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DISCONNECTED POSETS AND LD-IRREDUCIBLE POSETS

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Abstract. Using ld-irreducible posets, we can easily characterize posets with respect to linear discrepancy. However, it is difficult to have the list of all the irreducible posets with respect to a given linear discrepancy. In this paper, we investigate some properties of disconnected posets and connected posets with respect to linear discrepancy, respectively and then we find various relationships between ld-irreducibily and connectedness. From these results, we suggest some methods to construct ld-irreducible posets.

1. Introduction

For a poset $\mathbf{P} = (X, \leq_{\mathbf{P}})$, the *linear discrepancy* of a poset $\mathbf{P}$, denoted by $\mathrm{ld}(\mathbf{P})$, is defined as

$$\mathrm{ld}(\mathbf{P}) = \min_{f\in F} \max_{x||y\in X} |f(x) - f(y)|,$$

where F is the set of all injective order preserving maps from X to integers, and $x||y$ denotes that x and y are incomparable in $\mathbf{P}$.

An *ℓ-ld-irreducible poset* is the poset whose linear discrepancy is ℓ and decreases by at least one if any element is removed from it [1], which gives a characterization method of posets with respect to linear discrepancy. Using ld-irreducibility, we characterize posets with respect to linear discrepancy. In 2006, this characterization method was firstly suggested by G.-B. Chae, M. Cheong, and S.-M. Kim [1], in which posets of linear discrepancy 1 was characterized by providing all the 1- and 2-ld-irreducible posets.

To characterize a poset with respect to the linear discrepancy ℓ, we need all the ℓ- and $(\ell + 1)$-ld-irreducible posets [4]. For example, to characterize posets of linear discrepancy 2, D. M. Howard, et al. [4, 5] give all the 2- and 3-ld-irreducible posets.

However, for the characterization of posets of higher discrepancy ℓ, it is very difficult to find all the ℓ- and $(\ell + 1)$-ld-irreducible posets, respectively. Hence,

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in [2], for reducing this difficulty, simple posets are also used with ld-irreducible posets for characterization of posets with respect to the linear discrepancy.

In this paper, we investigate properties of disconnected posets and connected posets with respect to linear discrepancy. Using the properties which we found, we suggest some methods to construct ld-irreducible posets with providing some proper examples.

2. Some definitions and properties

Let $\mathbf{P} = (X, \leq_{\mathbf{P}})$ be a poset, where X has n elements. If there is no possibility of confusion, we write $x \in \mathbf{P}$ instead of $x \in X$. For a partial order relation $\leq_{\mathbf{P}} \subseteq X \times X$ of $\mathbf{P}$, and $(x, y) \in \leq_{\mathbf{P}}$, we write this as $x \leq_{\mathbf{P}} y$ for convenience. If x and y are incomparable, i.e., $(x, y) \notin \leq_{\mathbf{P}}$ and $(y, x) \notin \leq_{\mathbf{P}}$, then we write it as $x \|_{\mathbf{P}} y$. If there is no confusion, we just write it as $x \| y$.

The *chain* of order n, denoted by $\mathbf{n} = (X, \leq_{\mathbf{n}})$ (simply, $\mathbf{n}$), is a poset such that $|X| = n$ and $x \leq_{\mathbf{n}} y$ or $y \leq_{\mathbf{n}} x$ for all $x, y \in X$. And the *antichain* of order n, denoted by $\mathbf{A}(n) = (X, \leq_{\mathbf{A}(n)})$ (simply, $\mathbf{A}(n)$), is a poset such that $|X| = n$ and $x \| y$ for all x, $y \in X$. The linear discrepancy of a chain is defined as 0, i.e., $\mathrm{ld}(\mathbf{n}) = 0$. And the linear discrepancy of an antichain $\mathbf{A}(n)$ is clearly $n - 1$.

