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ON SEQUENCE SPACES DEFINED BY THE DOMAIN OF TRIBONACCI MATRIX IN c_0 AND c

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Abstract. In this article we introduce tribonacci sequence spaces $c_0(T)$ and $c(T)$ derived by the domain of a newly defined regular tribonacci matrix T. We give some topological properties, inclusion relations, obtain the Schauder basis and determine $\alpha-$, $\beta-$ and $\gamma-$ duals of the spaces $c_0(T)$ and $c(T)$. We characterize certain matrix classes $(c_0(T), Y)$ and $(c(T), Y)$, where Y is any of the spaces c_0, c or ℓ_∞. Finally, using Hausdorff measure of non-compactness we characterize certain class of compact operators on the space $c_0(T)$.

1. Introduction

Throughout the paper $\mathbb{N} = \{0, 1, 2, 3, \ldots\}$ and w is the space of all real valued sequences. By ℓ_∞, c_0 and c, we mean the spaces all bounded, null and convergent sequences, respectively. Also by ℓ_p, cs, cs_0 and bs, we mean the spaces of absolutely p-summable, convergent, null and bounded series, respectively, where $1 \leq p < \infty$. We write ϕ for the space of all sequences that terminate in zero. Moreover, we denote the space of all sequences of bounded variation by bv. A Banach space X is said to be a BK-space if it has continuous coordinates. The spaces ℓ_∞, c_0 and c are BK-spaces with norm $\|x\|_{\ell_\infty} = \sup_k |x_k|$. Here and henceforth, for simplicity in notation, the summation without limit runs from 0 to ∞. Also, we shall use the notation $e = (1, 1, 1, \ldots)$ and $e^{(k)}$ to be the sequence whose only non-zero term is 1 in the k^{th} place for each $k \in \mathbb{N}$.

Let X and Y be two sequence spaces and let $A = (a_{nk})$ be an infinite matrix of real entries. We write A_n to denote sequence in the n^{th} row of the matrix A. We say that a matrix A defines a matrix mapping from X to Y if for every sequence $x = (x_k)$, the $A-$transform of x i.e. $Ax = \{(Ax)_n\}_{n=0}^{\infty} \in Y$ where

$$(Ax)_n = \sum_k a_{nk}x_k, \quad n \in \mathbb{N}. \tag{1}$$

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By (X,Y), we denote the class of all matrices A from X to Y. Thus $A \in (X,Y)$ if and only if the series on the right hand side of the equation (1) converges for each $n \in \mathbb{N}$ and $x \in X$ such that $Ax \in Y$ for all $x \in X$.

The sequence space X_A defined by

$$X_A = \{x = (x_k) \in w : Ax \in X\}, \tag{2}$$

is called the domain of matrix A in the space X. Several authors in the literature have constructed sequence spaces using the domain of some special matrices. For instance, one may refer to these nice papers [2, 4, 22, 29, 32, 43]. For some recent publications dealing with the sequence spaces derived by the domain of some special triangular matrices, one may see [11, 18, 19, 21, 38, 39, 46–51].

1.1. Compact operators and Hausdorff measure of non-compactness. Throughout the paper, $B(X)$ will denote unit sphere in X. Let X and Y be two Banach spaces, then by $B(X,Y)$ we denote the class of all bounded linear operators $L : X \to Y$. $B(X,Y)$ itself is a Banach space with the operator norm defined by $\|L\| = \sup_{x\in B(X)} \|Lx\|$. We denote

$$\|a\|_X^* = \sup_{x\in B(X)} \left|\sum_k a_k x_k\right| \tag{3}$$

for $a \in w$, provided that the series on the right hand side is finite which is the case whenever X is a BK space and $a \in X^\beta$ [44]. Also L is said to be compact if the domain of L is all of X and for every bounded sequence (x_k) in X, the sequence $((Lx)_k)$ has a convergent subsequence in Y. We denote the class of all such operators by $C(X,Y)$.

The Hausdorff measure of noncompactness of a bounded set Q in a metric space X is defined by

$$\chi(Q) = \inf\left\{\varepsilon > 0 : Q \subset \cup_{l=0}^n B(x_l, r_l), x_l \in X, r_l < \varepsilon (l = 0, 1, 2, \ldots, n), n \in \mathbb{N}\right\},$$

where $B(x_l, r_l)$ is the open ball centered at x_l and radius r_l for each $l = 0, 1, 2, \ldots, n$. One may refer to [31] and the references mentioned therein for more details on Hausdorff measure of non-compactness.

The Hausdorff measure of non-compactness of an operator L, denoted by $\|L\|_\chi$, is defined by $\|L\|_\chi = \chi(L(B(X)))$, and the necessary and sufficient condition for the operator L to be compact is that $\|L\|_\chi = 0$. Using this relation several authors in the recent times have characterized compact operators using Hausdorff measure of non-compactness between BK spaces. For some relevant papers, one may see [18–20, 30, 33, 34].

1.2. Some definitions and notations. The following definitions are fundamental in our investigation:

Definition 1.1. [44] A matrix $A = (a_{nk})_{n,k\in\mathbb{N}}$ is said to be regular if and only if the following conditions hold:

(a) There exists $M > 0$ such that for every $n \in \mathbb{N}$, the inequality $\sum_k |a_{nk}| \le M$ holds.
(b) $\lim_{n\to\infty} a_{nk} = 0$ for every $k \in \mathbb{N}$.
(c) $\lim_{n\to\infty} \sum_k a_{nk} = 1$.

