Print ISSN: 2233-4165 / Online ISSN 2233-5382
JIDB website: http://www.jidb.or.kr
doi:http://dx.doi.org/10.13106/jidb.2021.vol12.no5.7

When VMI with Consignment Brings Benefit to Supply Chain Members?

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Received: February 28, 2021. Revised: March 30, 2021. Accepted: May 05, 2021.

Abstract

Purpose: This study intends to examine how VMI with consignment performs over time and find out its on-going impacts on the individual supply chain member's achievement. **Research design, data, and methodology:** This study formulates the mathematical model that represents two-stage supply chain system. By analyzing the numerical examples, this study compares VMI with consignment with the traditional system. **Results:** VMI with consignment eventually makes higher supply chain profit than the traditional system, even though it's early performance is poor. The influence of VMI with consignment on the performance of the supply chain member is distinct depending on the individual member and time. The consignment may not be helpful to increase the system profit, but it reduces the manufacturer's burden of costs. **Conclusions:** VMI with consignment improves the supply chain performance after all, and it still takes times until its benefit becomes fully realized. To be a successful collaboration program, VMI with consignment requires a carefully designed incentive scheme that provides the timely compensation to the individual supply chain members. This study also finds out that the consignment contract of this collaboration program plays a role of financially supporting the manufacturer at the early stage of its implementation.

Key words: Supply Chain Collaboration, Vendor-Managed Inventory, Consignment, Optimal Control

JEL Classifications: M11, M19, M21, M29

1. Introduction

In the history of business, there have been endless efforts to overcome the inherited problem of double marginalization and improve the entire supply chain performance (Egri & Vancza, 2013; Li, 2013). In the line with that movement, the supply chain collaboration programs have evolved into more advanced forms such as Efficient Consumer Response (ECR) and Vendor-Managed Inventory (VMI). In particular, these collaboration programs are not exclusively applied to the industries, and instead, they are operated as the proper combination of multiple programs.

VMI with consignment is the combined form of two famous collaboration programs – VMI and consignment, and it has been used in diverse business areas. The value of VMI with consignment has been evaluated by many past studies, and they commonly conclude that this collaboration program brings some benefits to the supply chain members (Cachon, 2004; Ferretti, Mazzoldi, Zanoni, & Zavanella, 2017; Zanoni & Jaber, 2015). On the other hand, VMI has been suspected to cause the vendor's loss during its early stage of implementation due to his extra burden of managing and paying for the buyer's inventories (Cooke, 1998). Meanwhile, most of past studies observe merely the final outcomes from VMI with consignment and fail to catch when this program improves the performance of its participants. By tracing the ongoing performance of VMI with consignment over time, this study intends to find out when its participants can obtain the benefit and provide the detailed insight about what kind of timely actions that they should take to properly manage this program.

This study focuses on VMI with consignment as the one of programs that are designed to achieve the supply chain collaboration. By comparing with the non-collaborative

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traditional system, this study intends to figure out the true value of this collaboration program. The mathematical model is formulated to describe two stage supply chain system to which a manufacturer and a retailer belong. The numerical examples of the proposed models are experimented to compare VMI with consignment with the traditional system.

Based on the numerical examples, this study obtains several critical findings that would give the valuable managerial implication. While VMI with consignment improves the supply chain performance after all, its early achievement is not profitable. In particular, the individual supply chain members need to endure the loss during the early or later period of this collaboration program. To be a successful collaboration program, VMI with consignment should equip the carefully designed incentive scheme that provides the timely compensation to the supply chain members.

The numerical analysis also points out the certain environment conditions when VMI with consignment improves the supply chain performance. In this hybrid collaboration program, the consignment contract plays the role of lessening the manufacturer's burden of costs, although the entire supply chain system obtains better performance with VMI only.

2. Research Subject and Related Literature

Once the idea of VMI (Vendor-Managed Inventory) was introduced in 1950's, it has been implemented by Procter& Gamble and Walmart in early 1980s as the pilot program to improve the inventory operation in the retail industry (Kannan, Grigore, Devika, & Senthilkumar, 2013). Under VMI, the vendor is responsible for managing the inventory stored at the retailer's warehouse (Fry, Kapuscinski, & Olsen, 2001). VMI allows the efficient inventory management, since the vendor is able to access to the demand and inventory information at the retail level (Savasaneril & Erkip, 2010). In addition, the vendor can have flexible production and delivery schedules due to his authority to determine replenishment for the retailer (Lee & Cho, 2014).

VMI has been applied to diverse business areas such as grocery (Campbell Soup, Barilla SpA, Nestle), electronics (Intel, H&P, Dell), and apparels (Fruits of the Loom, JC Penney, and Dillard Department Stores) (Hariga & Al-Ahmari, 2013; Lee & Cho, 2014; Savasaneril & Erkip, 2010; Sui, Gosavi, & Lin, 2010). As the popular business practices realizing the supply chain collaboration, VMI is well known to be provide the significant benefits including cost saving regarding inventory (Lee & Cho, 2014), shortened lead time (Dong, Xu & Dresner, 2007), and improved customer service (Almehdawe & Mantin, 2010).