The disjoint union of posets $\mathbf{U} = (X, \leq_{\mathbf{U}})$ and $\mathbf{V} = (Y, \leq_{\mathbf{V}})$ is the poset $\mathbf{U} + \mathbf{V} = (X \cup Y, \leq_{\mathbf{U}} \cup \leq_{\mathbf{V}})$. If a poset is a disjoint sum of two or more posets, then the poset is called *disconnected*. If a poset is not disconnected, we call it a *connected poset*.

In [6], Tanenbaum *et al.* dealt with a disjoint sum of chains which is a special disconnected poset. The linear discrepancy of the poset is given as follows.

Theorem 1 ([6]). *If P is a disjoint union $\mathbf{r_1} + \mathbf{r_2} + \cdots + \mathbf{r_t}$ of $t \geq 2$ chains with $r_1 \geq r_2 \geq \cdots \geq r_t$, then $\mathrm{ld}(\mathbf{P}) = \lceil \frac{r_1}{2} \rceil + r_2 + \cdots + r_t - 1$.*

A disjoint sum of chains is very specific. Cheong [3] determines the linear discrepancy of a disjoint sum of more general posets other than chains as follows.

Theorem 2 ([3]). *For two posets $\mathbf{Q}_1$ and $\mathbf{Q}_2$, let $\mathbf{P} = \mathbf{Q}_1 + \mathbf{Q}_2$, and let $l_i = \max\left\{ \left\lceil \frac{|\mathbf{Q}_i|}{2} \right\rceil + |\mathbf{P}| - |\mathbf{Q}_i| - 1, \mathrm{ld}(\mathbf{Q}_i) + |\mathbf{P}| - |\mathbf{Q}_i| \right\}$ for $i = 1$, 2. Then*

$$\mathrm{ld}(\mathbf{P}) = \min\{l_1, l_2\}.$$

From Theorem 2, we have the following useful inequality.

Lemma 3. *For two posets $\mathbf{Q}_1$ and $\mathbf{Q}_2$ with $|\mathbf{Q}_1| \geq |\mathbf{Q}_2|$, let $\mathbf{P} = \mathbf{Q}_1 + \mathbf{Q}_2$. Then*

$$\mathrm{ld}(\mathbf{P}) \geq \left\lceil \frac{|\mathbf{Q}_1|}{2} \right\rceil + |\mathbf{Q}_2| - 1.$$

Proof. Let $l_i = \max\left\{\left\lceil \frac{|\mathbf{Q}_i|}{2}\right\rceil + |\mathbf{P}| - |\mathbf{Q}_i| - 1, \mathrm{ld}(\mathbf{Q}_i) + |\mathbf{P}| - |\mathbf{Q}_i|\right\}$ for $i = 1$, 2. If $\mathrm{ld}(\mathbf{P}) = l_1$, then $\mathrm{ld}(\mathbf{P}) \geq \left\lceil \frac{|\mathbf{Q}_1|}{2}\right\rceil + |\mathbf{Q}_2| - 1$. If $\mathrm{ld}(\mathbf{P}) = l_2$, then $\mathrm{ld}(\mathbf{P}) \geq \left\lceil \frac{|\mathbf{Q}_2|}{2}\right\rceil + |\mathbf{Q}_1| - 1 \geq \left\lceil \frac{|\mathbf{Q}_1|}{2}\right\rceil + |\mathbf{Q}_2| - 1$. Therefore, the lemma holds. □

3. Investigation on posets with respect to irreducibility and connectedness

In this section, using Theorems 1 and 2, and Lemma 3, we investigate some relation between ld-irreducibility and connectedness, and we find a way to construct new irreducible posets from a given irreducible poset.

