Journal of the Korea Academia-Industrial cooperation Society
Vol. 22, No. 1 pp. 9-17, 2021

https://doi.org/10.5762/KAIS.2021.22.1.9
ISSN 1975-4701 / eISSN 2288-4688

Free Vibration Analysis of Circular Arches Considering Effects of Midsurface Extension and Rotatory Inertia Using the Method of Differential Quadrature

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미분구적법을 이용 중면신장 및 회전관성의 영향을 고려한 원형아치의 고유진동해석

강기준
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Abstract Curved beams are increasingly used in buildings, vehicles, ships, and aircraft, which has resulted in considerable effort being directed toward developing an accurate method for analyzing the dynamic behavior of such structures. The stability behavior of elastic circular arches has been the subject of a large number of investigations. One of the efficient procedures for the solution of ordinary differential equations or partial differential equations is the differential quadrature method DQM. This method has been applied to a large number of cases to overcome the difficulties of the complex computer algorithms, as well as excessive use of storage due to conditions of non-linear geometries, loadings, or material properties. This study uses DQM to analyze the in-plane vibration of the circular arches considering the effects of midsurface extension and rotatory inertia. Fundamental frequency parameters are calculated for the member with various parameter ratios, boundary conditions, and opening angles. The solutions from DQM are compared with exact solutions or other numerical solutions for cases in which they are available and given to analyze the effects of midsurface extension and rotatory inertia on the frequency parameters of the circular arches.

요 약 빌딩, 자동차, 선박, 항공기 등에서 원형 아치의 사용 증가로 인해 이러한 구조물의 동적 거동 해석에 있어 괄목할 만한 성과가 있어 왔다. 탄성 원형 아치의 안정성 거동 해석분야는 많은 연구자들의 관심분야였다. 전통적으로 미분방정식의 해법은 유한차분법 혹은 유한요소법으로 해결해왔다. 복잡한 기하학적 구조 및 하중으로 인한 과도한 컴퓨터 용량의 사용과 복합알고리즘 프로그램의 어려움을 극복하기 위하여 미분구적법(DQM)이 많은 분야에 적용되어왔다. 상미분방정식 혹은 편미분방정식의 해를 구하기 위한 효율적인 방법 중의 하나는 미분구적법이다. 또한 비선형 구조, 하중, 혹은 재료 물성 치로 인한 과도한 컴퓨터 용량의 사용과 복합알고리즘 프로그램의 어려움을 극복하기 위하여 미분구적법(DQM)이 지금도 많이 사용된다. 본 연구에서는, DQM을 이용하여 중면 신장 및 회전 관성의 영향을 고려한 원형 아치의 내 평면 진동을 분석하였다. 다양한 매개변수 비, 경계 조건, 그리고 열림 각에 따른 기본 진동수를 계산하였다. DQM 결과는 활용 가능한 다른 엄밀해 혹은 다른 수치해석과 비교하였다. 해석결과에 따르면 DQM은, 적은 격자점을 사용하고도, 엄밀해 결과와 일치함을 보여주었고, 중면 신장 및 회전 관성이 원형 아치의 기본 진동수에 미치는 영향을 분석할 수 있게 했다.

Keywords : DQM, Extensional Vibration, Inextensional Vibration, In-Plane, Rotatory Inertial

This research was supported by the Academic Research Fund of Hoseo University in 2018 (20180301).
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Received September 14, 2020 Revised November 6, 2020
Accepted January 8, 2021 Published January 31, 2021