DEFINITION 1.2. A sequence $x = (x_k)$ of a normed space $(X, \|\cdot\|)$ is called a Schauder basis if for every $u \in X$ there exists a unique sequence of scalars (a_k) such that

$$\lim_{n\to\infty} \left\| u - \sum_{k=0}^{n} a_k x_k \right\| = 0.$$

DEFINITION 1.3. The $\alpha-$, $\beta-$ and $\gamma-$duals of the subset $X \subset w$ are defined by

$$X^{\alpha} = \{a = (a_k) \in w : ax = (a_k x_k) \in \ell_1 \text{ for all } x \in X\},$$
$$X^{\beta} = \{a = (a_k) \in w : ax = (a_k x_k) \in cs \text{ for all } x \in X\},$$
$$X^{\gamma} = \{a = (a_k) \in w : ax = (a_k x_k) \in bs \text{ for all } x \in X\},$$

respectively.

2. Tribonacci sequence spaces $c_0(T)$ and $c(T)$

The studies on tribonacci numbers was first initiated by a 14 year old student, Mark Feinberg [16] in 1963. Define the sequence $(t_n)_{n\in\mathbb{N}}$ of tribonacci numbers given by third order recurrence relation

$$t_n = t_{n-1} + t_{n-2} + t_{n-3},\ n \geq 3 \text{ with } t_0 = t_1 = 1 \text{ and } t_2 = 2.$$

Thus, the first few numbers of tribonacci sequence are $1, 1, 2, 4, 7, 13, 24, \ldots$. Some basic properties of tribonacci sequence are:

$$\lim_{n\to\infty} \frac{t_n}{t_{n+1}} = 0.54368901\ldots,$$
$$\sum_{k=0}^{n} t_k = \frac{t_{n+2} + t_n - 1}{2},\ n \geq 0,$$
$$\lim_{n\to\infty} \frac{t_{n+1}}{t_n} = 1.83929\ (approx.).$$

Binet's formula for tribonacci sequence is given in [41]. For some nice papers related to tribonacci sequence, one may refer to [7–9, 12, 16, 17, 26, 37, 40, 41, 45].

Now, we define the infinite matrix $T = (t_{nk})$ given by

$$t_{nk} = \begin{cases} \frac{2t_k}{t_{n+2}+t_n-1} & (0 \leq k \leq n), \\ 0 & (k > n). \end{cases}$$

Equivalently

$$T = \begin{bmatrix} 1 & 0 & 0 & 0 & \ldots & \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & \ldots \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{2} & 0 & 0 & \ldots \\ \frac{1}{8} & \frac{1}{8} & \frac{1}{4} & \frac{1}{2} & 0 & \ldots \\ \frac{1}{15} & \frac{1}{15} & \frac{2}{15} & \frac{4}{15} & \frac{7}{15} & \ldots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}.$$

Clearly, Definition 1.1 implies that the matrix T is regular. Quite recently Yaying and Hazarika [47] studied the domain of the matrix T in the space ℓ_p and introduced the sequence spaces $\ell_p(T)$ for $1 \leq p \leq \infty$.

Now, we introduce the tribonacci sequence spaces $c_0(T)$ and $c(T)$ as the set of all sequences whose $T-$transform are in the spaces c_0 and c, respectively, that is

$$c_0(T) = \{x = (x_k) \in w : Tx \in c_0\} \text{ and } c(T) = \{x = (x_k) \in w : Tx \in c\}.$$

Using the notation (2), the above sequence spaces may be redefined as

$$c_0(T) = (c_0)_T \text{ and } c(T) = (c)_T. \tag{4}$$

Define the sequence $y = (y_n)$ which will be frequently used as the $T-$transform of the sequence $x = (x_k)$ by

$$y_n = (Tx)_n = \sum_{k=0}^{n} \frac{2t_k}{t_{n+2} + t_n - 1} x_k, \ n \in \mathbb{N}. \tag{5}$$

We also use the convention that any term with negative subscripts, eg. x_{-1} or t_{-1} shall be considered as naught. We begin with the following theorem:

THEOREM 2.1. *The spaces $c_0(T)$ and $c(T)$ are BK-spaces with the norm defined by*

$$\|x\|_{c_0(T)} = \|x\|_{c(T)} = \sup_{n \in \mathbb{N}} |(Tx)_n| . \tag{6}$$

Proof. The sequence spaces c_0 and c are BK spaces with their natural norms. Since equation (4) holds and T is a triangular matrix, therefore Theorem 4.3.12 of Wilansky [44] yields the fact that $c_0(T)$ and $c(T)$ are BK-spaces with the given norm. □

THEOREM 2.2. *The sequence spaces $c_0(T)$ and $c(T)$ are linearly isomorphic to c_0 and c, respectively.*

Proof. We prove the theorem for the space $c_0(T)$. Using the notion (5), we define the mapping $\Phi : c_0(T) \to c_0$ by $x \mapsto y = \Phi x = Tx$. Clearly Φ is linear and $x = 0$ whenever $\Phi x = 0$. Thus, Φ is injective.
Furthermore, let $y = (y_k) \in c_0$ and define the sequence $x = (x_k)$ by

$$x_k = \sum_{j=k-1}^{k} (-1)^{k-j} \frac{t_{j+2} + t_j - 1}{2t_k} y_j, \ (k \in \mathbb{N}). \tag{7}$$

Then

$$\begin{aligned} \lim_{k\to\infty} (Tx)_k &= \lim_{k\to\infty} \sum_{j=0}^{k} \frac{2t_j}{t_{k+2} + t_k - 1} x_j \\ &= \lim_{k\to\infty} \sum_{j=0}^{k} \frac{2t_j}{t_{k+2} + t_k - 1} \sum_{i=j-1}^{j} (-1)^{j-i} \frac{t_{j+2} + t_j - 1}{2t_j} y_j \\ &= \lim_{k\to\infty} y_k = 0. \end{aligned}$$

Thus, $x \in c_0(T)$. Hence, Φ is surjective and norm preserving. Thus, $c_0(T) \cong c_0$. □

Now, we give certain inclusion relations regarding the space $X(T)$, where $X = \{c_0, c\}$.