VMI with consignment is the hybrid program where the VMI is combined with the consignment (Ben-Daya, Hassini, Hariga, & AlDurgam, 2013). According to the consignment contract, the vendor sells the products to the buyer without receiving payment until they are sold or used (Kim, Kim, Jung, & Youn, 2014; Ru & Wang, 2010; Yi, Yeom, & Seo, 2015). Combined with VMI, the consignment is expected to induce the vendor and buyer to share the inventory holding cost (Lee & Cho, 2014). Many companies in retail stores (Target and Meijer), on-lie commerce (Amazon), and consumer goods (Sara Lee's corporation) adopt this combined collaboration program in these days (Chen, Lin, & Cheng, 2010; Ru & Wang, 2010).

Many past studies have accessed the value of VMI with consignment as the one of supply chain collaboration programs. Most of them evaluate the performance of VMI with consignment compared with the non-collaborative traditional system (Chen, 2017; Ferretti et al., 2017; Hariga & Al-Ahmari, 2013). In evaluation of this collaboration program, other studies consider the consignment only system (Gümüş, Jewkes, & Bookbinder, 2008; Ru & Wang, 2010), VMI without consignment (Chen et al., 2010; Nagarajan & Rajagopalan, 2008; Savasaneril & Erkip, 2010), and integrated system (Ben-Daya et al., 2013; Zanoni, Mazzoldi, & Jaber, 2014).

Lee and Cho (2014) examines the effect of VMI with consignment on the system cost by comparing with the conventional Retailer Managed Inventory system. Their numerical analysis on two stage supply chain model reveals that VMI with consignment results in cost saving for both retailer and supplier. By considering the case of the fixed and proportional stockout penalty, they also investigate the value of information sharing and find out that sharing information of retailer's stockout quantity has a significant impact on the system cost.

In Hariga and Al-Ahmari's study (2013), the proposed supply chain model accounts for the integrated decisions on retail shelf space allocation and inventory management. They investigate the result from the implementation of VMI with consignment agreement between a retailer and a supplier. The numerical examples of their study show that the supply chain system with VMI and consignment partnership gains greater profit than the one without that partnership.

With the assumption that stockout of perishable products is possible, Chen (2017) compares VMI with consignment with RMI with a price only contract. In his proposed models, the supplier keeps the ownership of the product and determines the retail price under VMI with consignment, while RMI with a price only contract represents that the retailer sets the retail price based on the supplier's wholesale price. The outcomes from his numerical analysis indicate that VMI with consignment outperforms RMI in terms of both service level and system wide profit.

Under two echelon supply chain system with a single vendor and multiple buyers, Ben-daya et al. (2013) investigate the benefit of VMI and consignment partnership by comparing with the independent case and vertical integrated system. According to the numerical examples of their proposed models, the VMI and consignment partnership results in significant cost savings for both vendor and buyers compared with the independent case. While the integrated system generate less system cost than the VMI and consignment partnership, only the vendor receives the benefit from the centralized decision making process.

Other research issues related with this collaboration program are the algorithm development for solving the complex inventory and distribution problems (Jemai, Rekik, & Kalaï, 2013; Sui et al., 2010), learning in production (Zanoni, Jaber, & Zavanella, 2012), production with defective items (Bazan, Jaber, Zanoni, & Zavanella, 2014), forward and backward stocking policy (Lee, Wang, & Chen, 2017), and emission control in production (Marchi, Zanoni, Zavanella, & Jaber, 2019; Zanoni et al., 2014).

Following the majority of studies about the supply chain collaboration, this study investigates how VMI with consignment improves the supply chain performance. Meanwhile, this study possesses some unique features that cannot be found in most past studies. First, the on-going performance of the supply chain system is the main interest of this study. While most past researchers access only the final outcomes, this study observes the on-going performance of the supply chain system over time and finds out that VMI with consignment has different impacts on the supply chain performance depending on times.

Second, this study focus on the content of the consignment contract and identifies its role in this collaboration program. By testing the different consignment ratio, this study finds out that the consignment, as a part of this hybrid program, plays an important role of supporting the stable initiation of supply chain collaboration by lessening the particular member's burden of cost.

3. Research Model

This study evaluates the financial performance of VMI with consignment over time and intends to find out when the supply chain members can realize the benefit from this collaboration program. To compare VMI with consignment with the traditional system, the supply chain model is formulated in the mathematical format. The proposed model represents two stage supply chain system. In the supply chain system, one manufacturer produces and supplies the single product item to a retailer. The retailer buys the products from the manufacturer and then sell them to the retail market. Table 1 shows the notation are used in the proposed mathematical models.