Lemma 4. *Let* $\mathbf{P}$ *be a disconnected poset with* $|\mathbf{P}| = n$*. Then we have*

$$\mathrm{ld}(\mathbf{P}) \geq \left\lceil \frac{n-1}{2} \right\rceil.$$

Proof. Since $\mathbf{P}$ is disconnected, there are two subposets $\mathbf{Q}_1$ and $\mathbf{Q}_2$ of $\mathbf{P}$ such that $\mathbf{Q}_1 + \mathbf{Q}_2 = \mathbf{P}$ so that $n = |\mathbf{P}| = |\mathbf{Q}_1| + |\mathbf{Q}_2|$. We may assume that $|\mathbf{Q}_1| \geq |\mathbf{Q}_2|$. Then

$$\mathrm{ld}(\mathbf{P}) \geq \frac{|\mathbf{Q}_1|}{2} + |\mathbf{Q}_2| - 1 = \frac{|\mathbf{Q}_1|}{2} + (n - |\mathbf{Q}_1|) - 1 = (n-1) - \frac{|\mathbf{Q}_1|}{2}$$

from Lemma 3. Since $|\mathbf{Q}_1| \geq |\mathbf{Q}_2|$, we have $\frac{n}{2} \leq |\mathbf{Q}_1| \leq n-1$. Therefore, $\mathrm{ld}(\mathbf{P}) \geq \frac{n-1}{2}$. □

For a poset $\mathbf{P}$, let $I(x)$ be a set of incomparable elements to x in $\mathbf{P}$. Then, $I(x)$ is a subposet of $\mathbf{P}$, and it is clear that $|I(x)| \leq 2\,\mathrm{ld}(\mathbf{P})$ for any $x \in \mathbf{P}$. For an ld-irreducible poset $\mathbf{P}$, we have the following useful lemma.

Lemma 5. *Let* $\mathbf{P}$ *be an* ℓ*-ld-irreducible poset. Then, for all* $x \in \mathbf{P}$*, we have* $|I(x)| \leq 2\ell - 1$*.*

Proof. Since $\mathrm{ld}(\mathbf{P}) = \ell$, we have $|I(x)| \leq 2\ell$ for all $x \in \mathbf{P}$. If $|I(x)| < 2\ell - 1$ for all $x \in \mathbf{P}$, then the result holds.

Assume that there is $x \in \mathbf{P}$ such that $|I(x)| = 2\ell$. If there is no $z \notin I(x)$ with $x \neq z$, then $\mathbf{P} = \{x\} + I(x)$ and $\mathbf{P}$ is not ld-irreducible since $\mathrm{ld}(\mathbf{P} \setminus \{y\}) = \ell$ for $y \in I(x)$. Hence, there is $z \in \mathbf{P}$ such that $z \notin I(x)$. Then $I(x)$ is a subposet of $\mathbf{P} \setminus \{z\}$.

Note that $\mathbf{P}$ is ld-irreducible and $\mathrm{ld}(\mathbf{P}) = \ell$. So, we have $\mathrm{ld}(\mathbf{P} \setminus \{z\}) < \ell$. It is known that the linear discrepancy of a poset is greater than or equal to $\frac{1}{2}|I(x)|$ for all x in the poset. Hence, we have

$$\ell > \mathrm{ld}(\mathbf{P} \setminus \{z\}) \geq \frac{1}{2}|I(x)| = \ell.$$

This is a contradiction. Therefore, $|I(x)| \leq 2l - 1$ so that the lemma holds. □

For a connected ld-irreducible poset $\mathbf{P}$ and $x \in \mathbf{P}$, we have another upper bound of $|I(x)|$ as follows.

Lemma 6. *Let* $\mathbf{P}$ *be a connected* l*-ld-irreducible poset. Then, we have* $|I(x)| \leq 2l-2$ *for every* $x \in \mathbf{P}$.

Proof. Suppose that there is $x_0 \in \mathbf{P}$ such that $|I(x_0)| = 2l-1$. Since $\mathbf{P}$ is connected, there is $y \in \mathbf{P}$ with $y \neq x_0$ and $y \notin I(x_0)$. Since $\mathbf{P}$ is l-ld-irreducible, we have $\mathrm{ld}(\mathbf{P} \setminus \{y\}) \leq l-1$. However, we have

$$l = \mathrm{ld}(\{x_0\} + I(x_0)) \leq \mathrm{ld}(\mathbf{P} \setminus \{y\}) \leq l-1$$

since $\{x_0\} + I(x_0)$ is a subposet of $\mathbf{P} \setminus \{y\}$. This is a contradiction. Therefore, we have $|I(x)| \leq 2l-2$ for all $x \in \mathbf{P}$. $\square$

4. How to make ld-irreducible posets

If an ld-irreducible poset $\mathbf{P}$ is given, we make another ld-irreducible poset, as follows.

