한국학교수학회논문집 제 24 권, 제 1 호
Journal of the Korean School Mathematics Society
Volume 24, Number 1, 59-81, March 2021
http://doi.org/10.30807/ksms.2021.24.1.004

Exploring the process of learning mathematics by repeated reading: Eye tracking and heart rate measurement[1]

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This study aimed to investigate how the learners' mathematics learning processes change with repeatedly reading mathematical text. As a way to teach and learn mathematics, we also wanted to examine the effect of repeated reading and to explore the implications for a more efficient teaching and learning strategy. To help us with this study, we mainly used eye tracking and heart rate (HR) measurement. There were four cycles in a cycle of repeated reading, and the number of repeated readings for all cycles was fixed to three times. Eight prospective mathematics teachers in the Department of Mathematics Education of a National University in South Korea participated. Data were analyzed in five aspects: (1) the total reading time per round, the total reading time per slide; (2) the change trends of total reading time per round and slide; (3) the order of slides read; (4) the change trends of HR per round. We found that most participants read in a similar pattern in the first reading, but the second and third reading patterns appeared more diverse for each learner. Also, the first reading required the most time regardless of the repeat cycle, and the time it took to repeatedly read afterward varied depending on the individual. Based on the findings of this study, the most primary conclusion is that self-directed mathematics learning by using repeated reading is effective regardless of cycle. In addition, we suggested four strategies to improve the efficiency of this teaching and learning method.

Key words: teaching and learning mathematics, repeated reading, eye tracking, heart rate (HR), prospective mathematics teachers

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* 2010 Mathematics Subject Classification: 97C70
1) This work was supported by the Ministry of Education of the Republic of Korea and the National Research Foundation of Korea (NRF-2018S1A5A2A01031370).
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Ⅰ. Introduction

Reading is a basic and essential skill in mathematics learning (Miller and Koesling, 2008; Yang and Yu, 2015) and is also emphasized as a subelement of mathematical communication ability (Kang, 2005; An, 2010). The requirements in mathematics reading might be different from those in linguistic reading because mathematical texts have characteristics that differ from those of texts other subjects. Mathematical texts contain more concepts and information than other domains do, as well as numbers, symbols, terms, and graphs to be interpreted (Kenney, Hancewicz, Heuer, Metsisto, and Tuttle, 2005). Mathematics reading is a positive and cognitive mental process aimed at acquiring meaning from mathematical texts, and the key elements in mathematics reading are interpreting characteristics (general terms, symbols, and graph), translating language, and understanding it (Yang and Yu, 2015).

One of the strategies that learners can commonly use to read text for learning purposes is repeated reading. Many learners, to increase their own understanding of the text, use this easiest and most common reading strategy (Baker and Brown, 1984; Ryu, 2010; Jang, 2011). To train learner's reading, Samuels (1979) endorsed the method of repeated reading. Samuels argued that this method is effective in raising vocabulary identification to an automatic level by enabling learners to gain more cognitive resources to aid understanding. We also know empirically that learners can check the information missing from the first reading through repeated reading. Repeated reading has a positive effect not only on the cognitive aspect but also on the affective aspect.

Repeated reading is an important strategy not only in language learning but also in math learning. However, it is difficult to conclude that the texts of mathematics textbooks have characteristics that differ from those of texts of linguistics, so that the process and effect of repeated reading may be the same. It means that the study of mathematics reading or repeated reading must be carried out separately. Studies focused on reading mathematics must be conducted considering the characteristics of mathematics textbooks and mathematical text in order to support mathematics learning. Teachers' understanding of how students generally read mathematical texts can help them realize their expectations for students. Given that there are many students with difficulties in reading mathematics in math classrooms, it is important for math teachers to understand students' overall form of math reading relative to educational intervention for their math reading ability.

Analyzing the repeated reading process allows us to divide the learner's learning process into parts, examine it, and check the effectiveness of prior knowledge based on the repetitive order of repeated reading. However, the research on reading comprehension has the limitation that it is impossible to confirm the reading process of the learner through the researcher's eyes. Many researchers used various methods such as reading error analysis, direct interview, post-interview, recall, and journal writing to overcome this kind of limitation (e.g., Newell and Simon, 1972; Goodman, Goodman, and Flores, 1979; Sim, 2014). It is difficult to avoid the disadvantage that the subjectivity of interviewers/examiners and interviewees/learners cannot be excluded even using these methods, and the objectivity of the results is somewhat insufficient. Campbell (2010) argued that physiological data such as eye tracking, pupillary response, electrocardiogram, electroencephalography, electrical skin response, and breathing could provide a deeper understanding of the psychological aspects of students in math teaching and learning.

Therefore, in this article, we defined “math reading” as reading for understanding and learning the mathematical content presented in the form of textbooks, and examined how the learners’ math reading processes change with repeated reading of mathematical texts. To show learners’ reading processes more objectively, we applied a research method that mainly employs eye tracking and heart rate (HR) measurement, as suggested in recent educational researches. On the basis of the objective and practical data collected, we intended to report that the repeated reading strategy could be used as an effective way to teach and learn mathematics.

Ⅱ. Theoretical Background

1. Reading and Mathematics Education

Reading can be misunderstood as simply looking at the text and speaking it out loud. In the early 1960s, pronunciation education was dominant in this perspective; however, looking at the history of reading theory in the linguistics field, the perspective on reading has changed (Shepherd, Selden, and Selden, 2012). Since the 1980s, under the influence of cognitive psychology, theorists have recognized reading as an active process in which readers construct new meaning by using and manipulating their own knowledge of language and the world (Rosenblatt, 1994). Reading is a dynamic process in which a reader's background knowledge and thinking system interact with the text in special situations (Grabe, 1991). By these changes, the reading concept has expanded to one that involves thinking and learning, from one that interprets a sign as a verbal expression by moving the eye on the page written with signs (Draper, 2002). From this point of view, readers integrate their existing knowledge structures with new information to create meaning in the reading process (Flood and Lapp, 1990).

In school education, reading plays an important role as a basic tool although it cannot be defined in one word. Given the reality of reading textbooks and reference books in all subject-learning, reading is an essential element of subject learning and a tool to improve thinking in any subject that one endeavors to learn (Kim, Park, Choi, and Kim, 2006). The lack of reading skills as a basic learning ability negatively affects learning in other subjects (Jenkins and O'Connor, 2002). Studies regarding reading in language education have been carried out extensively from various perspectives because reading is crucial to learning activities.