Table 1. Notation of Mathematical Models

Manufacturer		Retailer	
π_M	Profit	π_R	Profit
p_t	Price	r_t	Price
y_t	Production rate	q_0	Lot size
o_M	Setup cost	o_R	Ordering cost
h_M	Unit inventory holding cost	h_R	Unit inventory holding cost
υ	Unit manufacturing cost	h_R^F	Financing cost for inventory
δ	Transportation cost per price	h_R^S	Stocking cost for inventory
x_t^M	Estimated market demand	x_t^R	Market demand
$x_t^{M\prime}$	Rate of demand change	$x_t^{R\prime}$	Rate of demand change
N_M	Potential market size	N_R	Potential market size
d_M	Price elasticity of demand	d_R	Price elasticity of demand
		α	Profit margin
B	Initial market demand	γ	Discount rate
T	Time to leave the market		

3.1. Traditional System

The traditional system indicates the supply chain where no specific collaborative activities occur among the supply chain members. This study considers the traditional supply chain system to be a base case compared with the supply chain collaboration, which is VMI with consignment. In the traditional system, the manufacturer receives the orders from the retailer and then determines how much to produce and how much the retailer pays for each product. The retailer purchases the products from the manufacturer according to his order (q_0), and sell them at the retail price (r_t).

Equations (1) through (3) shows the manufacturer's problem in the traditional system. The manufacturer seeks to maximize his total profit (π_M) including the revenue from the sales to the retailer, setup cost, inventory holding cost, manufacturing cost, and transportation cost by optimally determining the transfer price (p_t) and production rate (y_t) as Equation (1) shows. His total profit is accumulated between the time between 0 and terminal time T and discounted at the certain rate (γ). The proposed model adopts the extended Economic Order Quantity model as the inventory policy (Tungalag, Erdenebat & Enkhbat, 2017). Equations (2) and (3) indicate the demand change at the retailer market and the initial retailer market size estimated by the manufacturer. Based on the naive model of innovation diffusion, the market demand is assumed to be dependent on the price (Kalish, 1985).

In the retailer's problem (Equations (4), (5), and (6)), the retailer decides the retail price (r_t) and order quantity (q_0) to maximize his total profit (π_R). The retailer's profit comprises the revenue from the sales to the retail market, cost of purchasing the products from the manufacturer, ordering cost, and inventory holding cost, as it is shown in Equation (4). Two constraints in Equations (5) and (6) indicate the demand change and initial market size observed by the retailer.

$$\max_{p_t, y_t} \ \pi_M = \int_0^T e^{-\gamma \cdot t} \left(p_t \cdot x_t^M - \frac{o_M \cdot x_t^M}{q_0} - q_0 \left(1 - \frac{t}{T}\right) h_M \cdot \frac{x_t^M}{y_t} - \upsilon \cdot y_t - \delta \cdot x_t^M \right) dt \tag{1}$$

$$\text{subject to } \ x_t^{M\prime} = N_M \cdot e^{-d_M \cdot p_t} \tag{2}$$

$$x_0^M = B \tag{3}$$

$$\max_{r_t, q_0} \ \pi_R = \int_0^T e^{-\gamma \cdot t} \left(r_t \cdot x_t^R - p_t \cdot x_t^R - \frac{o_R \cdot x_t^R}{q_0} - q_0 \left(1 - \frac{t}{T}\right) h_R \right) dt \tag{4}$$

$$\text{subject to } \ x_t^{R\prime} = N_R \cdot e^{-d_R \cdot r_t} \tag{5}$$

$$x_0^R = B \tag{6}$$

3.2. VMI with Consignment

Compared with the traditional system without any particular collaborative activities, VMI with consignment is defined as the supply chain system where VMI and consignment contracts are hold between the supply chain members to realize the supply chain collaboration. Equations (7) through (9) represent the manufacturer's problem under VMI with consignment. Similar to the traditional system, the manufacturer still tries to maximize his profit by deciding the transfer price and production rate. Meanwhile, the manufacturer's problem has the order quantity as the additional decision variable according to the VMI program. Due to the consignment contract, the manufacture pays financing cost of holding inventory at the retailer's warehouse (h_R^F) other than cost items that he pays in the traditional system (Valentini & Zavanella, 2003).

The retailer determines only the retailer price as Equation (10) shows, and he pays only portion of stocking cost to hold his inventory (h_R^S) according to the consignment contract. While the manufacturer relies on his estimated demand (Equation (2)) that can be different from the retailer's demand (Equation (5)) in the tradition system, he can receive the demand information directly from the retailer and they share the same demand information under VMI with consignment (Equations (8) and (11)).

$$\max_{p_t, y_t, q_0} \ \pi_M = \int_0^T e^{-\gamma \cdot t} \left(p_t \cdot x_t^M - \frac{o_M \cdot x_t^M}{q_0} - q_0 \left(1 - \frac{t}{T}\right) h_M \cdot \frac{x_t^M}{y_t} - \frac{o_R \cdot x_t^M}{q_0} - q_0 \left(1 - \frac{t}{T}\right) h_R^F - \upsilon \cdot y_t - \delta \cdot x_t^M \right) dt \tag{7}$$

$$\text{subject to } \ x_t^{M\prime} = N_R \cdot e^{-d_R \cdot \alpha \cdot p_t} \tag{8}$$

$$x_0^M = B \tag{9}$$

$$\max_{r_t} \ \pi_R = \int_0^T e^{-\gamma \cdot t} \left(r_t \cdot x_t^R - p_t \cdot x_t^R - q_0 \left(1 - \frac{t}{T}\right) h_R^S \right) dt \tag{10}$$

$$\text{subject to } \ x_t^{R\prime} = N_R \cdot e^{-d_R \cdot r_t} \tag{11}$$

$$x_0^R = B \tag{12}$$

The optimal solutions of decision variables are acquired by using the optimal control theory (Sethi & Thompson, 1981). The solution procedure and shooting process for the numerical examples are illustrated in Appendix.