1. Introduction

The increasing use of circular arches in many structures, such as buildings, bridges, and aircraft leads to significant effort into developing a more accurate method for the dynamic analysis of the structures. Accurate knowledge of the vibration response of circular arches is very important in many applications for the analysis of the design of the structures. The early researchers on the in-plane vibration of arches were Hoppe[1] and Love[2]. Love[2] developed on Hoppe's theory by permitting for stretching of the ring. Lamb[3] studied the statics of arches with various end conditions and the vibrations of arches of small curvature. Den Hartog[4] adopted the Rayleigh-Ritz method for calculating the lowest frequency of circular arches with various boundary conditions, and his research was improved by Volterra and Morell[5] for the dynamics of circular arches in the form of catenaries, parabolas, or cycloids. Archer[6] worked out for a mathematical research of the inextensional vibrations of the circular arche of a small cross section area with the differential equations given in Love[2] with damping effects. Nelson[7] used the Rayleigh-Ritz method with Lagrangian multipliers for the vibration of the circular ring segment with simply supported boundary conditions. Auciello and De Rosa[8] showed a critical brief review of the in-plane vibrations of circular arches and presented a number of other approaches.

A rather efficient procedure for the solution of differential equations is the differential quadrature method which was suggested by Bellman and Casti[9]. It was also used to the static research of structural members by Jang et al.[10] for the first time. The flexibility of the DQM to structural analysis is becoming progressively apparent by the publications of recent years. Kang and Han[11] used the DQM to the vibration analysis of a curved beam with shear deformable beam theories, and Kang[12] used the method for the vibration analysis of thin-wall curved beams. Kang and Kim[13] and Kang and Park[14] analyzed asymmetric curved beams for the vibration and the buckling using the DQM, respectively. Recently, Kang[15] studied extensional buckling analysis of curved beams, and Kang[16] also analyzed extensional vibrations of asymmetric curved beams using DQM, This method is applied to the study to analyze the in-plane vibrations of circular arches including the effects of midsurface extension, previously not considered, with various conditions, slenderness ratios, and opening angles. The results are compared with exact analytic solutions and numerical solutions by other methods (FEM, Galerkin, or C.D.M.) and also compared with solutions excluding the effects of mid surface extension and rotatory inertia.

2. Governing Differential Equations

The circular arch is shown in Fig. 1. A position on the centroidal axis is specified by the angle θ, started from the left support of the arch. The tangential and the radial displacements of an arch axis are v and w, respectively. The radius of the centroidal axis is a.

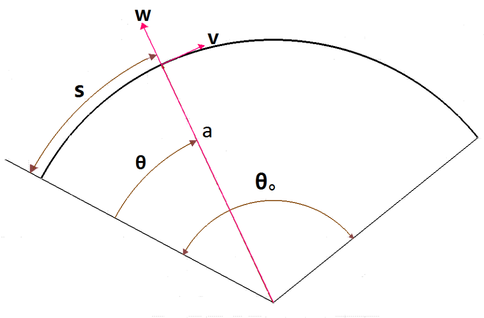


Fig. 1. Coordinate system for circular arch

2.1 In-Plane Inextensional Vibrations of Circular Arches

A mathematical research of the free in-plane inextensional vibrations of the circular arch with a small cross section area is worked out starting with the basic differential equations as given by Love[2]. From Love[2], the analysis is simplified by assuming focus to problems where there is no extensibility of the center line of the arche. This condition shows that v and w are related by

$$w = -\frac{\partial v}{\partial \theta} \tag{1}$$

If shear deformation and rotatory inertia are neglected, the governing differential equation for the free in-plane vibrations of the circular arch, in terms of the tangential displacement v, can be written as

$$\frac{EI}{a^4}\left(\frac{\partial^6 v}{\partial \theta^6}+2\frac{\partial^4 v}{\partial \theta^4}+\frac{\partial^2 v}{\partial \theta^2}\right)=m\frac{\partial^2}{\partial t^2}\left(v-\frac{\partial^2 v}{\partial \theta^2}\right) \tag{2}$$

or

$$\frac{v^{vi}}{\theta_0^6}+2\frac{v^{iv}}{\theta_0^4}+\frac{v''}{\theta_0^2}=\frac{ma^4\omega^2}{EI}\left(\frac{v''}{\theta_0^2}-v\right) \tag{3}$$

in which each prime represents one differentiation with respect to the dimensionless coordinate X.

$$X=\frac{\theta}{\theta_0} \tag{4}$$

E is the Young's modulus of elasticity, I is the moment of inertia of areas, m is the mass per unit length, θ_0 is the opening angle, and ω is the circular frequency.