Lemma 7. *For a positive integer* t*, let* $\mathbf{P}$ *be an ld-irreducible poset with* $|\mathbf{P}| = 2t-1$. *Then* $\mathbf{1} + \mathbf{P}$ *is also an ld-irreducible poset.*

Proof. Let $\mathbf{R} = \mathbf{1} + \mathbf{P}$. Then we have

$$\mathrm{ld}(\mathbf{R}) = \max\left\{\mathrm{ld}(\mathbf{P}) + 1, \left\lceil \frac{2t-1}{2} \right\rceil\right\}$$

from Theorem 2. Hence, we have $\mathrm{ld}(\mathbf{R}) \geq \mathrm{ld}(\mathbf{P}) + 1$.

In order to check the irreducibility of $\mathbf{R}$, we consider $\mathrm{ld}(\mathbf{R} \setminus \{x\})$ for any $x \in \mathbf{R}$. If $x \in \mathbf{1}$ of $\mathbf{R}$, then $\mathbf{R} \setminus \{x\} = \mathbf{P}$ so that we easily obtain $\mathrm{ld}(\mathbf{R} \setminus \{x\}) = \mathrm{ld}(\mathbf{P}) \leq \mathrm{ld}(\mathbf{R}) - 1 < \mathrm{ld}(\mathbf{R})$.

If $x \in \mathbf{P}$, then we have

$$\begin{aligned} \mathrm{ld}(\mathbf{R} \setminus \{x\}) &= \max\left\{\mathrm{ld}(\mathbf{P} \setminus \{x\}) + 1, \left\lceil \frac{2t-2}{2} \right\rceil\right\} \\ &\leq \max\{\mathrm{ld}(\mathbf{P}), t-1\} \qquad (1) \end{aligned}$$

since $\mathbf{P}$ is ld-irreducible. Suppose that $\mathrm{ld}(\mathbf{P}) \geq t-1$. Then, from (1),

$$\mathrm{ld}(\mathbf{R} \setminus \{x\}) \leq \mathrm{ld}(\mathbf{P}) < \mathrm{ld}(\mathbf{P}) + 1 \leq \mathrm{ld}(\mathbf{R}).$$

Suppose that $\mathrm{ld}(\mathbf{P}) < t-1$. Then we have

$$\mathrm{ld}(\mathbf{R}) = \max\{\mathrm{ld}(\mathbf{P}) + 1, t\} = t.$$

Since P is ld-irreducible, we have $\mathrm{ld}(\mathbf{P} \setminus \{x\}) + 1 \leq \mathrm{ld}(\mathbf{P}) < t-1$ so that

$$\mathrm{ld}(\mathbf{R} \setminus \{x\}) = \max\{\mathrm{ld}(\mathbf{P} \setminus \{x\}) + 1, t-1\} = t-1 < \mathrm{ld}(\mathbf{R}).$$

Therefore, for all $x \in \mathbf{R}$, we have $\mathrm{ld}(\mathbf{R} \setminus \{x\}) + 1 \leq \mathrm{ld}(\mathbf{R})$ so that $\mathbf{R}$ is ld-irreducible. $\square$

For a poset $\mathbf{P}$ with even cardinality, Lemma 7 could not hold. In fact, $\mathbf{1} + moth(10)$ is not ld-irreducible in spite of the irreducibility of $moth(10)$ (see Figure 1). In order that $\mathbf{1} + \mathbf{P}$ is ld-irreducible for $|\mathbf{P}| = 2t$ and a positive integer t, we needs more conditions than those of Lemma 7 as follows.