4. Numerical Analysis

This study identifies when VMI with consignment outperforms the traditional system by observing their ongoing performances over time in the numerical examples of the supply chain models. The parameters of the base case are illustrated in Table 2.

Table 2. Parameters of Base Case

N_M = 90	N_R = 100	d_M= 0.0012	d_R = 0.001
o_M = 4,000	o_R = 3,000	υ = 5	δ = 2
h_M = 3	h_R = 5	h_R^F = 3	h_R^S = 2
B = 1,000	α = 1.10	γ = 0.01	T = 30

In the numerical examples, the vales of six parameters including setup cost, ordering cost, manufacturer's inventory holding cost, retailer's inventory holding cost, potential market size, and profit margin are altered in five levels. The number of total cases experimented in the numerical examples is 15,625 (5^6=15,625).

4.1. Performance of VMI with Consignment

This study evaluates how VMI with consignment performs over time by applying the numerical analysis to the proposed supply chain models. Figure 1 shows the overall performance of VMI with consignment compared with the traditional system. In the beginning, VMI with consignment makes less supply chain profit than the traditional system. As time goes on, however, VMI with consignment becomes to overcome the early poor achievement and makes higher supply chain profit than the traditional system.

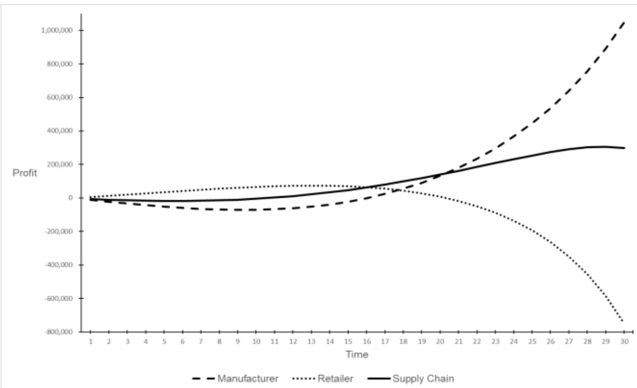

Figure 1. Profit Difference between VMI with Consignment and Traditional System

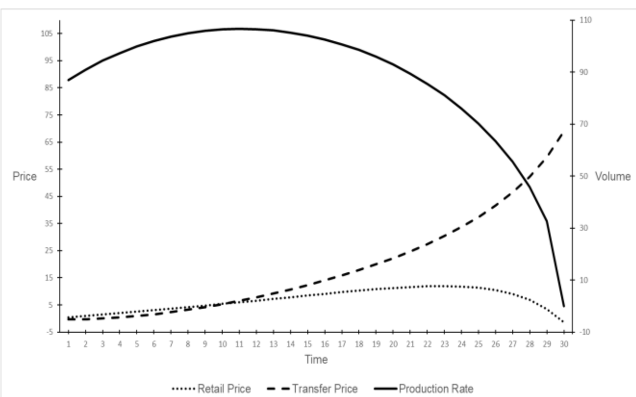

Figure 2. Difference in Decision Variables between VMI with Consignment and Traditional System

The manufacturer's profit loss during the early stage of VMI with consignment is mainly due to the large production rate and low transfer price, as Figure 2 shows. In the end, however, the manufacturer achieves much higher profit under VMI with consignment than the traditional system, because he saves the manufacturing cost by producing the small amount of products and increase the transfer price. In the beginning, the retailer saves the inventory holding cost due to VMI with consignment.

Meanwhile, his profit quickly decreases latter since he should purchase the product at high transfer price from the manufacturer and, at the same time, he needs to sustain low retail price to secure the sufficient retail market demand.

The impacts of VMI with consignment are quite different depending on the supply chain members. The manufacturer obtains less profit under VMI with consignment than the traditional system at the start, and then make much greater profit at the latter point. While the retailer's profit is higher at the early stage of VMI with consignment, he obtains less profit in the end compared with the traditional system.

4.2. Impacts of Conditional Factors on Performance of VMI with Consignment

This study conducts the additional analysis on the numerical examples to figure out how VMI with consignment performs under the different environments. Figure 3. shows that the profit difference between VMI with consignment and traditional system changes as the profit margin increases. The distinct levels of the profit margin do not make any significant difference in the supply chain profit in the early time. As time goes on, however, the profit difference with high profit margin becomes noticeably lower than the one with low profit margin.