From a linguistic anthropologic perspective, language acquisition is thoroughly empirical, and as there are more opportunities for input on the same target item, learning possibilities are higher (Lee, 2014). It is possible for learners to newly convert and strengthen the learning information stored in the first reading through repeated reading (Baker and Brown, 1984). Repeated reading has a positive effect on both cognitive and affective aspects (Blum and Koskinen, 1991). On the other hand, Rasinski (1989) raised concerns that students might be exhausted or lose interest through repetition. In a later study, Rasinski (1990) concluded that teachers should provide their students with practical reasons to expand their own knowledge and to participate in classes.

Everyone might agree that reading plays a fundamental role in mathematics education. Many students have difficulty in reading math because of the nature of mathematical text or statements, and these difficulties may be one of the obstacles to math learning. Yang and Yu (2014) reported that many students made errors or had learning difficulties because of misunderstanding and misuse of the literal, symbolic, and graphic languages in math classrooms. Shepherd, Selden, and Selden (2012) asserted that students might have difficulties in reading math because of the peculiar characteristics of the ways of describing it. Simonson (2012) detailed four features of mathematical text, and explained that cross-referencing, scanning, pausing, returning, and so on are needed to understand math because math reading does not happen linearly.

Having students read math textbooks repeatedly might be an effective strategy to teach and learn math. Mathematical texts have characteristics that differ from those of linguistics texts. Most textbooks explain mathematical concepts in a similar manner (Raman 2004). Mathematical concepts are generally defined in terms of correct and clear terms by using formal logical notation (Weber, Brophy, and Lin, 2008) and require effort for abstraction and visualization (Shepherd, Selden, and Selden, 2012). However, according to Weber, Brophy, and Lin (2008), few studies have been conducted on how students learn math by reading. The literature on math reading prior to 2008 mostly covered topics related to reading strategies (e.g., Adams 2003; Kenney, Hancewicz, Heuer, Metsisto, and Tuttle, 2005). Even after 2008, few studies have dealt with the processes by which students read math or read it repeatedly. This implies that it is necessary to investigate the changes that appear in the process of reading math repeatedly when compared with the first reading.

2. Eye Tracking

Eye tracking is a research method that helps analyze what learners are interested in by investigating where they look at and what they learn with their visual attention (Bergstrom and Schall, 2014). An eye tracker that records the eyes' movements carries out this investigation. The light emitted from the eye tracker measures the light reflected from the pupil part of a learner's cornea, and this reflected light is recorded on a high-resolution camera attached to the monitor (Bergstrom and Schall, 2014). Eye trackers used in recent studies employ the method of tracking infrared light reflected from the cornea (e.g., Mason, Scheiter, and Tornatora, 2017; Hautala, Kiili, Kammerer, Loberg, Hokkanen, and Leppänen, 2018; Kahraman, 2019).

Our eyes are ceaselessly moving while processing texts, and at this time the eyeballs allow effective cognitive processing by placing new information in the field of view that has high resolution (Choi, 2016). If the difficulty of text during learning is high, the fixation duration of the gaze becomes longer, the distance of the saccades becomes shorter, and the rate of regressions increases (Ashby, Rayner, and Clifton, 2005). In the process of recognizing and processing the object, the fixation of the gaze means that the learner pays visual attention (Wedel and Pieters, 2000). Hence, a greater duration of fixation indicates more intense cognitive processing activity. The higher number of fixations the more focused the learner on recognizing the object. Conversely, a small number of fixed times means that the search for the object was less efficient (Goldberg and Kotval, 1999). According to Rayner (1998), the number of fixations and the duration of fixation are closely related to the learning process and achievement. That is to say, an increase in the number of fixations and the extended duration of fixation during learning mean that more cognitive processing has been performed and that learning achievement increases more. Because the duration of fixing the eyes can be viewed as the total amount of perceiving and processing the object, the gaze-fixing data can provide useful information in analyzing learners' cognitive processing.

Therefore, tracking the learners' eyes in their learning process enables checking specifically at what stage they visually recognize what information. This eye tracking technique might be an alternative to overcome the disadvantage of the post-interview that cause deformation and distortion of memory among the existing research methods. By controlling exogenous variables that can affect learners, data can be collected relatively, effectively, and accurately (Kim, Boo, and Kim, 2007). Moreover, we can use these data to analyze more objectively and quantitatively how a student learns math by repeated reading.

3. Heart Rate

Human's emotion is expressed through a series of physiological changes as well as psychological actions, and physiological signals can more objectively reflect human's real feeling (Xiefeng, Wang, Dai, Zhao, and Liu, 2019). In addition, it is possible to assess the level of mental stress by interpreting physiological responses as mental stress cannot be measured directly (Yoo and Lee, 2011). Psychophysiological parameters include HR, heart rate variability (HRV), electroencephalogram, blood pressure, and so on (Yoo and Lee, 2011; Xiefeng, Wang, Dai, Zhao, and Liu, 2019). Hence, this study uses changes in HR of participants as a way to show the emotions they feel in real time during repeated reading of mathematical content.

HR is the number of times a person's heart beats per minute. It reflects the state of the autonomic nervous system including the sympathetic and parasympathetic nerves (Berntson, Quigley, and Lozano, 2007; Park, Lee, and Jeong, 2007). It is known that in the stable state, the parasympathetic nerve predominates and HR decreases to about 60~70 beats per minute. Moreover, when we have physical and emotional stress, the activity of the sympathetic nerve becomes dominant and HR increases (Choi and Noh, 2004; Park and Jeong, 2014; Jerčić, Sennersten, and Lindley, 2020). Reflecting these characteristics, HR has generally been used as one of the physiological measures for cognitive aspects (e.g., Darnell & Krieg, 2019; Solhjoo et al., 2019). Jerčić, Sennersten, and Lindley (2020) reported that HR consistently increased as tasks struck to be more difficult. This result is consistent with the results of other studies when comparing HR response to the difficulty of the task (e.g., Sosnowski, Krzywosz-Rynkiewicz, and Roguska, 2004; Boutcher and Boutcher, 2006).

Ⅲ. Methods

1. Design

This study was designed to proceed in the order of pretest, repeated reading by period, and, posttest. The questions of pretest and posttest were about mathematics content knowledge in mathematics learning material for repeated reading. The cycle of repeated reading consisted of 4 cycles of daily, 3 days, 5 days, and 7 days, and the number of repetitive readings of all cycles was fixed to three times. Two prospective mathematics teachers were assigned to each cycle group, and there was no limit to the learning time for each participant who used repeated reading. Hence, participants autonomously adjusted the learning time each time and decided to end learning by themselves.