THEOREM 2.3. *The inclusions $c_0 \subset c_0(T)$ and $c \subset c(T)$ strictly hold.*

Proof. Since the matrix T is regular, therefore the inclusions are obvious. To prove the strictness part, we consider the sequence $x = (1, 0, 1, 0 \ldots)$. Then, we have

$$(Tx)_n = \sum_{k=0}^{n} \frac{2t_k}{t_{n+2} - t_n + 1} x_k = \frac{2}{t_{n+2} + t_n - 1}\{t_0 + t_2 + \ldots + t_n\}, \ (n \in \mathbb{N})$$

which converges. Thus, $x \in c(T) \setminus c$. Similarly, one can prove the other case. □

Theorem 2.4. *The inclusion $c_0(T) \subset c(T)$ strictly holds.*

Proof. It is clear that the inclusion $c_0(T) \subset c(T)$ holds. To prove the strictness part, we consider the sequence $x = (x_k)$ given by $x_k = 1$ for all k. Then, we have

$$(Tx)_n = \sum_{k=0}^{n} \frac{2t_k}{t_{n+2} + t_n - 1} = 1 \text{ for all } n.$$

Thus, $Tx \in c$ but not in c_0. This implies that $x \in c(T) \setminus c_0(T)$. This establishes the result. □

We conclude this section by constructing a sequence of points of the spaces $c_0(T)$ and $c(T)$ which forms Schauder basis for that spaces. The mapping $\Phi : c_0(T) \to c_0$ defined in the proof of Theorem 2.2 is an isomorphism, therefore the inverse image of the basis $\{e^{(k)}\}_{k\in\mathbb{N}}$ of the space c_0 forms the basis of new space $c_0(T)$. Thus, we have the following result:

Theorem 2.5. *Define the sequence $b^{(k)} = (b_n^{(k)})$ for every fixed $k \in \mathbb{N}$ by*

$$b_n^{(k)} = \begin{cases} (-1)^{n-k} \frac{t_{k+2}+t_k-1}{2t_n} & (n-1 \le k \le n), \\ 0 & (0 \le k < n-1 \text{ or } k > n). \end{cases} \tag{8}$$

Then

(a) *the sequence $\{b^{(k)}\}_{k\in\mathbb{N}}$ is a basis for the space $c_0(T)$ and every $x \in c_0(T)$ has a unique representation of the form*

$$x = \sum_k \alpha_k b^{(k)}$$

where $\alpha_k = (Tx)_k$ for each $k \in \mathbb{N}$.

(b) *the sequence $\{e, b^{(0)}, b^{(1)}, \ldots\}$ is a basis for the space $c(T)$ and every $x \in c(T)$ has a unique representation of the form*

$$x = le + \sum_k (\alpha_k - l) b^{(k)},$$

where $\alpha_k = (Tx)_k$ for all $k \in \mathbb{N}$ and $l = \lim_{k\to\infty} (Tx)_k$.

Corollary 2.6. *The sequence spaces $c_0(T)$ and $c(T)$ are seperable spaces.*

Proof. The result is immediate from Theorem 2.1 and Theorem 2.5. □

3. $\alpha-$, $\beta-$ and $\gamma-$ duals

In this section we obtain $\alpha-$, $\beta-$ and $\gamma-$ duals of the spaces $c_0(T)$ and $c(T)$. Before proceeding, we recall certain results due to Stielglitz and Tietz [42] which are essential for our investigation. Throughout $\mathcal{N}$ will denote the collection of all finite subsets of $\mathbb{N}$.

Lemma 3.1. *$A = (a_{nk}) \in (c_0, \ell_1) = (c, \ell_1)$ if and only if*

$$\sup_{N\in\mathcal{N}} \sum_k \left| \sum_{n\in N} a_{nk} \right| < \infty.$$

Lemma 3.2. *$A = (a_{nk}) \in (c_0, c)$ if and only if*

$$\sup_n \left(\sum_k |a_{nk}| \right) < \infty; \tag{9}$$

$$\lim_{n\to\infty} a_{nk} \text{ exists for all } k \in \mathbb{N}. \tag{10}$$

Lemma 3.3. *$A = (a_{nk}) \in (c, c)$ if and only if (9) and (10) hold, and $\lim\limits_{n\to\infty} \sum_k a_{nk}$ exists.*

Lemma 3.4. *$A = (a_{nk}) \in (c_0, \ell_\infty) = (c, \ell_\infty)$ if and only if (9) holds.*

Theorem 3.5. *Define the set α_1 by*

$$\alpha_1 = \left\{ a = (a_k) \in w : \sup_{K\in\mathcal{F}} \sum_k \left| \sum_{n\in K} u_{nk} \right| < \infty \right\},$$

where the matrix $U = (u_{nk})$ is defined by

$$u_{nk} = \begin{cases} (-1)^{n-k} \frac{t_{k+2}+t_k-1}{2t_n} a_n & (n-1 \le k \le n), \\ 0 & (k > n \text{ or } 0 \le k < n-1), \end{cases}$$

for all $n, k \in \mathbb{N}$. Then,

$$[c_0(T)]^\alpha = [c(T)]^\alpha = \alpha_1.$$

Proof. Let $a = (a_k) \in w$ and $x = (x_k)$ is as defined in (7), then we have

$$\begin{aligned} a_n x_n &= \sum_{k=n-1}^{n} (-1)^{n-k} \frac{t_{k+2}+t_k-1}{2t_n} a_n y_k \\ &= (Uy)_n, \text{ for each } n \in \mathbb{N}. \end{aligned} \tag{11}$$

