be an additive map defined on the generators $x_1, \dots, x_n \in X$ by*

$$d_\lambda(x) = \begin{cases} 0 & \text{if } x = x_1, \dots, x_{\lambda-1}, x_{\lambda+1}, \dots, x_n; \\ \sum\limits_{i=1}^{n} x_i & \text{if } x = x_\lambda. \end{cases}$$

Then $d_\lambda = d$ is a non-zero derivation on X satisfying the identity $d(x) \circ y = d(xy)$ for all $x, y \in X$.

Proof. We prove the proposition only for generators $\{x_1, \dots, x_n\}$ of X. Other elements of the ring X will come by themselves. Clearly $d \neq 0$ by definition. We will prove that d is a derivation and that $d(x) \circ y = d(xy)$ for all $x, y \in \{x_1, \dots, x_n\}$. We do it in following four cases.

Case I: Let $x \neq x_\lambda$ and $y \neq x_\lambda$. Then there is nothing to prove.

Case II: Let $x = x_\lambda$ and $y \neq x_\lambda$. Then

$$d(x_\lambda y) = d(x_\lambda) = \sum_{i=1}^{n} x_i.$$

On the other hand

$$d(x_\lambda)y + x_\lambda d(y) = \left(\sum_{i=1}^{n} x_i\right) y + x_\lambda 0 = \sum_{i=1}^{n} x_i.$$

Hence d is a derivation. For the identity,

$$\begin{aligned} d(x_\lambda) \circ y = d(x_\lambda)y + yd(x_\lambda) &= \left(\sum_{i=1}^{n} x_i\right) y + y\left(\sum_{i=1}^{n} x_i\right) \\ &= \sum_{i=1}^{n} x_i + \sum_{i=1}^{n} yx_i = \sum_{i=1}^{n} x_i = d(x_\lambda y), \end{aligned}$$

where

$$\sum_{i=1}^{n} yx_i = ny = 0 \text{ because } n \text{ is even.}$$

Hence the identity also holds.

Case III: Let $y = x_\lambda$ and $x \neq x_\lambda$. Then

$$d(xx_\lambda) = d(x) = 0$$

and

$$d(x)x_\lambda + xd(x_\lambda) = 0 + x\sum_{i=1}^{n} x_i = 0.$$

Hence, again d is a derivation. For the identity,

$$d(x) \circ x_\gamma = d(x)x_\gamma + x_\gamma d(x) = 0 = d(x) = d(xx_\lambda).$$

Case IV: Finally, let $x = y = x_\lambda$. Then for the derivation

$$d(x_\lambda x_\lambda) = d(x_\lambda) = \sum_{i=1}^{n} x_i.$$

On the other hand,

$$\begin{aligned} d(x_\lambda)x_\lambda + x_\lambda d(x_\lambda) &= \left(\sum_{i=1}^{n} x_i\right) x_\lambda + x_\lambda\left(\sum_{i=1}^{n} x_i\right) \\ &= \left(\sum_{i=1}^{n} x_i x_\lambda\right) + \left(\sum_{i=1}^{n} x_\lambda x_i\right) \\ &= \sum_{i=1}^{n} x_i + nx_\lambda = \sum_{i=1}^{n} x_i. \end{aligned}$$

Hence d is a derivation. The identity

$$d(x_\lambda) \circ x_\gamma = d(x_\lambda x_\gamma)$$

trivially holds and the proposition is proved. □

Example 2.2. In case of the ring $K_{2^2} = \langle a, b\rangle = \{0, a, b, c\}$, we define

$$d_b = \begin{cases} 0 & \text{if } x = 0, a, \\ c & \text{if } x = b, c. \end{cases}$$

Then d_b is a non-zero derivation that satisfies the identity $d(x) \circ y = d(xy)$, $\forall x, y \in K_{2^2}$.

Theorem 2.2. *Let R be a non-zero prime ring of characteristic not 2. If R admits a generalized derivation F associated with a derivation d such that*

(i) $F(x \circ y) = F(x) \circ y - F(y) \circ x$ *for all* $x, y \in R$, *or*

(ii) $F[x, y] = F(x) \circ y - F(y) \circ x$ *for all* $x, y \in R$,

then $d = 0$.