Complementing the above design details were the following: we simultaneously evaluated the learning achievement of students in all cycle groups before starting learning and after three repeated readings. This evaluation was conducted to interpret changes in eye tracking and HR with their achievement results in the presence of certain patterns. To minimize the influence between tests and readings, the pretest was conducted 3 days before the start of repeated reading learning, and the posttest was conducted 3 days after the learning was over.

2. Participants

This study was conducted after IRB (2018-0103) approval. We openly recruited eight prospective mathematics teachers who were second-year students in the Department of Mathematics Education of any National University in South Korea. All participants were informed that we would follow research ethics throughout the experiment and that they could withdraw their participation in the experiment at any time even if they applied for participation, and then they filled out a written consent form.

Eight prospective mathematics teachers were divided into four groups, with two students assigned to each group. We randomly assigned each to their group, considering their pretest scores, schedule, and gender, and explained to them individually about the overall process of learning math using repeated reading. At this time, we informed them that learning had to be limited to the corresponding time, that there was to be no additional studying, and that they could not discuss with each other with regard to experiments including what they learned.

Meanwhile, the experiment was conducted with the first eight students, but it was difficult to track the eyes of one student who was in the 7 days cycle group. Hence, she stopped participating midway, and another student was recruited in order for the experiment to begin with the same procedure. <Table Ⅲ-1> provides detailed information regarding the eight students who finished the experiment, including their pretest and posttest dates, and the dates on which they learned using reading.

<Table Ⅲ-1> Participants and dates

Cycle	Participants		Dates(mm/dd)				
	ID	Gender	pretest	First Reading	Second Reading	Third Reading	posttest
1-day	1-A	F	03/19	03/22	03/23	03/24	03/27
	1-B	M	03/22	03/25	03/26	03/27	03/30
3-day	3-A	F	03/17	03/20	03/23	03/26	03/29
	3-B	M	03/18	03/21	03/24	03/27	03/30
5-day	5-A	F	03/17	03/20	03/25	03/30	04/02
	5-B	M	03/17	03/20	03/25	03/30	04/02
7-day	7-A	F	03/26	03/29	04/05	04/12	04/15
	7-B	M	03/16	03/19	03/26	04/02	04/05

3. Reading Material

Sindelar, Monda, and O'Shea (1990) reported that it is effective to use the reading text of an appropriate level for the target students in order not to make it overly difficult for them to understand and comprehend through reading. Hence, we intended to select the mathematical content for reading material by considering the level at which the participants could learn by themselves, on basis of the high school mathematics curriculum and the concepts of calculus at liberal arts for first grade. We reconstructed into the reading material by extracting somewhat easy content on number theory, on which none of the participants had taken the class until then.

The reading material consists of a total of five slides, and details of which are as follows: two definitions, six theorems and their proofs, and three examples (see Appendix). <Table Ⅲ-2> summarizes the composition of each slide.

<Table Ⅲ-2> The composition of reading material

Slide	Composition
1	Definition 1: Definition of divisor and multiple and their notations Theorems 1 and 2: Two propositions related to Definition 1 and their proofs
2	Definition 2: Definition of the modulus congruence and its notation Example 1: Examples on Definition 2
3	Theorems 3 and 4: Two propositions related to Definition 2 and their proofs
4	Corollary 5: Proposition related to Theorem 4 Theorem 6: Proposition about the modulus congruence of an integer and the remainder and its proof Example 2: Simple examples on Theorem 6
5	Example 3: Complex examples on Theorem 6

4. Test Tool

The pretest and posttest tools were identical. The pretest and posttest tool consisted of 10 questions, comprising five questions to judge true or false and five descriptive problems that asked both the answer and solution. For the true or false questions, the participants were asked not to guess the answer but only to answer when they were convinced of where the answer was true or false. For the descriptive problems, we asked them to leave all that they wrote without erasing. <Table Ⅲ-3> shows 10 items of the test tool.

<Table Ⅲ-3> The items of test tool

Sort	Items
True or False	1. Let a and b be positive integers. If $a\|b$, then $a \le b$. 2. Let a and b be integers. If $a\|b$, then $(-a)\|(-b)$. 3. Let a, b, and c be integers. If $a\|c$ and $b\|c$, then $c\|(a+b)$. 4. Let a, b, c, and m be integers with $m > 0$. If $a \equiv b \pmod{m}$ and $c \equiv d \pmod{m}$, then $a - c \equiv b - c \pmod{m}$. 5. Let a, b, and m be integers with $m > 0$. If $ab \equiv 0 \pmod{m}$ and $a \equiv 0 \pmod{m}$, then $b \equiv 0 \pmod{m}$.
Descriptive (solution and answer)	6. Find all integer c with $0 \le c \le 10$ such that c is congruent to 5 modulo 3. 7. Find all integer c with $-50 \le c < 50$ such that c is congruent to 27 modulo 23. 8. Find the least positive integer m that is congruent to $3 \times 10^3 + 4 \times 10$ modulo 9. 9. Find the remainder when 2^{20} is divided by 7. 10. Find the remainder when $3^{11} \times 7^{55}$ is divided by 5.

5. Data Collection and Analysis

Gazepoint's GP3 HD Eye Tracker synchronized with a biometrics kit was used to measure the eye movements and HR of the participants during learning via reading material. Gazepoint's GP3 HD Eye Tracker collects data from both eyes of the participants at up to 150 Hz units with 0.5–1.0 degree of visual angle accuracy. This eye tracker was installed under a 20 inch, 1600 × 900 pixels monitor approximately 65 cm away from the participant. A finger wearable HR sensor was positioned on the left of a desk for it to measure HR with the left middle finger of the participant. This sensor did not provide measures of HRV. A mini keyboard, controllable by the participant, was placed on the right of the desk. This mini keyboard consisted of Enter and Back keys.

Data were collected from eight participants whose uncorrected vision and corrected visual acuity with either lenses or glasses were more than 0.7. The participants were tested without having caffeine drinks before the experiment. They were seated in a chair, had calibrated viewpoints, and controlled their breath prior to the measurement. They used the mini keyboard to switch over the presented slide on the monitor, which they did by following their own individual learning pace, while proceeding with reading. In addition, they learned the content of the presented slide on the monitor screen with their eyes without any writing supplies. During their reading, eye tracking data and HR data were collected in 150 Hz units (i.e., 1 datum per millisecond or 150 data per second). However, some data for HR were missing in the 7-B participant's second reading process.