Thus, we deduce from (11) that $ax = (a_n x_n) \in \ell_1$ whenever $x \in c_0(T)$ or $c(T)$ if only if $Uy \in \ell_1$ whenever $y \in c_0$ or c. This yields that $(a_n) \in [c_0(T)]^\alpha$ or $[c(T)]^\alpha$ if and only if $U \in (c_0, \ell_1) = (c, \ell_1)$.
Thus, by using Lemma 3.1, we can conclude that

$$[c_0(T)]^\alpha = [c(T)]^\alpha = \alpha_1.$$

□

THEOREM 3.6. *Define the sets* α_2, α_3 *and* α_4 *by*

$$\alpha_2 = \left\{ a = (a_k) \in w : \sum_k \left| \Delta \left(\frac{a_k}{t_k} \right) \left(\frac{t_{k+2} + t_k - 1}{2} \right) \right| < \infty \right\};$$

$$\alpha_3 = \left\{ a = (a_k) \in w : \sup_k \left| \frac{t_{k+2} + t_k - 1}{2t_k} a_k \right| < \infty \right\};$$

and

$$\alpha_4 = \left\{ a = (a_k) \in w : \lim_k \frac{t_{k+2} + t_k - 1}{2t_k} a_k \text{ exists} \right\};$$

where $\Delta \left(\frac{a_k}{t_k} \right) \left(\frac{t_{k+2}+t_k-1}{2} \right) = \left(\frac{a_k}{t_k} - \frac{a_{k+1}}{t_{k+1}} \right) \left(\frac{t_{k+2}+t_k-1}{2} \right).$

Then, $[c_0(T)]^\beta = \alpha_2 \cap \alpha_3$ *and* $[c(T)]^\beta = \alpha_2 \cap \alpha_4$.

Proof. Let $a = (a_k) \in w$ and $x = (x_k)$ is as defined in (7). Then, we have

$$\begin{aligned}
\sum_{k=0}^{n} a_k x_k &= \sum_{k=0}^{n} a_k \left(\sum_{j=k-1}^{k} (-1)^{k-j} \frac{t_{j+2} + t_j - 1}{2t_k} y_j \right) \\
&= \sum_{k=0}^{n-1} \left(\frac{a_k}{t_k} - \frac{a_{k+1}}{t_{k+1}} \right) \left(\frac{t_{k+2} + t_k - 1}{2} \right) y_k + \frac{t_{n+2} + t_n - 1}{2t_n} a_n y_n \\
&= \sum_{k=0}^{n-1} \Delta \left(\frac{a_k}{t_k} \right) \left(\frac{t_{k+2} + t_k - 1}{2} \right) y_k + \frac{t_{n+2} + t_n - 1}{2t_n} a_n y_n \qquad (12)\\
&= (Vy)_n, \text{ for each } n \in \mathbb{N}, \qquad (13)
\end{aligned}$$

where the matrix $V = (v_{nk})$ is defined by

$$v_{nk} = \begin{cases} \Delta \left(\frac{a_k}{t_k} \right) \left(\frac{t_{k+2}+t_k-1}{2} \right) & (k < n), \\ \frac{t_{n+2}+t_n-1}{2t_n} & (k = n), \\ 0 & (k > 0). \end{cases}$$

Clearly the columns of the matrix V are convergent, since

$$\lim_{n \to \infty} v_{nk} = \Delta \left(\frac{a_k}{t_k} \right) \left(\frac{t_{k+2} + t_k - 1}{2} \right). \qquad (14)$$

Thus, we deduce from (13) that $ax = (a_k x_k) \in cs$ whenever $x = (x_k) \in c_0(T)$ if only if $Vy \in c$ whenever $y = (y_k) \in c_0$. This implies that $a = (a_k) \in [c_0(T)]^\beta$ if and only if $V \in (c_0, c)$.

Thus, using (12), (14) and Lemma 3.2, we get that

$$\sum_k \left| \Delta \left(\frac{a_k}{t_k} \right) \left(\frac{t_{k+2} + t_k - 1}{2} \right) \right| < \infty \text{ and } \sup_k \left| \frac{t_{k+2} + t_k - 1}{2t_k} \right| < \infty.$$

Therefore $[c_0(T)]^\beta = \alpha_2 \cap \alpha_3$.

Similarly, we can obtain the β-dual of the space $c(T)$ by using Lemma 3.3 and equation (14). □

THEOREM 3.7. $[c_0(T)]^\gamma = [c(T)]^\gamma = \alpha_2 \cap \alpha_3$.

Proof. The result can be obtained analogously to the previous theorem by using Lemma 3.4. □

4. Certain matrix transformations on the sequence spaces $c_0(T)$ and $c(T)$

In this section, we characterize the matrix classes $(c_0(T), Y)$ and $(c(T), Y)$ where Y is any of the spaces ℓ_∞, c and c_0. For brevity, we write,

$$\tilde{a}_{nk} = \left(\frac{a_{nk}}{t_k} - \frac{a_{n,k+1}}{t_{k+1}}\right)\left(\frac{t_{k+2} + t_k - 1}{2}\right) \tag{15}$$

for all $n, k \in \mathbb{N}$. Further, let $x, y \in w$ be connected by the relation $y = Tx$. Then, we have by (12)

$$\sum_{k=0}^{m} a_{nk}x_k = \sum_{k=0}^{m-1} \tilde{a}_{nk}y_k + \frac{t_{m+2} + t_m - 1}{2t_m} a_{nm}y_m \ (n, m \in \mathbb{N}). \tag{16}$$

Now let us consider following conditions before we proceed:

$$\sup_n \left(\sum_k |\tilde{a}_{nk}|\right) < \infty, \tag{17}$$

$$\left(\frac{t_{k+2} + t_k - 1}{2t_k} a_{nk}\right)_{k=0}^{\infty} \in \ell_\infty \text{ for every } n \in \mathbb{N}, \tag{18}$$

$$\left(\frac{t_{k+2} + t_k - 1}{2t_k} a_{nk}\right)_{k=0}^{\infty} \in c \text{ for every } n \in \mathbb{N}, \tag{19}$$

$$\sup_n \left|\sum_k a_{nk}\right| < \infty, \tag{20}$$

$$\lim_{n\to\infty} \left(\sum_k a_{nk}\right) = a, \ \text{for all } n, k \in \mathbb{N}, \tag{21}$$

$$\lim_{n\to\infty} \left(\sum_k a_{nk}\right) = 0, \ \text{for all } n, k \in \mathbb{N}, \tag{22}$$

$$\lim_{n\to\infty} \tilde{a}_{nk} = \tilde{a}_k; \ k \in \mathbb{N}, \tag{23}$$

$$\lim_{n\to\infty} \tilde{a}_{nk} = 0; \ k \in \mathbb{N}. \tag{24}$$

Now using the results in [42] and Theorem 3.6 together with (16), we deduce the following results:

Theorem 4.1. *(a) $A = (a_{nk}) \in (c_0(T)), \ell_\infty)$ if and only if (17) and (18) hold.*
(b) $A = (a_{nk}) \in (c_0(T), c_0)$ if and only if (17), (18), and (24) hold.
(c) $A = (a_{nk}) \in (c_0(T), c)$ if and only if (17), (18) and (23) hold.