Rao and Sundararajan[17] developed the free in-plane inextensional vibration of a circular ring with the effect of rotatory inertia. The differential equations with the effect of rotatory inertia can be written as

$$\frac{v^{vi}}{\theta_0^6}+2\frac{v^{iv}}{\theta_0^4}+\frac{v''}{\theta_0^2}=\frac{ma^4\omega^2}{EI}\Big(-\theta^{2_0}(\frac{r}{s})^2\frac{v^{iv}}{\theta^{4_0}}+ (-2\theta_0^2(\frac{r}{s})^2+1)\frac{v''}{\theta_0^2}+(-\theta_0^2(\frac{r}{s})^2-1)v\Big) \tag{5}$$

where $s(=a\theta_0)$ is the length of the arch axis, and r is the radius of gyration of areas ($r=\sqrt{I/A}$).

If the circular arch is clamped at θ=0 and θ=θ_0, the boundary conditions can be written as

$$v=0 \tag{6}$$

$$w-\frac{\partial v}{\partial \theta}=0 \tag{7}$$

$$\frac{\partial^2 v}{\partial \theta^2}+v=0 \tag{8}$$

or

$$v(0)=v'(0)=v''(0)=v(\theta_0)=v'(\theta_0)=v''(\theta_0)=0 \tag{9}$$

If the arch is simply supported at θ=0 and θ=θ_0, then the boundary conditions can be expressed in the following form as

$$v=0 \tag{10}$$

$$w-\frac{\partial v}{\partial \theta}=0 \tag{11}$$

$$\frac{\partial^2 w}{\partial \theta^2}+w=0 \tag{12}$$

or

$$v(0)=v'(0)=v'''(0)=v(\theta_0)=v'(\theta_0)=v'''(\theta_0)=0 \tag{13}$$

If the arch is clamped at θ=0 and simply supported θ=θ_0, then the boundary conditions can be expressed in the following form as

$$v(0)=v'(0)=v''(0)=v(\theta_0)=v'(\theta_0)=v'''(\theta_0)=0 \tag{14}$$

2.2 In-Plane Extensional Vibrations of Circular Arches

Veletsos et al.[18] applied the theory which considered the extension of the arch axis neglecting the effect of rotatory inertia to analyze the vibrations of the arches. The differential equations for the free in-plane vibrations of the system, described by specializing Flugge's equations for the cylindrical shell[19], are

$$\frac{w''''}{\theta_0^4}+2\frac{w''}{\theta_0^2}+[1+\frac{1}{\theta_0^2}(\frac{s}{r})^2]w+\frac{1}{\theta_0^3}(\frac{s}{r})^2v' =\frac{ma^4\omega^2}{EI}w \tag{15}$$

$$-(\frac{s}{r})^2[\frac{v''}{\theta_0^4}+\frac{w'}{\theta_0^3}]=\frac{ma^4\omega^2}{EI}v \quad (16)$$

in which each prime represents one differentiation with regard to the dimensionless distance X.

Austin and Veletsos[20] presented the analysis of the in-plane vibrational characteristics of circular arches and the simple approximate procedure for calculating the frequencies of the arches based on the theory which contained the effect of rotatory inertia.