The measured data from all participants were extracted through the Gazepoint Analysis program. Spreadsheet and graphical data were summarized and analyzed in Microsoft Excel. As for the primary objective of this study, data were analyzed for all eight participants in five aspects: the total reading time per round, the total reading time per slide, the change trends of total reading time per round and slide, the order of slides read, and the change trends of HR per round. The total reading time per round was calculated as the total time taken by each participant from the start to the end of reading, in the corresponding round. The total reading time per slide was calculated as the cumulative reading time for the corresponding slide, including the time at which the participant went back and read the corresponding slide again. The change trends of total reading time per round and slide were represented by graphs connected by lines. The order in which each participant in each round read the slides were listed without considering the time taken. The change trends of HR per round graphed the continuous change of participant's HR from the start of reading to the end of each round.

Ⅳ. Results and Discussion

1. Pretest and Posttest Scores

Each question was scored as 10 points if correct and 0 points as incorrect. It was a total of 10 questions, so 100 points was the perfect score. <Table Ⅳ-1> shows the pretest and posttest results of eight participants. All seven participants except one participant (5-B) scored more than 60 points. These results implicate that repeated reading could be used as a self-directed learning method for mathematics regardless of cycle. For the exceptional 5-B case, we need to look deeper at its cause or reason. On the other hand, the repeated reading cycles in which both members of each group scored 70 or higher on the posttest were a daily cycle and a 3-day cycle. In particular, 1-A not only solved the two questions that were solved in the patterns in the pretest by using the corresponding learning contents in the posttest but also answered all 10 questions correctly.

<Table Ⅳ-1> pretest and posttest scores

	Test	1-A	1-B	3-A	3-B	5-A	5-B	7-A	7-B
	Pre	20(1)	10(3)	0	0	0	20(7)	0	0
Post	T/F	50	40	40	20	40	20	40	30
	Descriptive	50(2)	30(4)	30(5)	50	40(6)	30(8)	30	30
	Total	100	70	70	70	80	50	70	60

(1) Descriptive problems Q9 and Q10 were solved by patterns.
(2) Q9 and Q10 were solved by the corresponding learning content.
(3), (5), (6) Q8 was solved by calculation, not by the corresponding learning content.
(4) Q9 was solved by the corresponding learning content but Q8 by calculation.
(7) Q9 and Q10 were solved by patterns.
(8) Q8 was solved by calculation, and Q9 and Q10 by patterns.

2. Total Reading Time by Round

Two hypotheses (H1 and H2) were set for the total reading time according to the round and cycle: (H1) as the reading was repeated, the total reading time per round would decrease. (H2) if the repeated read cycle became long, the total reading time per round would not decrease significantly when compared with the shorter cycle even though it was repeatedly read. The basis for these hypotheses was our experience with repeated reading. We spent a little more time to understand new content when we read something like a book for the first time. However, if the repeated reading cycle became long, it would take more time to understand the content again because one tends to forget more of the first reading content than one does when the cycle is short.

<Table Ⅳ-2> shows the total reading time of eight participants by round. As expected, seven participants, excluding one participant (5-A), devoted the most time to the first reading, regardless of the reading cycle. The 5-A student spent more time on the second reading than she did on the first. For 7-A, the total time of the first reading was longer than the second, but there was only a difference of about 30 s. On the other hand, when comparing the total time of the second reading with the total time of the third, five participants (1-A, 3-B, 5-A, 5-B, and 7-B) spent more time at the second reading than they did at the third. However, for 7-B, the total time of the second reading was longer, but there was only a difference of about 19 s. The other three participants took 2-15 s more for the third reading. These results suggest that when learning new content, the first reading requires the most time regardless of the repeat cycle, and the time it takes to repeatedly read afterward varies depending on the learner.

It is also worth noting that there was one participant with little change in the total reading time per round in both groups with 5 and 7 reading cycles (5-A and 7-A). 5-A and 7-A were both female students and scored higher on the posttest when compared with the other member of each group that they belonged to.

<Table Ⅳ-2> Total reading time per round (min:s)

Round	1-A	1-B	3-A	3-B	5-A	5-B	7-A	7-B
First	34:17	08:35	12:59	20:23	07:33	13:20	10:51	13:54
Second	17:52	05:50	07:37	12:04	08:34	06:31	10:22	05:49
Third	14:12	05:52	07:50	09:42	06:50	04:56	10:35	05:30
Total	66:21	20:17	28:26	42:09	22:57	24:47	31:47	25:13

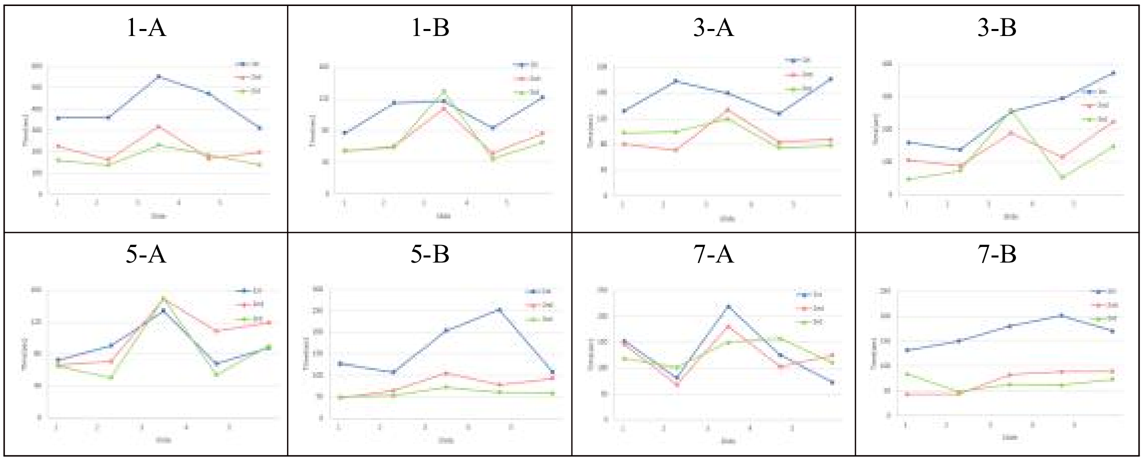
The four participants who had a relatively long total time for three readings were 1-A, 3-B, 7-A, and 3-A, and their posttest scores improved by more than 70 points when compared with their pretest scores. Of these, the reading cycles of three participants excluding 7-A were relatively short cycles of 1 day and 3 days. The total reading time of three readings of 1-B who repeated reading on a daily cycle, was the shortest among the eight, and the posttest score improved by 60 points from the pretest score. From these results, it can be suggested that the shorter the repeated reading cycle and the longer the reading time, the more positively mathematics learning is affected.