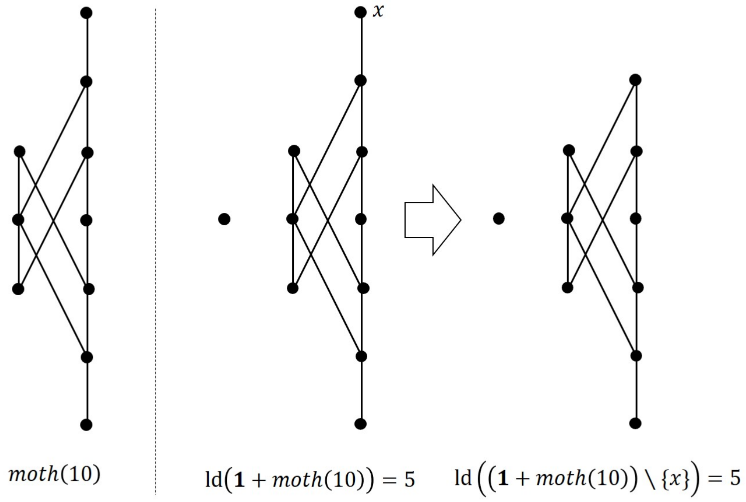

FIGURE 1. $moth(10)$ is 3-ld-irreducible, however, $\mathbf{1}+moth(10)$ is not ld-irreducible.

Lemma 8. *For a positive integer t, let $\mathbf{P}$ be an ld-irreducible poset with $|\mathbf{P}| = 2t$, and $\mathrm{ld}(\mathbf{P}) \geq t$. Then $\mathbf{1}+\mathbf{P}$ is also an ld-irreducible poset.*

Proof. This follows from the proof of Lemma 7 by noting that $\mathrm{ld}(\mathbf{1}+\mathbf{P}) = \mathrm{ld}(\mathbf{P})+1$ since $\mathrm{ld}(\mathbf{P}) \geq t$. $\square$

For a poset $\mathbf{P}$ with $\mathrm{ld}(\mathbf{P}) \geq \left\lceil \frac{|\mathbf{P}|}{2} \right\rceil$, we have $\mathrm{ld}(\mathbf{1}+\mathbf{P}) = \mathrm{ld}(\mathbf{P})+1$, and $\mathrm{ld}(\mathbf{1}+\mathbf{P}) \geq \left\lceil \frac{1+|\mathbf{P}|}{2} \right\rceil$. From this property, Lemmas 7 and 8, we easily obtain the following theorem.

Theorem 9. *For a positive integer w, and an ld-irreducible poset $\mathbf{P}$ with $\mathrm{ld}(\mathbf{P}) \geq \left\lceil \frac{|\mathbf{P}|}{2} \right\rceil$, it is true that $\mathbf{A}(w)+\mathbf{P}$ is ld-irreducible.*

With the following theorem, we can also construct an ld-irreducible poset from an ordinary poset.

Theorem 10. *For a positive integer t with $t \geq 2$, let $\mathbf{P}$ be a poset with $|\mathbf{P}| = 2t-1$ and $\mathrm{ld}(\mathbf{P}) \leq t-2$. Then $\mathbf{1}+\mathbf{P}$ is t-ld-irreducible.*

Proof. Since $\mathrm{ld}(P) \leq t-2$, we have

$$\mathrm{ld}(\mathbf{1}+\mathbf{P}) = \max\left\{\mathrm{ld}(P)+1, \left\lceil \frac{2t-1}{2} \right\rceil\right\} = t.$$

Hence, $\mathrm{ld}(\mathbf{1}+\mathbf{P}) = t > t-2 = \mathrm{ld}(\mathbf{P})$.