The differential equations with the effects of rotatory inertia, obtained from Federhofer's system, are

$$\frac{w''''}{\theta_0^4}+2\frac{w''}{\theta_0^2}+[1+(\frac{s}{\theta_0 r})^2]w+(\frac{s}{\theta_0 r})^2\frac{v'}{\theta_0}$$
$$=(-(\frac{r}{s})^2w''+w+(\frac{r}{s})^2\theta_0 v')\frac{ma^4\omega^2}{EI} \quad (17)$$

$$\frac{w'}{\theta_0^3}+\frac{v''}{\theta_0^4}$$
$$=(\theta_0(\frac{r}{s})^4w'-((\frac{r}{s})^2+(\theta_0^2(\frac{r}{s})^4))v\frac{ma^4\omega^2}{EI} \quad (18)$$

The boundary conditions for both ends clamped, both ends simply supported, and clamped - simply supported ends are, respectively,

$$v(0)=w(0)=w'(0)=v(\theta_0)$$
$$=w(\theta_0)=w'(\theta_0)=0 \quad (19)$$
$$v(0)=w(0)=w''(0)=v(\theta_0)$$
$$=w(\theta_0)=w''(\theta_0)=0 \quad (20)$$
$$v(0)=w(0)=w'(0)=v(\theta_0)$$
$$=w(\theta_0)=w''(\theta_0)=0 \quad (21)$$

3. Application

The DQM is used for the analysis of the free in-plane vibrations of a circular arche including the effects of mid surface extension and rotatory inertia. The DQM approximations of the differential equations and boundary conditions are presented.

Applying the DQM to Eqs. (15) and (16) gives

$$\frac{1}{\theta_0^4}\sum_{j=1}^{N}D_{ij}W_j+\frac{2}{\theta_0^2}\sum_{j=1}^{N}B_{ij}W_j+[1+\frac{1}{\theta_0^2}(\frac{s}{r})^2]W_i$$
$$+\frac{1}{\theta_0^2}(\frac{s}{r})^2\sum_{j=1}^{N}A_{ij}V_j=\frac{ma^4\omega^2}{EI}W_i \quad (22)$$
$$-(\frac{s}{r})^2[\frac{1}{\theta_0^3}\sum_{j=1}^{N}A_{ij}W_j+\frac{1}{\theta_0^4}\sum_{j=1}^{N}B_{ij}V_j]=\frac{ma^4\omega^2}{EI}V_i \quad (23)$$

Similarly, applying the DQM to Eqs. (17) and (18) gives

$$\frac{1}{\theta_0^4}\sum_{j=1}^{N}D_{ij}W_j+\frac{2}{\theta_0^2}\sum_{j=1}^{N}B_{ij}W_j+[1+\frac{1}{\theta_0^2}(\frac{s}{r})^2]W_i$$
$$+(\frac{1}{\theta_0^3}(\frac{s}{r})^2\sum_{j=1}^{N}A_{ij}V_j=\frac{ma^4\omega^2}{EI}$$
$$(-(\frac{r}{s})^2\sum_{j=1}^{N}B_{ij}W_j+W_i+\theta_0(\frac{r}{s})^2\sum_{j=1}^{N}A_{ij}V_j) \quad (24)$$
$$\frac{1}{\theta_0^3}\sum_{j=1}^{N}A_{ij}W_j+\frac{1}{\theta_0^4}\sum_{j=1}^{N}B_{ij}V_j=$$
$$\frac{ma^4\omega^2}{EI}(\theta_0(\frac{r}{s})^4\sum_{j=1}^{N}A_{ij}W_j-(\theta_0^2(\frac{r}{s})^6V_i) \quad (25)$$

The boundary conditions for both ends clamped, shown by Eq. (19), can be written in differential quadrature form as below:

$$v_1=0 \quad \text{at } X=0 \quad (26)$$
$$v_N=0 \quad \text{at } X=1 \quad (27)$$
$$w_1=0 \quad \text{at } X=0 \quad (28)$$
$$w_N=0 \quad \text{at } X=1 \quad (29)$$
$$\sum_{j=1}^{N}A_{2j}w_j=0 \quad \text{at } X=0+\delta \quad (30)$$
$$\sum_{j=1}^{N}A_{(N-1)j}w_j=0 \quad \text{at } X=1-\delta \quad (31)$$