On the other hand, in addition to the above four participants, 5-A was the only participant who improved by 80 points on the posttest. In the case of 5-A, the repeated reading cycle was relatively long, and the total reading time was short, but in what is common with 7-A is that her total reading time per round was similar in the process of repeatedly reading three times. From the results of 5-A and 7-A, it may be inferred that the same effort was made in each round during repeated reading, resulting in positive learning results.

3. Total Reading Time by Slide

We now look in more detail by linking the test scores on the basis of the total reading time for each slide in relation to the two hypotheses (H1 and H2) about the total reading time by round and cycle. To that end, let us examine the change trends of the total reading time of each slide per round of repeated reading (see [Figure Ⅳ-1]). For the two groups that learned on a 1-day and 3-day cycle, it was found that all four participants almost met the first hypothesis (H1). In other words, all of them spent the most time on the first reading and the second and third are similar (1-A and 3-B) or reduced (1-B and 3-A). Furthermore, they all scored 70 points or more on the posttest. Participant 1-A, who had a long learning time overall and the highest posttest score, spent the most time on slide 3 in each round while repeatedly reading three times. Slide 3 contained the theorems of *congruence modulo* m with their proofs that participants had never learned before. This seems to be because they learned the definition of *congruence modulo* m and its properties for the first time, unlike the contents of factor which they were familiar from having learned in secondary school. The other three participants, 1-B, 3-A and 3-B, spent the most time on slide 5 in the first reading but spent relatively increased time on slide 3 when compared with other slides in the second and third readings. In particular, the time when both of 1-B and 3-B spent on slide 3 was longer in the third reading than it was in the first and second.

In addition, the time when 3-A spent on slides 1 and 2 was longer in the third reading than it was in the second. These change trends imply that they spent more time and effort on what they thought was more important or needed more attention as the reading was repeated.



[Figure Ⅳ-1] The change trends of total reading time per round and slide

On the other hand, each participant in the two groups, learning on a 5-day and 7-day cycle, showed different reading trends. 5-A and 7-A did not meet the first hypothesis (H1), but met the second hypothesis (H2). Conversely, 5-B and 7-B met (H1) and did not meet (H2). That is to say, the total reading times per round of 5-A and of 7-A almost did not decrease in the process of reading repeatedly, as their repeated reading cycles were rather long. The total reading times per round of 5-B and 7-B were reduced in the process of reading repeatedly even though their repeated reading cycles are rather long. They showed a different tendency in the posttest scores as well as a different of change trend in repeated reading. 5-A and 7-A got the same or higher improvement scores as 3-A and 3-B, whereas 5-B and 7-B had lower scores on the posttest when compared with the other member of their groups. In particular, it is worth considering that the posttest score of 5-B increased by only 30 points when compared with the pretest score, and the posttest score of 7-B was also somewhat lower when compared with the other six participants.

In the case of 5-A and 7-A, the total reading times per round were almost the same, but the slides that took more time changed as the reading was repeated. This trend reveals that they were not learning by the same pattern every round during repeated reading even though the cycles were rather long. 5-A showed a tendency to learn for a longer duration afterward on the slides that initially learned shorter duration, and a shorter duration afterward on the slides that initially took more time, but she spent the longest on slide 3 in every round. 7-A had a similar tendency for first and second readings, including spending the most time on slide 3, whereas in the last reading, she spent relatively longer times on slides 3 and 4 but a similar time overall. As in the case of four participants who learned on 1-day and 3-day cycles, 5-A and 7-A generally focused on slide 3 and the more time-consuming slides were changed according to round. However, unlike the case of the other six participants, the repeated reading trends of 5-B and 7-B did not show any slides that they took noticeably longer times in the second and third readings. In their case, the time spent on each slide was reduced in three readings for all four slides except slide 1. These two contrasting aspects of the two groups learning on 5-day and 7-day cycles suggest the importance of grasping what needs to be learned more and practicing it.

Overall, let us take a look at which slide the eight participants concentrated on by investigating their total reading times per slide. <Table Ⅳ-3> shows the total reading time trends of all eight participants for each slide by adding up the time allotted to each slide. Of the six participants who scored 70 or higher on the posttest spent, all five except 3-B spent the longest time learning slide 3. In the case of 3-B, more time was allocated to slide 5; however, the time spent on slide 3 was relatively longer when compared with that spent by other participants on that slide. The two participants (5-B and 7-B) with a slightly lower score read slide 4 the longest. Most participants who scored 70 or higher on the posttest spent the most time learning slide 3, but the reading pattern of the two participants with slightly lower scores appeared differently. It was interpreted that each participant spent more time and effort on the content that they thought was more necessary for their own learning, and there was somewhat of a difference in the learning results accordingly. These results suggest that it is essential to learn by grasping which content requires more time and effort or is more important, when learning by using repeated reading.

<Table Ⅳ-3> Total reading time per slide (min:s)

Slide	1-A	1-B	3-A	3-B	5-A	5-B	7-A	7-B
1	12:25	3:06	5:11	5:14	3:24	3:46	6:55	4:19
2	11:02	3:54	5:49	5:01	3:31	3:45	4:11	4:02
3	18:22	5:55	6:52	11:43	7:14	6:23	9:10	5:26
4	13:43	2:59	4:46	7:44	3:51	6:32	6:25	5:52
5	10:49	4:24	5:47	12:27	4:58	4:20	5:07	5:33

4. The Order of Slides Read

In general, math learners will not read in one order without going back from beginning to end because of the unique features of mathematics that consist of definitions, theorems and their proofs, examples, and so forth. That is to say, the readers learn the entire contents while repeating the process of rechecking the previous contents as necessary. Considering this aspect, we established two hypotheses (H3 and H4) for the order of slides read along rounds and cycles: (H3) the more learners repeat the reading, the less frequent the process of going back to the previous slide and rechecking. (H4) if the repeated read cycle becomes longer, the frequency of the process of going back to the previous slide and rechecking will not reduce significantly when compared with that of the shorter cycle.

<Table Ⅳ-4> shows the order of slides on which the eight participants read the reading material. Before examining the two hypotheses, let us view overall how participants read when learning math content. In the first reading, when all participants except for 7-B read slide 3 for the first time, they returned to slide 2 to read again. Given that the contents of slide 3 are the theorems associated with the modulus congruence defined in slide 2, this demonstrates that most math learners studied the propositions of a math concept on the basis of its definition and recognized the importance of defining and notation math concepts. They went back and rechecked slide 2 at different speeds: less than 1 s (3-B and 7-A), 1-2 s (1-A, 5-A, and 5-B), 4 s (3-A), 11 s (1-B). This indicates that the time taken to go back and check the previous slide again was relatively short. Subsequently, most of them (1-A, 1-B, 3-B, 5-A, 5-B, and 7-A) returned to slide 1 and read again from the beginning. 3-A returned to slide 3 again, but after that, she read slide 2 once more and then read again from the beginning like other participants. As we have seen, in the first reading, most participants read in a similar pattern. However, the second and third reading patterns appeared more diverse for each learner.