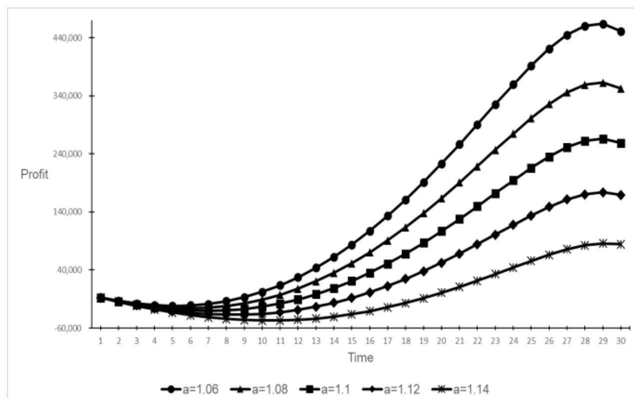

Figure 3. Impact of Profit Margin on Profit Difference

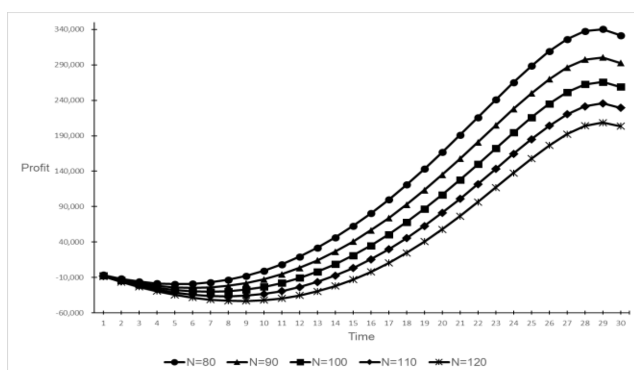

Figure 4. Impact of Potential Market Size on Profit Difference

The potential market size also has the similar effect on the performance of VMI with consignment, as it is seen in Figure 4. While the profit difference with the large potential market size is not considerably different from the one with the small market size in the beginning, the large market size decreases the profit difference compared with small market size at the latter point.

Furthermore, this study examines how the consignment ratio affects the performance of VMI with consignment. The consignment ration represents the portion of financial cost in the whole cost of holding inventory at the retailer's warehouse $\left(\frac{h_R^F}{h_R^F+h_R^S}\right)$. As Figure 5 shows, higher consignment ratio makes bigger profit difference between VMI with consignment and traditional system. Meanwhile, as the consignment ratio becomes larger, the change in the profit differences gets smaller.

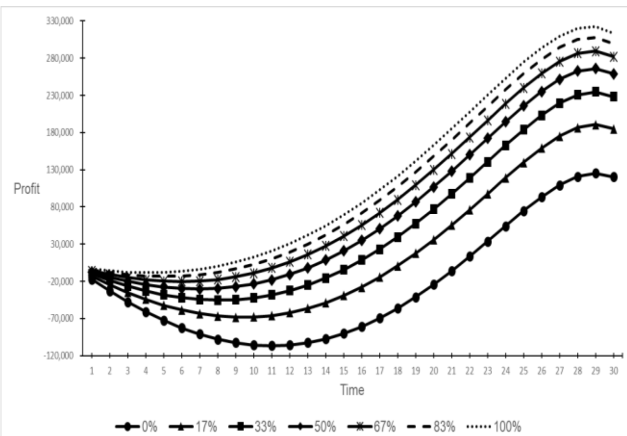

Figure 5. Impact of Consignment Ratio on Profit Difference

5. Discussion

In this study, VMI with consignment, as the one of supply chain collaboration programs, is evaluated to identify whether it makes the expected level of achievement. In particular, this study focuses on the on-going performance of VMI with consignment compared with the non-collaborative supply chain system. The numerical analysis on the supply chain models reveal the following outcomes and they provides the managerial implications to any managers who may consider the implementation of VMI with consignment to their businesses.

First, VMI with consignment obviously improves the supply chain performance after all. This result can be found in the majority of past studies that examine the value of this collaboration program (Ben-Daya et al., 2013; Chen, 2017). Nevertheless, the achievement of VMI with consignment is poor at the early stage of implementation, and even its early supply chain profit is less than the one under the traditional system. This result implies that the supply chain system cannot expect to obtain the immediate fruitful outcomes right after VMI with consignment is executed. Instead, the supply chain system should go through a painful progress for quite a time until this collaboration program produces its real value.

Second, the effect of VMI with consignment is different depending on the supply chain member. The past studies made different conclusions about who receives the benefit from VMI with consignment. A group of researchers finds that this collaboration program is beneficial to both vendor and buyer (Ben-Daya et al., 2013; Ferretti et al., 2017; Hariga & Al-Ahmari, 2013; Lee & Cho, 2014; Nagarajan & Rajagopalan, 2008). According to the other studies, VMI with consignment improves the performance only at the particular stage of supply chain system (Chen et al., 2010; Chen, 2017; Lee & Cho, 2014).