Theorem 4.2. *(a) $A = (a_{nk}) \in (c(T), \ell_\infty)$ if and only if (17), (19) and (20) hold.*
(b) $A = (a_{nk}) \in (c(T), c_0)$ if and only if (17), (19), (22) and (24) hold.
(c) $A = (a_{nk}) \in (c(T), c)$ if and only if (17), (19), (21) and (23) hold.

The following lemma gives the necessary and sufficient conditions for matrix mappings between any two sequence spaces:

Lemma 4.3. [5, Lemma 5.3] *Let X and Y be any two sequence spaces. Let A be an infinite matrix and B be a triangle. Then $A \in (X, Y_B)$ if and only if $BA \in (X, Y)$.*

Now, combining Lemma 4.3 with Theorem 4.1 and Theorem 4.2 and choosing B as one of the special matrices, Fibonacci matrix F [10, 13], Euler matrix E^r [2, 4, 32] and Riesz matrix R^t [3, 29], we deduce the following corollaries:

Corollary 4.4. *Define the matrix $D = (d_{nk})$ by $d_{nk} = \sum_{j=0}^{n} \frac{f_{j+1}}{f_{n+3}-1} c_{jk}$ for all $n, k \in \mathbb{N}$. Then, we have*

(a) $C = (c_{nk}) \in (c(T), \ell_\infty(F))$ if and only if (17), (19) and (20) hold with d_{nk} instead of a_{nk}.

(b) $C = (c_{nk}) \in (c(T), c_0(F))$ if and only if (17), (19), (22) and (24) hold with d_{nk} instead of a_{nk}.

(c) $C = (c_{nk}) \in (c(T), c(F))$ if and only if (17), (19), (21) and (23) hold with d_{nk} instead of a_{nk}.

Corollary 4.5. *Define the matrix $D = (d_{nk})$ by $d_{nk} = \sum_{j=0}^{n} \binom{n}{j} (1-r)^{n-j} r^j c_{jk}$ for all $n, k \in \mathbb{N}$. Then, we have*

(a) $C = (c_{nk}) \in (c(T), e^r_\infty)$ if and only if (17), (19) and (20) hold with d_{nk} instead of a_{nk}.

(b) $C = (c_{nk}) \in (c(T), e^r_0)$ if and only if (17), (19), (22) and (24) hold with d_{nk} instead of a_{nk}.

(c) $C = (c_{nk}) \in (c(T), e^r_c)$ if and only if (17), (19), (21) and (23) hold with d_{nk} instead of a_{nk}.

Corollary 4.6. *Define the matrix $D = (d_{nk})$ by $d_{nk} = \frac{1}{T_n} \sum_{j=0}^{n} t_j c_{jk}$ for all $n, k \in \mathbb{N}$. Then, we have*

(a) $C = (c_{nk}) \in (c(T), r^t_\infty)$ if and only if (17), (19) and (20) hold with d_{nk} instead of a_{nk}.

(b) $C = (c_{nk}) \in (c(T), r^t_0)$ if and only if (17), (19), (22) and (24) hold with d_{nk} instead of a_{nk}.

(c) $C = (c_{nk}) \in (c(T), r^t_c)$ if and only if (17), (19), (21) and (23) hold with d_{nk} instead of a_{nk}.

5. Hausdorff measure of non-compactness

In this section, we obtain necessary and sufficient condition for an operator to be compact from $c_0(T)$ to $Y \in \{c_0, c, \ell_\infty, \ell_1, cs_0, cs, bs, bv\}$ using Hausdorff measure of non-compactness. First, we recall certain results and notations that are essential for our investigation.

Lemma 5.1. $\ell_\infty^\beta = c^\beta = c_0^\beta = \ell_1$. *Further, for $X \in \{\ell_\infty, c, c_0\}$, then $\|x\|_X^* = \|x\|_{\ell_1}$.*

Lemma 5.2. [44, Theorem 4.2.8] *Let X and Y be any two $BK-$spaces. Then we have $(X, Y) \subset B(X, Y)$, that is, every $A \in (X, Y)$ defines a linear operator $L_A \in B(X, Y)$, where $L_A x = Ax$ for all $x \in X$.*

Lemma 5.3. [31, Theorem 1.23] *Let $X \supset \phi$ be a BK space. If $A \in (X, Y)$ then*

$$\|L_A\| = \|A\|_{(X,Y)} = \sup_n \|A_n\|_X^* < \infty.$$

LEMMA 5.4. [31, Theorem 2.15] *Let Q be a bounded subset in c_0 and $P_r : c_0 \to c_0$ is the operator defined by $P_r(x_0, x_1, x_2 \ldots) = (x_0, x_1, x_2 \ldots, x_r, 0, 0, \ldots)$ for all $x = (x_k) \in c_0$, then*

$$\chi(Q) = \lim_{r\to\infty} \left(\sup_{x\in Q} \|(I - P_r)(x)\| \right),$$

where I is the identity operator on c_0.