<Table Ⅳ-4> The order of slides read

Participant	Round	The order of slides
1-A	First	1→2→3→2→1→2→3→4→5→4→3→2→1→2→3→4→5
	Second	1→2→3→2→3→4→5
	Third	1→2→3→4→5
1-B	First	1→2→3→2→1→2→3→4→5
	Second	1→2→3→2→3→4→5
	Third	1→2→3→2→3→4→5
3-A	First	1→2→3→2→3→2→1→2→3→2→1→2→3→4→5
	Second	1→2→3→2→1→2→3→4→5
	Third	1→2→3→2→1→2→3→4→5
3-B	First	1→2→3→2→1→2→3→2→1→2→3→2→1→2→3→4→3→4→5→4→5
	Second	1→2→3→4→5
	Third	1→2→3→4→5
5-A	First	1→2→3→2→1→2→3→4→3→2→3→4→5
	Second	1→2→3→4→5→4→5
	Third	1→2→3→4→5
5-B	First	1→2→3→2→1→2→3→4→5
	Second	1→2→3→4→5
	Third	1→2→3→4→5
7-A	First	1→2→3→2→1→2→3→2→1→2→3→4→5→4→3→4→5
	Second	1→2→3→2→1→2→3→2→3→2→1→2→3→4→5→4→5→4→5
	Third	1→2→3→2→1→2→3→4→5
7-B	First	1→2→3→4→5→4→5
	Second	1→2→3→4→5
	Third	1→2→3→4→5

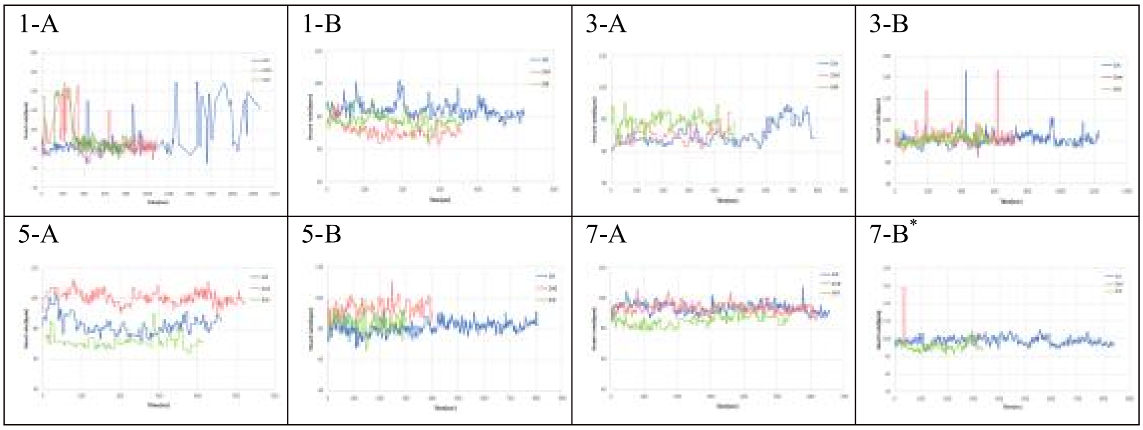
Next, let us look at the results related to hypothesis (H3). All participants except for 7-A had the most frequency of returning to the previous slide during the first reading process. In the case of 7-A, the frequency of returning to the previous slide and rechecking was more in the second reading process than it was in the first reading. Upon comparing the second and third reading processes, both courses were the same (1-B, 3-A, 3-B, 5-B, and 7-B) or the frequency in the third was reduced (1-A, 5-A, and 7-A). These results generally reveal that the more learners repeat their reading, the less they go back to the previous slide to recheck regardless of the repeat cycle. In particular, it is worth noting that the repeated reading learning processes of 5A and 7-A, which actively support this hypothesis (H3). For 5-A, the frequency of returning to the previous slide to recheck was reduced although the total time of the second reading was longer than that of the first. For 7-A, the frequency of returning to the previous slide was noticeably reduced despite the similar total reading times of the second and third readings.

As for hypothesis (H4), the results were different from the prediction. That is to say, the length of repeated reading cycles did not necessarily mean that the frequency of returning to the previous slide and reading again was higher. Participants (e.g., 5-B and 7-B) who learned with the longer repeated reading cycles did not return more frequently to the previous slide, and participants (e.g., 1-A and 3-A) who learned with shorter cycles returned to the previous slide more often. From these results, it can be said that returning to the previous content and checking it again is more related to the learner's tendency or attitude than it is to the repeated reading cycle.

On the other hand, in both the second and third reading processes, there were three learners (3-B, 5-B, and 7-B) who read five slides in sequence without returning to the previous slide and reviewing them again. Of these, 5-B had the lowest posttest score, and 7-B had the next lowest. It is notable that both of these learners belonged to groups with longer repeated reading cycles and had the lowest frequency of returning to the previous slide in the first reading process. 1-B was the learner with the least frequency of returning to the previous slide in the first reading process. Thus, 1-B and 3-B, who got higher scores on the posttest, had one common thing, but they also had the differences. Both of 1-B and 3-B had relatively short repeated reading cycles. 1-B returned to the previous slides in both the second and the third readings, but 3-B did not so. Instead, 3-B rechecked by returning to the previous slides most frequently in the first reading when compared with the other seven participants. These differences reveal that they learned new contents by returning to the previous content and rechecking more frequently, despite the fact that their cycles for repeated reading were shorter.

5. Change Trends of HR by Round

The primary objective of our study was to examine how the learners' mathematics learning processes change with repeated reading. In this section, as an aid to assess this question, we examined the trends of HR change during times when learners were involved in repeated reading. It was assumed that when the learners repeated the reading, if they discovered the newly perceived content as though seeing the previously read content for the first time, HR would tend to increase because of tension. In addition, we supposed that such tension would have the effect of making the learners more focused. Under these assumptions, we want to examine and interpret the trend of change in HR of learners. [Figure Ⅳ-2] shows the trends of HR change per round for the eight learners.