This study shows that the retailer obtains greater profit under VMI with consignment than at the traditional system in the beginning, but he makes much less profit latter. Meanwhile, VMI with consignment makes smaller profit for the manufacturer in the early stage of its implementation, and then significantly increases his profit in the end. After all, VMI with consignment brings unequal benefits to the supply chain members at different times. By implication, on the purpose of sustainable collaboration program, VMI with consignment needs to run together with the proper incentive system that gives timely compensation to the supply chain members who have loss due to this program. In particular, the incentive program should be carefully designed in the way that the manufacturer can endure the early financial damage and the retailer remains in VMI with consignment until the end.

Third, VMI with consignment brings greater benefits to the supply chain system under the certain environment conditions. The numerical analysis indicates that VMI with consignment makes better performance when the manufacturer holds lower profit margin. The supply chain profit under VMI with consignment is also greater with the lower potential market size. Although the high profit margin and huge market size may have the positive impact on the manufacturer's revenue, they result in a heavy burden of the increased purchasing cost for the retailer.

Fourth, the consignment ratio should be properly determined in the consignment contract among the supply chain members in order to obtain the best outcome from VMI with consignment. According to the numerical analysis, VMI with consignment makes the greatest profit when the consignment ratio is 100%. 100% consignment ratio represents the VMI without consignment case where the retailer does not pay any inventory holding cost.

Certainly, the supply chain system achieves its supreme performance when VMI is solely applied to the system where both decision authority and cost payment responsibility regarding inventory control are concentrated in the hands of the manufacturer. Meanwhile, the past studies address the issue that VMI may increase the vendor's cost (Bernstein, Chen & Federgruen, 2006; Sui et al., 2010). The numerical examples in this study imply that the consignment contract with the proper amount of consignment ratio allows the retailer to bear the some portion of the inventory holding cost, and then it mitigates the early loss that the manufacturer should make to participate in VMI.

Other than the practical implications, this study also provides the academic researchers with some research directions. While the collaboration program is expected to require a certain warming-up period until it makes the noticeable benefits to the supply chain members (Cooke, 1998), most past studies rely on the final outcome when they evaluate the performance of the collaboration programs. By showing that VMI with consignment makes quite different achievements depending on the times, this study implies that the researchers should observe the ongoing performance of the collaboration program in order to figure out its true value and provide useful managerial guidelines to the business managers.

6. Conclusion

In both academia and industries, VMI with consignment has been widely accepted as the effective combination of two supply chain collaboration programs due to its potential for improving the supply chain performance. Even though it is necessary to observe the ongoing performance of the collaboration program, most past studies evaluate only its final outcome.

This study examines how VMI with consignment performs over time and figures out when and which supply chain member obtains the benefit from this collaboration program. By focusing on the consignment function, this study also identifies which role that the consignment contract plays in this collaboration program.

With the assumption of two-stage supply chain system where a manufacturer and a retailer trade a single type of products, the profit optimization model is proposed and analyzed to examine the performance of VMI with consignment. Focusing on VMI with consignment, this study makes the direct comparison with the traditional system by using the numerical examples.

The numerical analysis reveals several important findings that provide the valuable managerial implications. First, even though VMI with consignment outperforms the traditional system, the supply chain system needs to endure the loss during the early stage of its implementation. By implication, once the supply chain members participates in VMI with consignment, they need to wait patiently until its value is fully realized in the supply chain system.

Second, the successful VMI with consignment requires a carefully designed incentive scheme, because this collaboration program has distinct impacts on the individual supply chain members' profits at different times. In particular, the well prepared incentive program should provide the initial financial support to the manufacturer and compensate the retailer's loss during the latter part of time in this collaboration program.

Third, VMI with consignment would work best for the supply chain system under the specific environmental conditions. Lower profit margin for the manufacturer and smaller potential market size provide the best situation that this collaboration program improves the supply chain performance.

Finally, the consignment function plays a particular role of supporting this collaboration program. The consignment contract reduces the initial burden of loss that the manufacturer needs to have and provide the motivation to participate in this program at the beginning of its implementation.

Some points are addressed as the limitations that this study possesses, and they enlighten the new research issues to which future studies need to pay attention. First, this study conducts the numerical analysis based on only the arbitrarily determined parameters. By using the data that are empirically collected from the real industry cases, the future studies can result in the practically acceptable outcomes.

Second, the supply chain system that this study assumes in the numerical analysis is the restricted form of the dyadic relationship between one manufacturer and one retailer. The researchers can examine the value of VMI with consignment under rather diverse forms of the supply chain system, such as more than two echelons (Sari, 2007; Sohrabi, Fattahi, Kheirkhah, & Esmaeilian, 2016) or multiple buyers (Ben-Daya et al., 2013; Kannan et al., 2013), and they are expected to provide more generalizable results.