Similarly, the boundary conditions for both ends simply supported, given by Eq. (20), can be written in differential quadrature form as below:

$$v_1=0 \quad \text{at } X=0 \quad (32)$$
$$v_N=0 \quad \text{at } X=1 \quad (33)$$
$$w_1=0 \quad \text{at } X=0 \quad (34)$$
$$w_N=0 \quad \text{at } X=1 \quad (35)$$

$$\sum_{j=1}^{N} B_{2j} w_j = 0 \qquad \text{at } X = 0+\delta \tag{36}$$

$$\sum_{j=1}^{N} B_{(N-1)j} w_j = 0 \qquad \text{at } X = 1-\delta \tag{37}$$

Those linear algebraic equations with the proper boundary conditions can be calculated for the free in-plane extensional vibrations of a circular arch with the effects of rotatory inertia.

4. Results and Comparisons

On the basis of the applications, the fundamental frequency parameter $\lambda = \omega(ma^4/EI)^{1/2}$ of the free in-plane inextensional and extensional vibrations of the circular arch with the effects of rotatory inertia is calculated by the DQM and is showed together with exact solutions and other numerical solutions for cases in which those are available. All results are calculated with thirteen grid points along the dimensionless coordinate axis, and the value of δ is 1×10^{-6}. Figs. 2 and 3 show the convergence studies with regard to the number of discrete grid points N and the small distance parameter δ, respectively. Fig. 2 presents that the accuracy of the solution by the DQM increases with increasing the number of grid points N and passes over a maximum. Then, the instabilities of the solutions arise if the gird point N becomes too big. The optimum value for N is to be 11 and 13 points for this case. Fig. 3 presents the sensitivity of the solutions by the DQM to the choice of δ. The optimal choice of value for δ is to be 1×10^{-5} to 1×10^{-6} which is gained by trial and error. The accuracy of the solution by the DQM decreases due to instabilities if a small distance δ becomes too small.

The frequency parameter of the free in-plane inextensional vibrations of the circular arch neglecting the effects of rotatory inertia is calculated by the DQM and is showed together with solutions from other methods: the exact solution by Archer[6], he Lagrangian multiplier technique by Nelson[7], Galerkin, or finite element methods. The results are summarized in Tables 1 and 2. Auciello and De Rosa[8] calculated the natural frequencies of the arch using the SAP IV and the SAP 90 finite element methods (FEM) with 60 elements. Tables 1 and 2 also show that the numerical solutions by the DQM are in good agreement with the solutions by exact or by other numerical methods. The frequency parameters of the free in-plane inextensional vibrations of the arch including the effects of rotatory inertia with simply supported and clamped ends are also calculated by the DQM, and the results are presented in Tables 3 and 4.

In Tables 5 and 6, the frequency parameters of the extensional vibrations excluding the effects of rotatory inertia with simply supported and clamped ends are showed.

Finally, the frequency parameters of the extensional vibrations of the arch including the effects of rotatory inertia with simply supported, clamped, and clamped-simply supported ends are summarized in Tables 7~9. Fig. 4 shows the comparisons of fundamental frequency parameters for the inextensional and extensional vibrations of the arch with or without the effects of rotatory inertia.

From Tables 1~9 and Fig 4, the fundamental frequency parameters of the arche with both ends clamped are much higher than those of the arch with simply supported ends and clamped-simply supported ends, and the fundamental frequency parameters are increased by decreasing the angles for both inextensional and extensional vibrations. The fundamental frequencies of the free in-plane inextensional vibrations are higher than those of the in-plane extensional vibrations, and the frequency parameters of the in-plane vibrations neglecting the effects of rotatory inertia are also little higher than those of the in-plane vibrations including the effects of rotatory inertia for both inextensional and extensional vibrations. The difference of fundamental frequency values is reduced by increasing the slenderness ratio. As the slenderness ratio becomes higher than 300, the difference of the fundamental frequency values becomes less than 0.1 percent. The frequency parameters with clamped boundary conditions are much affected by the slenderness ratio than any other boundary conditions. In Table 10, the first four frequency parameter, $\lambda=\omega(ma^4/EI)^{1/2}$, of the free in-plane extensional vibrations of the arch neglecting the effects of rotatory inertia is calculated by the DQM.