LEMMA 5.5. [33, Theorem 3.7] *Let $X \supset \phi$ be a $BK-$space. Then, the following statements hold:*

(a) If $A \in (X, c_0)$, then $\|L_A\|_\chi = \limsup_{n\to\infty} \|A_n\|_X^*$ and L_A is compact if and only if $\lim_{n\to\infty} \|A_n\|_X^* = 0$.

(b) If X has AK and $A \in (X, c)$, then

$$\frac{1}{2} \limsup_{n\to\infty} \|A_n - a\|_X^* \le \|L_A\|_\chi \le \limsup_{n\to\infty} \|A_n - a\|_X^*$$

and L_A is compact if and only if $\lim_{n\to\infty} \|A_n - a\|_X^* = 0$, where $a = (a_k)$ with $a_k = \lim_{n\to\infty} a_{nk}$ for all $k \in \mathbb{N}$.

(c) If $A \in (X, \ell_\infty)$, then $0 \le \|L_A\|_\chi \le \limsup_{n\to\infty} \|A_n\|_X^*$ and L_A is compact if $\lim_{n\to\infty} \|A_n\|_X^* = 0$.

In the rest of the paper, $\mathcal{N}_r$ is the subcollection of $\mathcal{N}$ consisting of subsets of $\mathbb{N}$ with elements that are greater than r.

LEMMA 5.6. [33, Theorem 3.11] *Let $X \supset \phi$ be a $BK-$space. If $A \in (X, \ell_1)$, then*

$$\lim_{r\to\infty} \left(\sup_{N\in\mathcal{N}_r} \left\| \sum_{n\in N} A_n \right\|_X^* \right) \le \|L_A\|_\chi \le 4 \cdot \lim_{r\to\infty} \left(\sup_{N\in\mathcal{N}_r} \left\| \sum_{n\in N} A_n \right\|_X^* \right)$$

and L_A is compact if and only if $\lim_{r\to\infty} \left(\sup_{N\in\mathcal{N}_r} \left\| \sum_{n\in N} A_n \right\|_X^* \right) = 0$.

LEMMA 5.7. [33, Theorem 4.4, Corollary 4.5] *Let $X \supset \phi$ be a $BK-$space and let*

$$\|A\|_{bs}^{[n]} = \left\| \sum_{m=0}^{n} A_m \right\|_X^*.$$

Then, the following statements hold:

(a) If $A \in (X, cs_0)$, then $\|L_A\|_\chi = \limsup_{n\to\infty} \|A\|_{(X,bs)}^{[n]}$ and L_A is compact if and only if $\lim_{n\to\infty} \|A\|_{(X,bs)}^{[n]} = 0$.

(b) If X has AK and $A \in (X, cs)$, then

$$\frac{1}{2} \limsup_{n\to\infty} \left\| \sum_{m=0}^{n} A_m - \tilde{a} \right\|_X^* \le \|L_A\|_\chi \le \limsup_{n\to\infty} \left\| \sum_{m=0}^{n} A_m - \tilde{a} \right\|_X^*$$

and L_A is compact if and only if $\lim_{n\to\infty} \|\sum_{m=0}^{n} A_m - \tilde{a}\|_X^* = 0$, where $\tilde{a} = (\tilde{a}_k)$ with $\tilde{a}_k = \lim_{n\to\infty} \sum_{m=0}^{n} a_{mk}$ for all $k \in \mathbb{N}$.

(c) *If* $A \in (X, bs)$, *then* $0 \leq \|L_A\|_\chi \leq \limsup_{n\to\infty} \|A\|_{(X,bs)}^{[n]}$ *and* L_A *is compact if* $\lim_{n\to\infty} \|A\|_{(X,bs)}^{[n]} = 0$.

LEMMA 5.8. [33, Theorem 4.4, Corollary 4.6] *Let* $X \supset \phi$ *be a* $BK-$*space and let*

$$\|A\|_{bv}^{(n)} = \sup_{N\in\mathcal{N}_r} \left\| \sum_{n\in N} A_n - A_{n-1} \right\|_X^* .$$

Then if $A \in (X, bv)$, *then*

$$\lim_{r\to\infty} \|A\|_{bv}^{(r)} \leq \|L_A\|_\chi \leq 4 \cdot \lim_{r\to\infty} \|A\|_{bv}^{(r)}$$

and L_A *is compact if and only if* $\lim_{r\to\infty} \|A\|_{bv}^{(r)} = 0$.

LEMMA 5.9. *Let* X *be a sequence space and* $A = (a_{nk})$ *and* $\tilde{A} = (\tilde{a}_{nk})$ *be related by (15). If* $A \in (c_0(T), X)$, *then* $\tilde{A} \in (c_0, X)$ *and* $Ax = \tilde{A}y$ *for all* $x \in c_0(T)$.

THEOREM 5.10. *The following statements hold:*

(a) *if* $A \in (c_0(T), c_0)$, *then*

$$\|L_A\|_\chi = \limsup_{n\to\infty} \sum_k |\tilde{a}_{nk}| .$$

(b) *If* $A \in (c_0(T), c)$, *then*

$$\frac{1}{2} \limsup_{n\to\infty} \sum_k |\tilde{a}_{nk} - \alpha_k| \leq \|L_A\|_\chi \leq \limsup_{n\to\infty} \sum_k |\tilde{a}_{nk} - \alpha_k| ,$$

where $\alpha = (\alpha_k)$ *and* $\alpha_k = \lim_{n\to\infty} \tilde{a}_{nk}$ *for each* $k \in \mathbb{N}$.

(c) *If* $A \in (c_0(T), \ell_\infty)$, *then*

$$0 \leq \|L_A\|_\chi \leq \limsup_{n\to\infty} \sum_k |\tilde{a}_{nk}| .$$

(d) *if* $\Omega \in (c_0(T), \ell_1)$, *then*

$$\lim_{r\to\infty} \|A\|_{(c_0(T),\ell_1)}^{[r]} \leq \|L_A\|_\chi \leq 4 \lim_{r\to\infty} \|A\|_{(c_0(T),\ell_1)}^{[r]} ,$$

where $\|A\|_{(c_0(T),\ell_1)}^{[r]} = \sup_{N\in\mathcal{N}_r} \sum_k \left|\sum_{n\in N} \tilde{a}_{nk}\right|$, $r \in \mathbb{N}$.