[Figure Ⅳ-2] The change trends of HR (bpm) per round (*in the second reading process of 7-B, some data were lost and excluded from the graph)

First, it seems that it is difficult to find common features when the repeat reading cycles were daily and 3 days. In the case of both 1-B and 3-A, HR in the third reading process was generally higher than that in the second reading process. However, when looking at HR in the first read, 1-B was generally higher than the other two reading processes, and the 3-A was the lowest. For 1-A, high HR was found in the latter part of the first reading and in the former parts of the second and third readings. For 3-B, HR indicated high singularity in the first and second readings, but generally, all three rounds showed similar HR. From this, it can be seen that the trend of HR change in the rounds is slightly different for each learner but that repeated reading provided learners with the opportunity to tense up and concentrate.

Next, the participants who repeatedly read in 5-day and 7-day cycles revealed some commonalities. The data for the second reading process of 7-B were partially missing and were considered on the basis of some measured data (maximum of 157 bpm and minimum of 97 bpm). For all four participants who learned by these cycles, HR appeared higher in the second reading process than it was in the third. In particular, for 5-A, 5-B and 7-B, it can be noted that HR in the second reading process was noticeably higher than it was in the first. We interpret this phenomenon as a result of embarrassment and tension with a new feeling about what they learned in the first reading because this phenomenon did not appear for the participants belonged to the groups with a rather short cycles. This interpretation is also supported by the result of 7-A. She spent almost the same amount of time in both the first and second readings, and the change patterns of HR in both readings were similar, too. Thus, it is important to re-state at this point that repeated reading, regardless of the cycles, provides all learners with the opportunity to concentrate more on learning contents.

Ⅴ. Conclusion and Suggestions

We measured and investigated the process of eight prospective mathematics teachers who learned math by using repeated reading, by employing two contrivances: eye tracking and HR measurement.

In addition, we conducted the pretest and posttests to examine the effect of the repeated reading learning method and explored the implications for a more efficient repeated reading teaching-learning strategy by variegating repeated cycle into four types. Based on the findings of this study, the conclusions on the effectiveness of the repeated reading mathematics teaching-learning strategy and suggestions for improving the efficiency of this strategy are summarized as follows.

First, the most primary conclusion in our study is that self-directed mathematics learning using repeated reading is possible. The improved scores on the posttest differed, depending on the individual, but all the prospective mathematics teachers who participated in this study presented evidence that learning mathematics by using repeated reading had positive results. From this, although the degree of the effect might be different depending on the individual's tendency or attitude toward mathematics, it can be concluded that repeated reading can be used as an effective method of teaching-learning mathematics, Of course, the comprehension ability to understand language and mathematical symbols, as well as prior knowledge related to the current learning content, should be given priority for self-directed mathematics teaching-learning using repeated reading to be successful. Considering these prerequisites, mathematics repeated reading learning might work as a more effective strategy for higher grades or prospective mathematics teachers than it might for lower grades.

Second, for more efficient self-directed mathematics learning, we propose to respect the repeated reading cycle and total reading time determined by the learner autonomously. Hence, it is not necessary to uniformly set the repeated reading cycle or the total reading time and apply it to all learners. This is based on the results of two learners who improved by 80 points on the posttest over the pretest. One learner had a repeated reading cycle of 1 day but the longest total reading time, whereas the other had a 5-day cycle, but the total reading time was the second shortest. From this, we interpreted that the effect of repeated reading could appear differently depending on the learner's individual learning ability and affective attitude toward mathematics learning, not on the reading cycle or total reading time. Nevertheless, it is expected that the cycle would be shorter and that the total reading time would be longer if the learner's desire to learn was strong.

Third, for more efficient repeated reading for mathematical content, it is suggested that learners be guided with information on learning elements that are more important and require more attention among the gamut of learning content in the appropriate teaching-learning stage. In other words, it is necessary to provide an opportunity for learners to check and reflect on whether they have understood more important learning content in the repeated reading process of teaching-learning. This is a suggestion based on the results of two learners who improved their scores, which were relatively low on the posttest. This is because most learners with relatively high posttest scores spent a lot of time and effort on the contents in slide 3 during repeated reading, whereas these trends did not appear in the two learners. Therefore, we interpreted that it is necessary for the instructor to directly inform some learners of what learning factors are important and to provide them with the opportunity to read more carefully. However, a careful approach might be required as to when instructor will provide students with this information. This is because providing such information from the beginning may cause negligent reading of other learning content. We would like to add that it is necessary to design with sufficient consideration in the planning stage of the repeated reading teaching - learning.

Fourth, we suggest that the repeated cycle be made shorter than 5 days for more efficient second and third readings. In other words, it is more effective to do the second reading before forgetting more by comparing the content learned in the first reading with the shorter repeated reading cycle. It is a suggestion based on the results of the total reading time and HR for each round of two learners who obtained 70 points or more scores on the posttest among the learners who repeatedly read every 5 or 7 days. These two learners achieved scores of improvement that were similar to those achieved by learners reading repeatedly on daily or 3-day cycles. However, for them, the total reading times of the second and third readings were overall longer or similar when compared with those of the first, and HR in the second reading was not only relatively higher than that in the third but also even higher or not lower than that in the first. From these findings, we interpreted that a learner with a long repeated reading cycle would forget more of what was learned in the first or second readings, and become more embarrassed as a result. Therefore, we suggest that a learner reads repeatedly the mathematics textbooks by the cycle that is as short as possible to remember longer the content that they previously read and use the time more effectively in subsequent readings.

Studies on repeated reading have been conducted mainly in the field of language education. This study was conducted in the field of mathematics education, and we investigated the mathematics learning process of prospective mathematics teachers using repeated reading with eye tracking and HR measurement. In addition, experiments were conducted with different repeated reading cycles, and evaluation was conducted to further examine the effect. In the mathematics learning process, various patterns were observed according to learners, repeated reading cycles, and so forth. Although reading patterns may vary, it has been shown that repeated reading is consistently an effective math learning strategy for students who have prior knowledge of the learning contents of learning. On the other hand, mathematics learning using repeated reading may vary depending on the student's age and achievement level; hence, various follow-up studies are needed to verify it. In addition, we expect follow-up studies that explore various variables to maximize the effectiveness of teaching - learning using repeated reading.