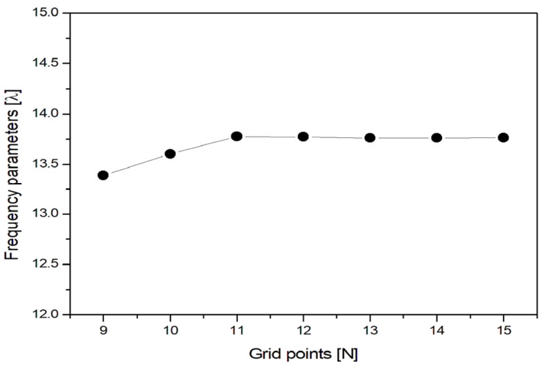


Fig. 2. Fundamental frequency parameter $\lambda=\omega(ma^4/EI)^{1/2}$ for free in-plane inextensional vibrations of the circular arch with both ends simply supported and a range of N; θ_0 =90^0 and $\delta=1\times10^{-6}$

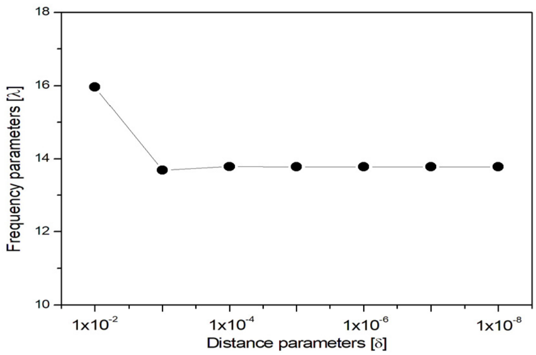


Fig. 3. Fundamental frequency parameter $\lambda=\omega(ma^4/EI)^{1/2}$ for free in-plane inextensional vibrations of the circular arch with both ends simply supported and a range of δ ; $\theta_0=90^0$ and N=11

Table 1. Fundamental frequency parameter $\lambda=\omega(ma^4/EI)^{1/2}$ for in-plane inextensional vibrations of the circular arch with simply supported ends

θ_0, Deg.	$\lambda=\omega(ma^4/EI)^{1/2}$				
	Archer[6] (Exact)	Galerkin	Rayleigh-Ritz	SAP IV FEM	DQM
30		228.1	222.3	222.3	222.3
60		55.22			53.73
90		23.29			22.62
120		12.22			11.84
150		7.19			6.95
180	4.38	4.53			4.38

Table 2. Fundamental frequency parameter $\lambda=\omega(ma^4/EI)^{1/2}$ for in-plane inextensional vibrations of the circular arch with clamped ends

θ_0	$\lambda=\omega(ma^4/EI)^{1/2}$				
	Nelson [7]	Galerkin	Rayleigh-Ritz	SAP IV FEM	DQM
30		141.5	141.53	141.53	141.5
60	33.63	33.72			33.62
90	13.76	13.76			13.76
120		6.928			6.927
150		3.860			3.859
180	2.267	2.268			2.267

Table 3. Fundamental frequency parameter $\lambda=\omega(ma^4/EI)^{1/2}$ for free in-plane inextensional vibrations of the circular arch with simply supported ends including effects of rotatory inertia

θ_0	s/r			
	30	50	100	300
30	138.71	140.47	141.27	141.51
60	32.999	33.397	33.569	33.621
90	13.543	13.683	13.744	13.762
120	6.8364	6.8939	6.9188	6.9261
150	3.8177	3.8430	3.8550	3.8583
180	2.2474	2.2599	2.2652	2.2667