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Appendix

1. Traditional System

In Equation (A1), the Hamiltonian equation of the manufacturer's model contains two decision variables (the transfer price (p_t) and production rate (y_t)), one state variable (the market demand (x_t^M)), and the adjoint variable (λ_t^M).

$$H_M = e^{-\gamma\cdot t}\left(p_t\cdot x_t^M - \frac{o_M\cdot x_t^M}{q_0} - q_0\left(1-\frac{t}{T}\right)h_M\cdot\frac{x_t^M}{y_t} - \upsilon\cdot y_t - \delta\cdot x_t^M\right) + \lambda_t^M\cdot N_M\cdot e^{-d_M\cdot p_t} \quad \text{(A1)}$$

Based on the optimal control theory, the following four equations represent the necessary conditions for optimality.

$$\frac{\partial H_M}{\partial p_t} = 0 \quad \text{(A2)}$$
$$\frac{\partial H_M}{\partial y_t} = 0 \quad \text{(A3)}$$
$$\frac{\partial H_M}{\partial x_t^M} = -\lambda_t^{M\prime} \quad \text{(A4)}$$
$$\frac{\partial H_M}{\partial \lambda_t^M} = x_t^{M\prime} \quad \text{(A5)}$$

Equation (A6) shows the Hamiltonian equation of the retailer's model with the retail price (r_t) as the decision variable, the market demand (x_t^R), as the state variable, and λ_t^R as the adjoint variable. The optimal conditions are represented in Equations (A7) through (A9).

$$H_R = e^{-\gamma\cdot t}\left(r_t\cdot x_t^R - p_t\cdot x_t^R - \frac{o_R\cdot x_t^R}{q_0} - q_0\left(1-\frac{t}{T}\right)h_R\right) + \lambda_t^R\cdot N_R\cdot e^{-d_R\cdot r_t} \quad \text{(A6)}$$
$$\frac{\partial H_R}{\partial r_t} = 0 \quad \text{(A7)}$$
$$\frac{\partial H_R}{\partial x_t^R} = -\lambda_t^{R\prime} \quad \text{(A8)}$$
$$\frac{\partial H_R}{\partial \lambda_t^R} = x_t^{R\prime} \quad \text{(A9)}$$

The optimal solutions of decision variables are obtained as Equations (A10) through (A17).

$$q_0 = \sqrt{\frac{2\cdot O_R\cdot x_T^r}{h_R}} \quad \text{(A10)}$$
$$p_t = \frac{\gamma\cdot t}{d_M}\cdot ln\left(\frac{x_t^M}{d_M\cdot\lambda_t^M\cdot N_M}\right) \quad \text{(A11)}$$
$$y_t = \sqrt{\frac{q_0\left(1-\frac{t}{T}\right)\cdot h_M\cdot x_t^M}{\upsilon}} \quad \text{(A12)}$$
$$\lambda_t^M = \lambda_T^M + \int_t^T e^{-\gamma\cdot t}\left(p_t - \frac{o_M}{q_0} - q_0\left(1-\frac{t}{T}\right)\frac{h_M}{y_t} - \delta\right)dt \quad \text{(A13)}$$
$$x_t^M = B + \int_0^T N_M\cdot e^{-d_M\cdot p_t}\,dt \quad \text{(A14)}$$
$$r_t = \frac{\gamma\cdot t}{d_R} ln\left(\frac{x_t^R}{d_R\cdot\lambda_t^R\cdot N_R}\right) \quad \text{(A15)}$$
$$\lambda_t^R = \lambda_T^R + \int_t^T e^{-\gamma\cdot t}\left(r_t - p_t - \frac{o_R}{q_0}\right)dt \quad \text{(A16)}$$
$$x_t^R = B + \int_0^T N_R\cdot e^{-d_R\cdot r_t}\,dt \quad \text{(A17)}$$

2. VMI with Consignment

In the VMI with consignment case, the identical solution procedure applied to the traditional system is used. Equations (A18) and (A23) indicate the Hamiltonian equations of the manufacturer and retailer's problems. The necessary conditions for optimality are represented in Equations (A19) - (A22) and (A24) - (A26).

$$H_M = e^{-\gamma\cdot t}\left(p_t\cdot x_t^M - \frac{o_M\cdot x_t^M}{q_0} - q_0\left(1-\frac{t}{T}\right)h_M\cdot\frac{x_t^M}{y_t} - \frac{o_R\cdot x_t^M}{q_0} - q_0\left(1-\frac{t}{T}\right)h_R^F - \upsilon\cdot y_t - \delta\cdot x_t^M\right) + \lambda_t^M\cdot N_R\cdot e^{-d_R\cdot\alpha\cdot p_t} \quad \text{(A18)}$$
$$\frac{\partial H_M}{\partial p_t} = 0 \quad \text{(A19)}$$
$$\frac{\partial H_M}{\partial y_t} = 0 \quad \text{(A20)}$$
$$\frac{\partial H_M}{\partial x_t^M} = -\lambda_t^{M\prime} \quad \text{(A21)}$$
$$\frac{\partial H_M}{\partial \lambda_t^M} = x_t^{M\prime} \quad \text{(A22)}$$
$$H_R = e^{-\gamma\cdot t}\left(r_t\cdot x_t^R - p_t\cdot x_t^R - q_0\left(1-\frac{t}{T}\right)h_R^S\right) + \lambda_t^R\cdot N_R\cdot e^{-d_R\cdot r_t} \quad \text{(A23)}$$
$$\frac{\partial H_R}{\partial r_t} = 0 \quad \text{(A24)}$$
$$\frac{\partial H_R}{\partial x_t^R} = -\lambda_t^{R\prime} \quad \text{(A25)}$$
$$\frac{\partial H_R}{\partial \lambda_t^R} = x_t^{R\prime} \quad \text{(A26)}$$