(e) *if* $A \in (c_0(T), cs_0)$, *then*

$$\|L_A\|_\chi = \limsup_{n\to\infty} \left(\sum_k \left| \sum_{m=0}^n \tilde{a}_{mk} \right| \right) .$$

(f) *if* $A \in (c_0(T), cs)$, *then*

$$\frac{1}{2} \limsup_{n\to\infty} \left(\sum_k \left| \sum_{m=0}^n \tilde{\omega}_{mk} - \tilde{\alpha}_k \right| \right) \leq \|L_A\|_\chi \leq \limsup_{n\to\infty} \left(\sum_k \left| \sum_{m=0}^n \tilde{a}_{mk} - \tilde{\alpha}_k \right| \right) ,$$

where $\tilde{\alpha} = (\tilde{\alpha}_k)$ *with* $\tilde{\alpha}_k = \lim_{n\to\infty} \left(\sum_{m=0}^n \tilde{a}_{mk}\right)$ *for each* $k \in \mathbb{N}$.

(g) *If* $A \in (c_0(T), bs)$, *then*

$$0 \leq \|L_A\|_\chi \leq \limsup_{n\to\infty} \left(\sum_k \left| \sum_{m=0}^n \tilde{a}_{mk} \right| \right) .$$

(h) $\Omega \in (c_0(T), bv)$, *then*

$$\lim_{r\to\infty} \|A\|^{(r)}_{(c_0(T),bv)} \leq \|L_A\|_\chi \leq 4 \lim_{r\to\infty} \|A\|^{(r)}_{(c_0(T),bv)},$$

where $\|A\|^{(r)}_{(c_0(T),bv)} = \sup_{N\in\mathcal{N}_r} \sum_k \left|\sum_{n\in N} \tilde{a}_{nk} - \tilde{a}_{n-1,k}\right|$, $r \in \mathbb{N}$.

Proof. (a) Let $A \in (c_0(T), c_0)$. One can notice that

$$\|A_n\|^*_{c_0(T)} = \left\|\tilde{A}_n\right\|^*_{c_0} = \left\|\tilde{A}_n\right\|_{\ell_1} = \sum_k |\tilde{a}_{nk}|$$

for $n \in \mathbb{N}$. Hence using Part (a) of Lemma 5.5, we conclude that

$$\|L_A\|_\chi = \limsup_{n\to\infty} \left(\sum_k |\tilde{a}_{nk}|\right).$$

(b) Observe that

$$\left\|\tilde{A}_n - \alpha_k\right\|^*_{c_0} = \left\|\tilde{A}_n - \alpha_k\right\|_{\ell_1} = \sum_k |\tilde{a}_{nk} - \alpha_k| \tag{25}$$

for each $n \in \mathbb{N}$. Now, let $A \in (c_0(T), c)$, then from Lemma 5.9, we have $\tilde{A} \in (c_0, c)$. Applying Part (b) of Lemma 5.5, we deduce that

$$\frac{1}{2} \limsup_{n\to\infty} \left\|\tilde{A}_n - \alpha\right\|^*_{c_0} \leq \|L_A\|_\chi \leq \limsup_{n\to\infty} \left\|\tilde{A}_n - \alpha\right\|^*_{c_0},$$

which on using (25) gives us

$$\frac{1}{2} \limsup_{n\to\infty} \sum_k |\tilde{a}_{nk} - \alpha_k| \leq \|L_A\|_\chi \leq \limsup_{n\to\infty} \sum_k |\tilde{a}_{nk} - \alpha_k|$$

which is the desired result.

(c) This is similar to the proof of Part (a) with Part (b) except that we employ Part (c) of Lemma 5.5 instead of Part (a) of Lemma 5.5.

(d) Observe that

$$\left\|\sum_{n\in\mathbb{N}} \tilde{A}_n\right\|^*_{c_0} = \left\|\sum_{n\in\mathbb{N}} \tilde{A}_n\right\|_{\ell_1} = \sum_k \left|\sum_{n\in\mathbb{N}} \tilde{a}_{nk}\right|. \tag{26}$$

Let $A \in (c_0(T), \ell_1)$ then by Lemma 5.9, we get that $\tilde{A} \in (c_0, \ell_1)$. Hence, by applying Lemma 5.6, we get

$$\lim_{r\to\infty} \left(\sup_{N\in\mathcal{N}_r} \left\|\sum_{n\in N} \tilde{A}_n\right\|^*_{c_0}\right) \leq \|L_A\|_\chi \leq 4 \cdot \lim_{r\to\infty} \left(\sup_{N\in\mathcal{N}_r} \left\|\sum_{n\in N} \tilde{A}_n\right\|^*_{c_0}\right)$$

which further reduces on using (26) to

$$\lim_{r\to\infty} \|A\|^{[r]}_{(c_0(T),\ell_1)} \leq \|L_A\|_\chi \leq 4 \lim_{r\to\infty} \|\Omega\|^{[r]}_{(c_0(T),\ell_1)},$$

as desired.