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반복 읽기를 이용한 수학 학습의 과정 분석: 시선의 움직임 추적과 심박수 측정을 중심으로[1)]

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초 록

이 연구에서는 학습자가 수학 텍스트를 반복하여 읽을 때 나타나는 수학 학습 과정이 어떻게 변하는지를 조사하였다. 또한 수학 학습 방법으로써 반복 읽기의 효과를 점검하고 보다 효율적인 반복 읽기 교수·학습 전략에 대한 시사점을 모색하였다. 반복 읽기 수학 학습에는 국립대학교 수학교육과에 재학 중인 예비 수학교사 8명이 참가하였다. 예비 수학교사는 각각 4개의 그룹으로 구성되어 그룹에 따라 서로 다른 4개의 주기로 총 3회 반복 읽기를 시행하였다. 수학 학습 자료 읽기 과정에서 나타나는 예비 수학교사의 시선의 움직임을 추적하고 심박수를 측정하였다. 수집한 자료를 회차별 총 읽기 시간, 슬라이드별 총 읽기 시간, 각 회차와 슬라이드별 총 읽기 시간의 변화 추세, 슬라이드 읽기 순서, 회차별 심박수 변화 추세 등의 다섯 가지 측면에서 분석하였다. 첫 번째 읽기에서는 참가자의 대부분이 비슷한 양상을 보였으나, 두 번째와 세 번째 읽기에서는 개별 학습자에 따른 읽기 패턴의 변화가 보다 다양하게 드러났다. 또한, 첫 번째 읽기에서 반복 주기와 무관하게 가장 많은 시간이 소요되었고, 이후 반복적 읽기 시간에서는 개인별로 차이가 나타났다. 연구 결과에서 도출한 가장 중요한 결론은 반복 읽기를 통한 자기 주도적 수학 학습은 주기와 관계없이 효과적이라는 것이다. 추가적으로 반복 읽기 교수·학습 전략의 효율성을 증진시키기 위한 네 가지 전략을 제안하였다.

주요용어: 수학 교수·학습, 반복 읽기, 시선추적, 심박수 측정, 예비 수학교사

Received January 28, 2021
Revised February 26, 2021
Accepted February 26, 2021

* 2010 Mathematics Subject Classification: 97C70

1) 이 논문은 2018년 대한민국 교육부와 한국연구재단의 지원을 받아 수행된 연구임(NRF-2018S1A5A2A01031370).

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<Appendix> A Part of the Reading Material

Slide 1	**정의 1)** 두 정수 a, d에 대하여 $a = de$ 인 정수 e가 존재할 때, d를 a의 약수 또는 인수라 하고, a를 d의 배수라고 한다. 이것을 $d \mid a$로 나타낸다. 한편 $d \mid a$가 아닐 때, $d \nmid a$로 나타낸다. **정리 1)** 두 정수 a, b에 대하여, $a \mid b$이면 $a \mid (-b)$, $(-a) \mid b$, $(-a) \mid (-b)$이다. **(증명)** $a \mid b$이면, 적당한 정수 k에 대하여 $b = ak$이다. 이때 $-b = a(-k)$, $b = (-a)(-k)$, $-b = (-a)k$ 이므로, $a \mid (-b)$, $(-a) \mid b$, $(-a) \mid (-b)$이다. **정리 2)** 세 정수 a, b, c에 대하여 $c \mid a$, $c \mid b$이면, 임의의 두 정수 x, y에 대하여 $c \mid (ax+by)$이고, 특히 $c \mid (a+b)$, $c \mid (a-b)$이다. **(증명)** $c \mid a$, $c \mid b$이면, 적당한 정수 k, m에 대하여 $a = ck$, $b = cm$이다. 이때 $ax+by = c(kx+my)$이므로, $c \mid (ax+by)$이다. 특히 $x=1, y=1$일 때 $c \mid (a+b)$이고, $x=1, y=-1$일 때 $c \mid (a-b)$이다.
Slide 2	**정의 2)** 고정된 양의 정수 m에 대하여, 두 정수 a, b의 차가 m의 배수일 때, 즉 $m \mid (a-b)$일 때, a와 b는 법(modulus) m에 관하여 합동이라 하고, 이것을 $a \equiv b \pmod{m}$으로 나타낸다. 즉, $a \equiv b \pmod{m} \Leftrightarrow m \mid (a-b)$ 또한 $a \equiv b \pmod{m}$이 아닐 때, 이것을 $a \not\equiv b \pmod{m}$로 나타낸다. 그리고 합동 기호 $\equiv$가 들어 있는 식을 합동식이라고 한다. 정의에 의하여 $a \equiv 0 \pmod{m} \Leftrightarrow m \mid a$이고, 또한 다음이 성립한다. $a \equiv b \pmod{m} \Leftrightarrow a - b \equiv 0 \pmod{m}$ **보기 1)** 정수 a에 대하여 $a \equiv -a \pmod{2}$이다. 또한 다음이 성립한다. $7 \equiv 4 \pmod{3}$, $6 \equiv 0 \pmod{3}$, $5 \equiv -4 \pmod{3}$, $10 \equiv 6 \pmod{4}$, $5 \equiv 1 \pmod{4}$, $7 \equiv -1 \pmod{4}$ 시계에서 시간을 가리키는 긴 바늘은 12 또는 24를 법으로 하여 같은 시간을 나타내고, 달력에서는 7을 법으로 하여 같은 요일이 되풀이된다. 일 | 월 | 화 | 수 | 목 | 금 | 토 1 | 2 | 3 | 4 | 5 | 6 | 7 8 | 9 | 10 | 11 | 12 | 13 | 14 15 | 16 | 17 | 18 | 19 | 20 | 21 22 | 23 | 24 | 25 | 26 | 27 | 29 30 | 31 
⋮	⋮
Slide 5	**보기 3)** $2^{56} \times 19^{41}$을 13으로 나누었을 때의 나머지는 5이다. 실제로, $2^2 \equiv 4 \pmod{13}$, $2^4 \equiv 3 \pmod{13}$, $2^8 \equiv 3^2 \equiv 9 \pmod{13}$ $2^{12} \equiv 3 \times 9 \equiv 27 \equiv 1 \pmod{13}$ $2^{56} \equiv (2^{12})^4 \times 2^8 \equiv 1^4 \times 9 \equiv 9 \pmod{13}$ $19 \equiv 6 \pmod{13}$, $19^2 \equiv 10 \pmod{13}$ $19^3 \equiv 60 \equiv 8 \pmod{13}$, $19^5 \equiv 80 \equiv 2 \pmod{13}$ $19^6 \equiv 12 \equiv -1 \pmod{13}$, $19^{12} \equiv (-1)^2 \equiv 1 \pmod{13}$ $19^{41} \equiv (19^{12})^3 \times 19^5 \equiv 1^3 \times 2 \equiv 2 \pmod{13}$ 이다. 따라서 $2^{56} \times 19^{41} \equiv 9 \times 2 \equiv 18 \equiv 5 \pmod{13}$이다