The optimal solutions of decision variables are obtained as Equations (A27) through (A34).

$$q_0 = \sqrt{\frac{2\cdot(o_M+o_R)\cdot x_T^M}{h_M+h_R^F}} \quad \text{(A27)}$$

$$p_t = \frac{\gamma\cdot t}{d_R\cdot\alpha}\cdot ln\left(\frac{x_t^M}{d_R\cdot\alpha\cdot\lambda_t^M\cdot N_R}\right) \quad \text{(A28)}$$

$$y_t = \sqrt{\frac{q_0\left(1-\frac{t}{T}\right)\cdot h_M\cdot x_t^M}{v}} \quad \text{(A29)}$$

$$\lambda_t^M = \lambda_T^M + \int_t^T e^{-\gamma\cdot t}\left(p_t - \frac{o_M}{q_0} - q_0\left(1-\frac{t}{T}\right)\frac{h_M}{y_t} - \frac{o_R}{q_0} - \delta\right)dt \quad \text{(A30)}$$

$$x_t^M = B + \int_0^T N_R\cdot e^{-d_R\cdot\alpha\cdot p_t}\,dt \quad \text{(A31)}$$

$$r_t = \frac{\gamma\cdot t}{d_R} ln\left(\frac{x_t^R}{d_R\cdot\lambda_t^R\cdot N_R}\right) \quad \text{(A32)}$$

$$\lambda_t^R = \lambda_T^R + \int_t^T e^{-\gamma\cdot t}(r_t - p_t)dt \quad \text{(A33)}$$

$$x_t^R = B + \int_0^T N_R\cdot e^{-d_R\cdot r_t}\,dt \quad \text{(A34)}$$

The following shooting method is used to conduct the numerical analysis on the VMI with consignment case.

Step 1. Set the arbitrary inputs to compute sum_x1, sum_x2, sum_x3, and sum_x4. Estimate the integral parts of Equations (A30), (A31), (A33), and (A34) by using the following discrete summation forms.

$$sum\_x1 = \sum_{t=\tau+1}^{T} e^{-\gamma\cdot t}\left(p_t - \frac{o_M}{q_0} - q_0\left(1-\frac{t}{T}\right)\frac{h_M}{y_t} - \frac{o_R}{q_0} - \delta\right) \quad \text{(A35)}$$

$$sum_{x2} = \sum_{t=1}^{T} N_R\cdot e^{-d_R\cdot\alpha\cdot p_t} \quad \text{(A36)}$$

$$sum_{x3} = \sum_{t=\tau+1}^{T} e^{-\gamma\cdot t}(r_t - p_t) \quad \text{(A37)}$$

$$sum_{x4} = \sum_{t=1}^{t} N_R\cdot e^{-d_R\cdot r_t} \quad \text{(A38)}$$

Step 2. Calculate the optimal solutions of λ_t^M, x_t^M, λ_t^R, x_t^R by using Equations (A30), (A31), (A33), and (A34) with the values of sum_x1, sum_x2, sum_x3, and sum_x4 from Step 1. Compute q_0, p_t, y_t, and r_t by using Equations (A27), (A28), (A29), and (A32). Set o_q_0=q_0

Step 3. Obtain the new values of sum_x1, sum_x2, sum_x3, and sum_x4 (n_sum_x1, n_sum_x2, n_sum_x3, and n_sum_x4) with the values of λ_t^M, x_t^M, λ_t^R, x_t^R, q_0, p_t, y_t, and r_t from Step 2. Set n_$q_0 = q_0$.

Step 4. If |n_sum_x1 - sum_x1| > tol_x1, |n_sum_x2 - sum_x2| > tol_x2, |n_sum_x3 - sum_x3| > tol_x3, |n_sum_x4 - sum_x4| > tol_x4, with the small tolerances for tol_x1, tol_x2, tol_x3, and tol_x4,

set sum_x1 = n_sum_x1×ω+sum_x1 × (1-ω)

sum_x2 = n_sum_x2×ω+sum_x2 × (1-ω)

sum_x3 = n_sum_x3×ω+sum_x3 × (1-ω)

sum_x4 = n_sum_x4×ω+sum_x4 × (1-ω)

where 0 < ω < 1

and go to Step 2

otherwise continue.

Step 5. If |n_q_0 - o_q_0| > tol_q_0, with the small tolerance for tol_q_0,

set o_q_0=n_q_0×ω+o_q_0×(1-ω), where 0 < ω < 1

and go to Step 2

otherwise exit.

The identical shooting procedure is applied to the numerical examples of the traditional system, except for the distinct solution eqautions of the variables.