Table 4. Fundamental frequency parameter $\lambda=\omega(ma^4/EI)^{1/2}$ for free in-plane inextensional vibrations of the circular arche with clamped ends including effects of rotatory inertia

θ_0	s/r			
	30	50	100	300
30	217.06	220.39	221.86	222.30
60	52.547	53.0	53.627	53.725
90	22.184	22.463	22.584	22.620
120	11.653	11.776	11.829	11.845
150	6.8644	6.9241	6.9497	6.9574
180	4.3361	4.3668	4.380	4.3839

Table 5. Fundamental frequency parameter $\lambda=\omega(ma^4/EI)^{1/2}$ for free in-plane extensional vibrations of the circular arch with simply supported ends

θ_0	s/r			
	30	50	100	300
30	62.23	92.40	141.5	141.5
60	26.75	33.55	33.61	33.62
90	13.61	13.71	13.75	13.76
120	6.835	6.895	6.919	6.926
150	3.086	3.840	3.854	3.858
180	2.240	2.257	2.264	2.266

Table 6. Fundamental frequency parameter $\lambda=\omega(ma^4/EI)^{1/2}$ for free in-plane extensional vibrations of the circular arch with clamped ends

θ_0	s/r			
	30	50	100	300
30	93.871	113.09	175.75	222.40
60	30.753	43.565	53.692	53.743
90	17.625	22.437	22.584	22.624
120	11.419	11.713	11.817	11.846
150	6.6836	6.8678	6.9374	6.9571
180	4.2121	4.3256	4.3706	4.3835

Table 7. Fundamental frequency parameter $\lambda=\omega(ma^4/EI)^{1/2}$ for free in-plane extensional vibrations of the circular arch with simply supported ends including effects of rotatory inertia

θ_0	s/r			
	30	50	100	300
30	61.896	92.224	141.25	141.51
60	26.619	33.329	33.554	33.621
90	13.406	13.635	13.732	13.761
120	6.7501	6.8629	6.9113	6.9257
150	3.7686	3.8260	3.8507	3.8580
180	2.2216	2.2505	2.2629	2.2666

Table 8. Fundamental frequency parameter $\lambda=\omega(ma^4/EI)^{1/2}$ for free in-plane extensional vibrations of the circular arch with clamped ends including effects of rotatory inertia

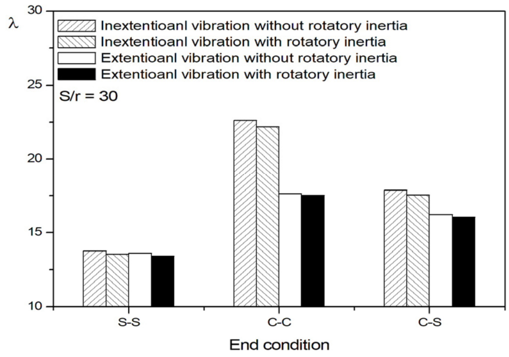
θ_0	s/r			
	30	50	100	300
30	93.239	112.82	175.63	222.34
60	30.548	43.450	53.580	53.730
90	17.509	22.282	22.543	22.619
120	11.262	11.646	11.799	11.844
150	6.6077	6.8354	6.9288	6.9561
180	4.1718	4.3081	4.3661	4.3830

Table 9. Fundamental frequency parameter $\lambda=\omega(ma^4/EI)^{1/2}$ for free in-plane extensional vibrations of the circular arch with clamped-simply supported ends including effects of rotatory inertia