Proof. By hypothesis, we have

$$F(x \circ y) = F(x) \circ y - F(y) \circ x \text{ for all } x, y \in R. \tag{2.7}$$

Replacing y by yx in (2.7) and using (2.7), we get

$$(x \circ y)d(x) = -y[F(x), x] - (y \circ x)d(x) - y[d(x), x] \text{ for all } x, y \in R. \tag{2.8}$$

Again replacing y by zy in (2.8) and using (2.8), we get

$$[x, z]yd(x) = [z, x]yd(x) \text{ for all } x, y, z \in R.$$

That is

$$2[x, z]yd(x) = 0 \text{ for all } x, y, z \in R.$$

Since $char(R) \neq 2$ we get

$$[x, z]Rd(x) = 0 \text{ for all } x, z \in R.$$

Applying the same arguments after Equation (2.6) in the proof of Theorem 2.1, we get the required result.

(ii) We have

$$F[x, y] = F(x) \circ y - F(y) \circ x \text{ for all } x, y \in R. \tag{2.9}$$

Replacing y by yx in (2.9) and using (2.9), we get

$$[x, y]d(x) = -y[F(x), x] - (y \circ x)d(x) - y[d(x), x] \text{ for all } x, y \in R. \tag{2.10}$$

Again replacing y by zy in (2.10) and using (2.10), we get

$$[x, z]yd(x) = [z, x]yd(x) \text{ for all } x, y, z \in R.$$

That is

$$2[x, z]yd(x) = 0 \text{ for all } x, y, z \in R.$$

Since $char(R) \neq 2$ we get

$$[x, z]Rd(x) = 0 \text{ for all } x, z \in R.$$

Now applying the same arguments as used after Equation (2.6) in the proof of Theorem 2.1, we get the required result. □

The following example demonstrates that the primeness of R and characteristic different from 2 in the hypothesis in above theorem is necessary.

Remark 2.3. We leave it as an exercise to find a generalized derivation F on K_{2^n} such that the condition (i) or (ii) of Theorem 2.2 is satisfied but $d \neq 0$. As a hint we provide example for $n = 2$.

Example 2.3. Let X and d be as in Examples (2.1) and (2.2). Define $F : X \to X$ by

$$F = \begin{cases} 0 & \text{if } x = 0, c, \\ \text{c} & \text{if } x = a, b. \end{cases}$$

It is easy to check that F is an additive generalized derivation associated with an additive mapping d on X which is a derivation satisfying (i) $F(x \circ y) = F(x) \circ y - F(y) \circ x$ or (ii) $F[x, y] = F(x) \circ y - F(y) \circ x \ \forall\ x, y \in X$, but $d \neq 0$.

Theorem 2.3. *Let R be a prime ring of characteristic not* 2. *If R admits a generalized derivation F associated with a non-zero derivation d such that $F[x, y] = [F(x), y] + [F(y), x]$ for all $x, y \in R$, then R is commutative.*

Proof. We have

$$F[x, y] = [F(x), y] + [F(y), x] \text{ for all } x, y \in R. \tag{2.11}$$

Replacing y by yx in (2.11) and using (2.11), we get

$$[x, y]d(x) = y[F(x), x] + [y, x]d(x) + y[d(x), x] \text{ for all } x, y \in R. \tag{2.12}$$

Again replacing y by zy in (2.12) and using (2.12), we get

$$[x, z]yd(x) = [z, x]yd(x) \text{ for all } x, y, z \in R.$$

That is

$$2[x, z]yd(x) = 0 \text{ for all } x, y, z \in R.$$

Since $char(R) \neq 2$ we get

$$[x, z]Rd(x) = 0 \text{ for all } x, z \in R.$$

Since R is prime so either $[x, z] = 0$ for all $x, z \in R$ or $d(x) = 0$ for all $x \in R$. If $[x, z] = 0$ for all $x, z \in R$, then R is commutative. On the other hand if $d(x) = 0$ for all $x \in R$, then $d(R) = 0$ and hence $d = 0$, a contradiction.

The following example demonstrates that the primeness of R and characteristic different from 2 in the hypothesis in the above theorem is necessary. □

Example 2.4. Let X, d and F be as in above example. Then it is easy to check that F is an additive generalized derivation associated with an additive mapping d on X which is a derivation satisfying $F[x, y] = [F(x), y] + [F(y), x]$ for all $x, y \in X$, but X is not commutative. Hence, in Theorem 2.3 the hypothesis of primeness and characteristic different from 2 cannot be omitted.