(e) It is clear that

$$\left\|\sum_{m=0}^{n} A_m\right\|_{c_0(T)}^{*} = \left\|\sum_{m=0}^{n} \tilde{A}_m\right\|_{c_0}^{*} = \left\|\sum_{m=0}^{n} \tilde{A}_m\right\|_{\ell_1} = \sum_k \left|\sum_{m=0}^{n} \tilde{a}_{mk}\right|.$$

Hence by using Part (a) Lemma 5.7, we get

$$\|L_A\|_\chi = \limsup_{n\to\infty}\left(\sum_k \left|\sum_{m=0}^{n} \tilde{a}_{mk}\right|\right).$$

(f) We have

$$\left\|\sum_{m=0}^{n} \tilde{A}_m - \tilde{\alpha}\right\|_{c_0}^{*} = \left\|\sum_{m=0}^{n} \tilde{A}_m - \tilde{\alpha}\right|_{\ell_1} = \sum_k \left|\sum_{m=0}^{n} \tilde{a}_{mk} - \tilde{\alpha}_k\right| \tag{27}$$

for each $n \in \mathbb{N}$. Now, let $A \in (c_0(T), cs)$, then by Lemma 5.9, we have $\tilde{A} \in (c_0 : cs)$. Thus by applying Part (b) of Lemma 5.7, we deduce that

$$\frac{1}{2}\limsup_{n\to\infty}\left\|\sum_{m=0}^{n} \tilde{A}_m - \tilde{\alpha}_k\right\|_{c_0}^{*} \le \|L_A\|_\chi \le \limsup_{n\to\infty}\left\|\sum_{m=0}^{n} \tilde{A}_m - \tilde{\alpha}_k\right\|_{c_0}^{*},$$

which on using (27) gives us

$$\frac{1}{2}\limsup_{n\to\infty}\left(\sum_k \left|\sum_{m=0}^{n} \tilde{a}_{mk} - \tilde{\alpha}_k\right|\right) \le \|L_A\|_\chi \le \limsup_{n\to\infty}\left(\sum_k \left|\sum_{m=0}^{n} \tilde{a}_{mk} - \tilde{\alpha}_k\right|\right),$$

as desired.

(g) This is similar to Part (e) except that we employ Part (c) of Lemma 5.7 instead of Part (a) of Lemma 5.7.

(h) We have

$$\left\|\sum_{n\in\mathbb{N}} \left(\tilde{A}_n - \tilde{A}_{n-1}\right)\right\|_{c_0}^{*} = \left\|\sum_{n\in\mathbb{N}} \left(\tilde{A}_n - \tilde{A}_{n-1}\right)\right\|_{\ell_1} = \sum_k \left|\sum_{n\in\mathbb{N}} \tilde{a}_{nk} - \tilde{a}_{n-1,k}\right|. \tag{28}$$

Let $A \in (c_0(T), bv)$ then by Lemma 5.9, we get that $\tilde{A} \in (c_0, bv)$. Hence, by applying Lemma 5.8, we get

$$\lim_{r\to\infty}\left(\sup_{N\in\mathcal{N}_r}\left\|\sum_{n\in N}\left(\tilde{A}_n - \tilde{A}_{n-1}\right)\right\|_{c_0}^{*}\right) \le \|L_A\|_\chi \le 4\cdot\lim_{r\to\infty}\left(\sup_{N\in\mathcal{N}_r}\left\|\sum_{n\in N}\left(\tilde{A}_n - \tilde{A}_{n-1}\right)\right\|_{c_0}^{*}\right)$$

which on using (28) gives us the desired result.

□

Now, we have the following corollaries:

Corollary 5.11. *The following statements hold:*

(a) Let $A \in (c_0(T), c_0)$, then L_A is compact if and only if $\lim_{n\to\infty}\sum_k |\tilde{a}_{nk}| = 0$.

(b) Let $A \in (c_0(T), c)$, then L_A is compact if and only if $\lim_{n\to\infty}\left(\sum_k |\tilde{a}_{nk} - \alpha_k|\right) = 0$.

(c) Let $A \in (c_0(T), \ell_\infty)$, then L_A is compact if $\lim_{n\to\infty}\sum_k |\tilde{a}_{nk}| = 0$.

(d) *Let* $A \in (c_0(T), \ell_1)$*, then* L_A *is compact if and only if*

$$\lim_{r\to\infty}\left(\sup_{N\in\mathcal{N}_r}\left(\sum_k\left|\sum_{n\in N}\tilde{a}_{nk}\right|\right)\right)=0.$$

(e) *Let* $A \in (c_0(T), cs_0)$*, then* L_A *is compact if and only if* $\limsup_{n\to\infty}\left(\sum_k\left|\sum_{m=0}^{n}\tilde{a}_{mk}\right|\right) = 0.$

(f) *Let* $A \in (c_0(T), cs)$*, then* L_A *is compact if and only if*

$$\limsup_{n\to\infty}\left(\sum_k\left|\sum_{m=0}^{n}\tilde{a}_{mk}-\tilde{\alpha}\right|\right)=0.$$

(g) *Let* $A \in (c_0(T), bs)$*, then* L_A *is compact if* $\limsup_{n\to\infty}\left(\sum_k\left|\sum_{m=0}^{n}\tilde{a}_{mk}\right|\right) = 0.$

(h) *Let* $A \in (c_0(T), bv)$*, then* L_A *is compact if and only if*

$$\lim_{r\to\infty}\left(\sup_{N\in\mathcal{N}_r}\left(\sum_k\left|\sum_{n\in N}\tilde{a}_{nk}-\tilde{a}_{n-1,k}\right|\right)\right)=0.$$

6. Conclusion

Tribonacci numbers has been studied by several authors in the past and investigated tribonacci identities, recurrence relations, generating functions, Binet's formula for tribonacci numbers, modified and generalized tribonacci numbers etc. Recently some authors, for instance, İlkhan et al. [18, 19, 21], Roopaei [39] and Yaying et al. [51] studied interesting sequence spaces using the domain of Euler totient matrix, Jordan totient matrix, Copson matrix and q-Cesàro matrix, respectively. Quite recently Yaying and Hazarika [47] studied the domain of tribonacci matrix in the space ℓ_p of p-absolutely summable sequences. We follow their approach and study the domain of tribonacci matrix in the spaces c_0 and c. We expect that our results might be a reference for further studies in this field. For further study, one can study the domain of tribonacci matrix in the Maddox's spaces, cs, bs, etc.

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