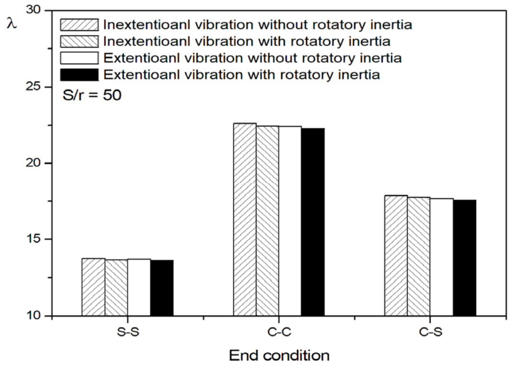
θ_0	s/r			
	30	50	100	300
30	73.583	98.367	166.05	178.81
60	27.454	54.601	42.748	42.927
90	16.037	17.565	17.804	17.865
120	8.7782	9.0693	9.1766	9.2068
150	5.0839	5.2239	5.2811	5.2975
180	3.1366	3.2122	3.2439	3.2530

Table 10. Frequency parameter $\lambda=\omega(ma^4/EI)^{1/2}$ for free in-plane extensional vibrations of the circular arch with simply supported ends including higher frequencies

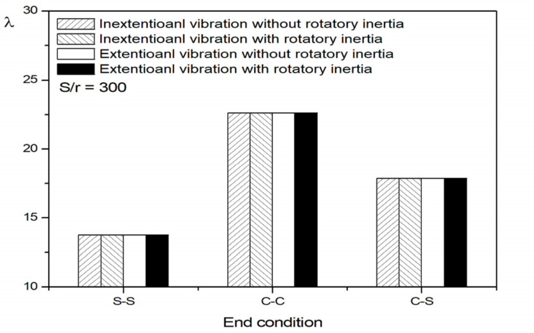
s/r	θ_0	n=1	n=2	n=3	n=4
11.78	90	18.08	71.53	89.78	148.6
17.28		25.25	83.19	113.8	215.1
23.56		33.32	81.49	153.9	226.0
47.12		33.82	144.9	171.5	351.4
117.8		33.94	151.9	345.6	414.4
251.3		33.96	152.2	349.5	652.7
377.0		33.96	152.3	349.8	627.0
7.85	180	18.26	39.18	74.73	-
15.71		21.36	66.53	133.2	167.5
47.12		22.27	133.6	205.8	339.9



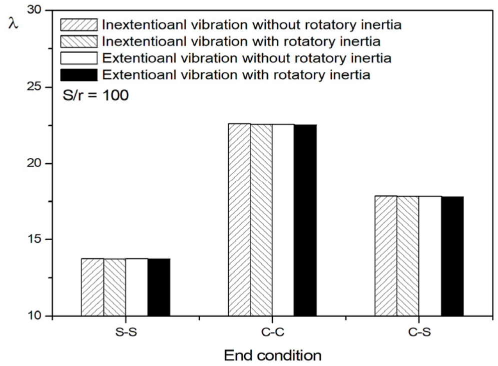
(a) S/r = 30



(b) S/r = 50



(c) S/r = 100



(d) S/r = 300

Fig. 4. Comparison of fundamental frequency parameter $\lambda = \omega(ma^4/EI)^{1/2}$ for free in-plane inextensional and extensional vibrations of the circular arch by DQM with $\theta_0 = 90^0$

5. Conclusions

The DQM was applied to calculate the eigenvalues of the differential equations for the free in-plane inextensional and extensional vibrations of the circular arch including the effects of rotatory inertia. The results including the effects of midsurface extension and rotatory inertia, previous not presented, are showed with the various boundary conditions, opening angles, or slenderness ratios. For some cases, the effects of midsurface extension and rotatory inertia can affect the frequencies of the circular arch significantly. Therefor, the research for the extensional vibrations of circular arches including the effects of rotatory inertia is import for the analysis of the circular arches.

The differential quadrature method also presents the solutions which agree excellent with the exact solutions and the numerical solutions by other methods and which require only a few number of grid points (thirteen points) for this study. The DQM may also be extended to arches of other profiles. However, the DQM can not be used for the complicated structures which have not the differential equations.

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〈Areas studied〉

Structural and Numerical Analysis, Buckling, Vibration