Theorem 2.4. *Let R be a non-zero prime ring of characteristic not 2. If R admits a generalized derivation F associated with a derivation d such that*

(i) $F(x) \circ x = -d(x^2)$ *for all $x \in R$, or*
(ii) $[F(x), x] = d(x^2)$ *for all $x \in R$,*

then $d = 0$.

Proof. We have

$$F(x) \circ x = -d(x^2) \text{ for all } x \in R. \tag{2.13}$$

Replacing x by $x + y$ in (2.13) and using (2.13), we get

$$F(x) \circ y + F(y) \circ x = -d(x \circ y) \text{ for all } x, y \in R. \tag{2.14}$$

Replacing y by yx in (2.14) and using (2.14), we get

$$-y[F(x), x] + y(d(x) \circ x) - [y, x]d(x) = -(x \circ y)d(x) \text{ for all } x, y \in R. \tag{2.15}$$

Now, replacing y by zy in (2.15) and using (2.15), we get

$$[z, x]yd(x) = [x, z]yd(x) \text{ for all } x, y, z \in R.$$

That is,

$$2[x, z]yd(x) = 0 \text{ for all } x, y, z \in R.$$

Since $char(R) \neq 2$ we get

$$[x, z]Rd(x) = 0 \text{ for all } x, z \in R.$$

Applying the same arguments after Equation (2.6) in the proof of Theorem 2.1, we get the required result:

(ii) We have

$$[F(x), x] = d(x^2) \text{ for all } x \in R. \tag{2.16}$$

Replacing x by $x + y$ in (2.16) and using (2.16), we get

$$[F(x), y] + [F(y), x] = d(x \circ y) \text{ for all } x, y \in R. \tag{2.17}$$

Replacing y by yx in (2.17) and using (2.17), we get

$$y[F(x), x] + y[d(x), x] + [y, x]d(x) = (x \circ y)d(x) \text{ for all } x, y \in R. \tag{2.18}$$

Now, replacing y by zy in (2.18) and using (2.18), we get

$$[z, x]yd(x) = [x, z]yd(x) \text{ for all } x, y, z \in R.$$

That is,

$$2[x, z]yd(x) = 0 \text{ for all } x, y, z \in R.$$

Since $char(R) \neq 2$ we get

$$[x, z]Rd(x) = 0 \text{ for all } x, z \in R.$$

Notice that the arguments given in the proof of Theorem 2.1 after Equation (2.6) are still valid in the present situation and hence repeating the same process, we get the required result. □

The following example illustrates that if R is not a prime ring and $char(R) \neq 2$, then the hypothesis in Theorem 2.4 are not superfluous.

Example 2.5. Let $(K_{2^2})^{op} = R = \{\left(\begin{smallmatrix} a & b \\ 0 & 0 \end{smallmatrix}\right) \mid a, b \in \mathbb{Z}_2\}$. Define $d : R \to R$ by $d\left(\begin{smallmatrix} a & b \\ 0 & 0 \end{smallmatrix}\right) = \left(\begin{smallmatrix} 0 & b \\ 0 & 0 \end{smallmatrix}\right)$ and $F : R \to R$ by $F\left(\begin{smallmatrix} a & b \\ 0 & 0 \end{smallmatrix}\right) = \left(\begin{smallmatrix} a & 0 \\ 0 & 0 \end{smallmatrix}\right)$. Clearly R is not prime as $\left(\begin{smallmatrix} 0 & 1 \\ 0 & 0 \end{smallmatrix}\right) R \left(\begin{smallmatrix} 1 & 0 \\ 0 & 0 \end{smallmatrix}\right) = \left(\begin{smallmatrix} 0 & 0 \\ 0 & 0 \end{smallmatrix}\right)$ and it is easy to see that d is a derivation of R and F is a generalized derivation of R associated with d such that they satisfy any one of the following identities: (i) $F(x) \circ x = -d(x^2)$, (ii) $[F(x), x] = d(x^2)$ for all $x \in R$, but $d \neq 0$. Hence, in Theorem 2.4 the hypothesis of primeness and characteristic different than 2 cannot be omitted.

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