Let x be an element in $\mathbf{P}$. Then

$$\mathrm{ld}(\mathbf{P} \setminus \{x\} + \mathbf{1}) = \max\left\{\mathrm{ld}(\mathbf{P} \setminus \{x\}) + 1, \left\lceil \frac{2t-2}{2} \right\rceil\right\}.$$

Since $\mathrm{ld}(\mathbf{P} \setminus \{x\}) \leq \mathrm{ld}(\mathbf{P}) \leq t-2$, we have $\mathrm{ld}(\mathbf{P} \setminus \{x\} + \mathbf{1}) = t-1$.

Therefore, $\mathbf{1} + \mathbf{P}$ is t-ld-irreducible. $\square$

For a disconnected poset $\mathbf{P}$, we can obtain the following lemma.

Lemma 11. *Let* $\mathbf{P}$ *be a disconnected ld-irreducible poset with* $|\mathbf{P}| = n \geq 2$. *Then* $\mathrm{ld}(\mathbf{P}) \geq \lceil \frac{n}{2} \rceil$.

Proof. Since $\mathbf{P}$ is disconnected, we have $\mathrm{ld}(\mathbf{P}) \geq \lceil \frac{n-1}{2} \rceil$ from Lemma 4. If n is even, then $\lceil \frac{n}{2} \rceil = \lceil \frac{n-1}{2} \rceil$. Hence, the result holds. Otherwise, i.e., suppose that n is odd. Since $\mathbf{P}$ is disconnected, it holds that $\mathbf{P} \setminus \{x\}$ is also disconnected for some $x \in \mathbf{P}$. Then, from Lemma 4, we have $\mathrm{ld}(\mathbf{P} \setminus \{x\}) \geq \frac{n-2}{2}$. Since $\mathbf{P}$ is ld-irreducible, we have $\mathrm{ld}(\mathbf{P}) \geq \mathrm{ld}(\mathbf{P} \setminus \{x\}) + 1$. Hence, we have

$$\mathrm{ld}(\mathbf{P}) \geq \mathrm{ld}(\mathbf{P} \setminus \{x\}) + 1 \geq \frac{n-2}{2} + 1 = \frac{n}{2}.$$

Therefore, the result holds. $\square$

From a connected ld-irreducible poset $\mathbf{P}$, we can construct an ld-irreducible poset whose linear discrepancy is higher than that of $\mathbf{P}$. The following theorem shows that it is possible.

Theorem 12. *For a positive integer* t, *let* $\mathbf{P}$ *be an ld-irreducible and connected poset with* $|\mathbf{P}| = 2t$ *and* $\mathrm{ld}(\mathbf{P}) < t$. *For* $x \in \mathbf{P}$, *let* $\mathbf{U} = \mathbf{P} \setminus \{x\} + \{x\}$. *Then* $\mathbf{U}$ *is* t*-ld-irreducible.*

Proof. For any $x \in \mathbf{P}$, let $\mathbf{U} = \mathbf{P} \setminus \{x\} + \{x\}$. Then $\mathbf{P}$ is an extension of $\mathbf{U}$. Note that $\mathrm{ld}(\mathbf{P} \setminus \{x\}) \leq \mathrm{ld}(\mathbf{P}) - 1 < t - 1$ since $\mathbf{P}$ is ld-irreducible and $\mathrm{ld}(\mathbf{P}) < t$. From Theorem 2, we have

$$\begin{aligned}\mathrm{ld}(\mathbf{P} \setminus \{x\} + \{x\}) &= \max\left\{\left\lceil \frac{2t-1}{2} \right\rceil, \mathrm{ld}(\mathbf{P} \setminus \{x\}) + 1\right\} \\ &= t.\end{aligned}$$

Take any $z \in \mathbf{P}$. If $z = x$, then $\mathrm{ld}(\mathbf{U} \setminus \{z\}) = \mathrm{ld}(\mathbf{P} \setminus \{x\}) \leq \mathrm{ld}(\mathbf{P}) - 1 < t - 1$. Suppose that $z \neq x$. Then $\mathbf{U} \setminus \{z\} = \mathbf{P} \setminus \{x, z\} + \{x\}$ so that

$$\begin{aligned}\mathrm{ld}(\mathbf{U} \setminus \{z\}) &= \max\left\{\left\lceil \frac{2t-2}{2} \right\rceil, \mathrm{ld}(\mathbf{P} \setminus \{x, z\}) + 1\right\} \\ &= t - 1\end{aligned}$$

since $\mathrm{ld}(\mathbf{P} \setminus \{x, z\}) \leq \mathrm{ld}(\mathbf{P} \setminus \{x\}) < t - 1$. Thus, $\mathrm{ld}(\mathbf{U}) = t$, and $\mathrm{ld}(\mathbf{U} \setminus \{z\}) \leq \mathrm{ld}(\mathbf{U}) - 1$ for $z \in X$. Therefore, $\mathbf{U}$ is t-ld-irreducible. $\square$

Example 13. Figure 2(a) is called a ladder, which is a 3-ld-irreducible connected poset from [4]. We can produce 4-ld-irreducible posets from the ladder, as seen in Figure 2. In fact, Figures 2(b), 2(c), and 2(d) are 4-ld-irreducible.

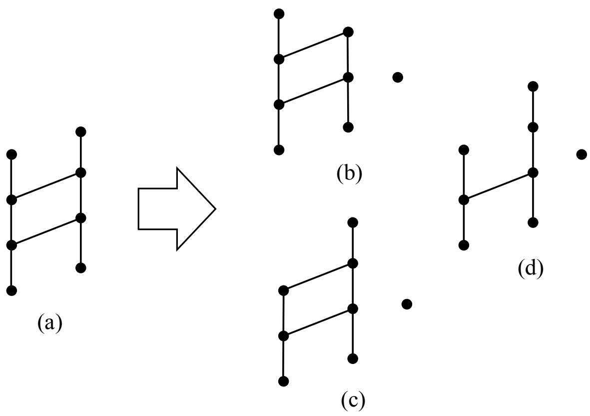

FIGURE 2. Making ld-irreducible posets from a ladder

TABLE 1. The conditions and methods for constructing an ld-irreducible poset from a given poset.

Conditions				Method
$\|P\|$	ld($\mathbf{P}$)	Connectness	Irreducibility	
$2t-1$	-	-	◯	$\mathbf{1}+\mathbf{P}$
$2t-1$	$< t-2$	-	-	$\mathbf{1}+\mathbf{P}$
$2t$	$\geq t$	-	◯	$\mathbf{1}+\mathbf{P}$
n	$\geq \lceil \frac{n}{2} \rceil$	-	◯	$\mathbf{A}(w)+\mathbf{P}$
$2t$	$< t$	◯	◯	$\mathbf{P}\setminus\{x\}+\{x\}$

• t, n, $w \in \mathbb{N}$, and $x \in \mathbf{P}$.

5. Conclusion

An ld-irreducible poset plays an important role to determine the linear discrepancy of a poset and to characterize a poset with respect to the given linear discrepancy. Hence, if we can collect various ld-irreducible posets, then we can characterize more posets and determine the ld of more posets. However, finding ld-irreducible posets is very difficult. In this paper, we suggest some methods to construct an ld-irreducible poset from a given irreducible poset and an ordinary poset. We summarize these as Table 1. In addition, we give a property for the linear discrepancy of a disconnected poset. From these results, we can suggest lower bounds for more posets.

Contract with disconnected posets, we guess that connected ld-irreducible posets has the following property.

Conjecture 14. *Let* $\mathbf{P}$ *be a connected and ld-irreduble poset. Then, for* $x \in \mathbf{P}$*, we have*

$$\mathrm{ld}(\mathbf{P}) = \mathrm{ld}(\mathbf{P} \setminus \{x\}) + 1.$$

For example, the linear discrepancies of both $moth(10)$ in Figure 1 and the ladder in Figure 2 are 3, and we can easily check the fact that the linear discrepancies of posets obtained by removing any one element from $moth(10)$ and the ladder are always 2. This is a different property from that of disconnected posets.

However, we expect that our results in this paper could help to prove Conjecture 14 is true.

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